# Sliding Mode Observer-based Sensor and Actuator Fault Reconstruction for Nonlinear System

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**Abstract.** This paper presents a fault reconstruction scheme based on sliding mode observers for sensor and actuator faults detection and isolation for a class of uncertain nonlinear systems. A coordinate transformation is first designed for the output equation of the system followed by a first-order low-pass filter in order to convert the sensor faults into equivalent actuator faults. The original system then is transformed into three subsystems through linear transformation. The simulation examples verify the effectiveness of such method.

## 1. Introduction

System faults and unknown input disturbances are inevitable during operation of a complicated power system. With regard to physical position, faults can be classified into actuator and sensor faults. Faults can damage the normal system operation and make the system unstable, therefore, the fault detection and isolation (FDI) technique plays an important role in system operation. In the past decades, the study on FDI made great progress, especially for the model-based fault detection [1–3]. Various approaches have been proposed to solve FDI problems, such as differential geometry method, self-adaptive control method, and sliding mode observer technique. Moreover, research on fault tolerance control and stability analysis considerably promoted the FDI [4–5].

Research on faults reconstruction method has become an important field that mainly includes actuator fault reconstruction [6], sensor fault reconstruction, and fault reconstruction considering unknown input disturbances. Different from the actuator, the sensor is a passive element that merely provides the information of the operating system and does not directly affect system behavior.

## 2. Problem description

A nonlinear system with actuator and sensor faults is given by

$$\begin{cases} \dot{x}(t) = Ax + \Phi(x,u) + Ef_a(t) + Dd(t) + Bu(t) \\ y(t) = Cx(t) + F_s f_s(t) \end{cases} \tag{1}$$

where $x \in R^n$ , $u \in R^r$ and $y \in R^p$ denote, respectively, the state variables, inputs and outputs. The nonlinear continuous term $\Phi(x,u) \in R^n$ is assumed to be known. The unknown nonlinear term $d(t) \in R^q$ models the lumped uncertainties and disturbances experienced by the system, which is assumed to be bounded, i.e., a positive constant $\gamma_1$ exists such that $\|d(t)\| \le \gamma_1$ .The unknown nonlinear terms $f_a(t) \in R^q$ and $f_s(t) \in R^m$ denote, respectively, actuator faults and sensor faults, which are also bounded, i.e., two constant $\gamma_2$ and $\gamma_3$ exist such that $\|f_a(t)\| \le \gamma_2$ and $\|f_s(t)\| \le \gamma_3$ . $A \in R^{n\times n}$ , $B \in R^{n\times r}$ , $C \in R^{p\times n}$ , $D \in R^{n\times q}$ , $E \in R^{n\times q}$ and $F_s \in R^{p\times m}$ are known constant matrices with $n > p > q + m$ .

***Assumption* 1.** $D$ is a column full rank matrix and $rank(CD)=rank(D)$.

***Remark* 1.** If the disturbance distribution matrix $D$ is not a column full rank matrix, for example, $rank(D)=q_1<q$, then a rank decomposition $D=D_1'D_2'$ could be applied, where $D_1'$ is a column full

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rank matrix and $d_1'$(t)=$D_2'd_2'$(t) could be considered as a new unknown disturbance.

***Assumption* 2.** The matrix pair ($A$,$C$) is observable.

The matrix obtain the following equation:

$$\dot{x}(t)=\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix}=\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+\begin{bmatrix}\Phi_1(x,u)\\ \Phi_2(x,u)\end{bmatrix}+\begin{bmatrix}E_1\\ E_2\end{bmatrix}f(t)+\begin{bmatrix}D_1\\ D_2\end{bmatrix}d(t)+\begin{bmatrix}B_1\\ B_2\end{bmatrix}u(t) \quad (2)$$

where $x_1(t)\in R^{n-q}$, $x_2(t)\in R^q$, $A_{11}\in R^{(n-q)\times(n-q)}$, $A_{12}\in R^{(n-q)\times q}$, $A_{21}\in R^{q\times(n-q)}$, $A_{22}\in R^{q\times q}$, $\Phi_1(x,u)\in R^{n-q}$, $\Phi_2(x,u)\in R^q$, $E_1\in R^{(n-q)\times q}$, $E_2\in R^{q\times q}$, $D_1\in R^{(n-q)\times q}$, and $D_2\in R^{q\times q}$ is a nonsingular matrix.

Based on Assumption 1, two transform matrices, namely, $T$ and $S$, such that

$$x(t)=T^{-1}z(t)=T^{-1}\begin{bmatrix}z_1(t)\\ z_2(t)\end{bmatrix}\qquad y_1(t)=S^{-1}\begin{bmatrix}v_1(t)\\ v_2(t)\end{bmatrix}$$

Therefore, Eq.(2) could be converted as follows:

$$\begin{cases}\dot{z}(t)=\begin{bmatrix}\dot{z}_1(t)\\ \dot{z}_2(t)\end{bmatrix}=TAT^{-1}z(t)+T\Phi(z,u)+TEf_a(t)+TDd(t)+TBu(t)\\ v(t)=\begin{bmatrix}v_1(t)\\ v_2(t)\end{bmatrix}=SC_1T^{-1}z(t)\end{cases}$$

where $SCT_1^{-1}=\begin{bmatrix}C_{11} & 0\\ 0 & C_{22}\end{bmatrix}$ and $C_{22}$ is a nonsingular matrix.

The following nonsingular transformation matrix $T$ is then constructed :

$$T=\begin{bmatrix}I_{n-q} & -D_1D_2^{-1}\\ 0 & I_q\end{bmatrix}$$

Therefore, the coefficient matrices in (10) are as follows:

$$TAT^{-1}=\begin{bmatrix}\bar{A}_{11} & \bar{A}_{12}\\ \bar{A}_{21} & \bar{A}_{22}\end{bmatrix},\quad TD=\begin{bmatrix}0\\ \bar{D}_2\end{bmatrix},\quad TB=\begin{bmatrix}\bar{B}_1\\ \bar{B}_2\end{bmatrix}$$

$$TE=\begin{bmatrix}\bar{E}_1\\ \bar{E}_2\end{bmatrix},\quad T\Phi(x,u)=\begin{bmatrix}\bar{\Phi}_1(x,u)\\ \bar{\Phi}_2(x,u)\end{bmatrix}$$

where $\bar{A}_{11}=A_{11}-D_1D_2^{-1}A_{12}$, $\bar{A}_{12}=(A_{11}-D_1D_2^{-1}A_{21})D_1D_2^{-1}+A_{12}-D_1D_2^{-1}A_{22}$, $\bar{A}_{21}=A_{21}$, $\bar{A}_{22}=A_{21}D_1D_2^{-1}+A_{22}$, $\bar{\Phi}_1(x,u)=\Phi_1(x,u)-D_1D_2^{-1}\Phi_2(x,u)$, $\bar{\Phi}_2(x,u)=\Phi_2(x,u)$, $\bar{E}_1=E_1-D_1D_2^{-1}E_2$, $\bar{E}_2=E_2$, $\bar{B}_1=B_1-D_1D_2^{-1}B_2$, $\bar{B}_2=B_2$, $z_1(t)\in R^{n-q}$, $z_2(t)\in R^q$, $\bar{A}_{11}\in R^{(n-q)\times(n-q)}$, $\bar{A}_{12}\in R^{(n-q)\times q}$, $\bar{A}_{21}\in R^{q\times(n-q)}$, $\bar{A}_{22}\in R^{q\times q}$, $\bar{E}_1\in R^{(n-q)\times q}$, $\bar{E}_2\in R^{q\times q}$, $\bar{\Phi}_1(x,u)\in R^{n-q}$, $\bar{\Phi}_2(x,u)\in R^q$, $\bar{D}_2\in R^{q\times q}$, $C_{11}\in R^{(p-q-m)\times(n-q)}$, $C_{22}\in R^{q\times q}$, $v_1\in R^{p-q-m}$ and $v_2\in R^q$.

System (2) is then transformed into the following subsystems:

$$\begin{cases}\dot{z}_1(t)=\bar{A}_{11}z_1(t)+\bar{A}_{12}z_2(t)+\bar{\Phi}_1(z,u)+\bar{E}_1f_a(t)+\bar{B}_1u(t)\\ v_1(t)=C_{11}z_1(t)\end{cases}\quad (3)$$

$$\begin{cases}\dot{z}_2(t)=\bar{A}_{21}z_1(t)+\bar{A}_{22}z_2(t)+\bar{\Phi}_2(z,u)+\bar{E}_2f_a(t)+\bar{D}_2d(t)+\bar{B}_2u(t)\\ v_2(t)=C_{22}z_2(t)\end{cases}\quad (4)$$

System (3) is converted as follows:

$$\begin{cases}\dot{z}_a(t)=A_sz_a(t)+B_sC_2T^{-1}z(t)+B_sFf_s(t)\\ y_a=z_a(t)\end{cases}\quad (5)$$

where $z=(z_1 \quad z_2)^T$, $B_s\in R^{m\times m}$, $C_2\in R^{m\times n}$, and $T^{-1}\in R^{n\times n}$, therefore $B_sC_2T^{-1}\in R^{m\times n}$. Afterward, we assume that $B_sC_2T^{-1}=(A_1 \quad A_2)$, where $A_1\in R^{m\times(n-q)}$ and $A_2\in R^{m\times q}$. System (5) could be converted as follows:

$$\begin{cases}\dot{z}_a(t)=A_sz_a(t)+A_1z_1(t)+A_2z_2(t)+B_sFf_s(t)\\ y_a=z_a(t)\end{cases} \qquad (6)$$

The details will be presented in the following sections.

## 3. Simulation example

Consider the following 6-order nonlinear system with multi-fault and multi-disturbance:

$$\begin{cases}\dot{x}(t)=Ax+\Phi(x,u)+Ef_a(t)+Dd(t)+Bu(t)\\ y(t)=Cx(t)+F_sf_s(t)\end{cases} \qquad (7)$$

where

$$x=\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\\x_6\end{bmatrix},\quad A=\begin{bmatrix}0&1&0&0&0&0\\-0.15&-0.05&1.5&0&-1&-0.03\\0.4&-1&-6&0&0.4&-1.6\\0.5&0&0&-4&0&0\\0&0&0&0&-20&0\\0&0&0&0&0&-25\end{bmatrix},\quad \Phi(x,u)=\begin{bmatrix}\sin x_2\\0\\x_1+\sin x_6\\0\\0\\0\end{bmatrix},$$

$$E=\begin{bmatrix}1&0\\0&1.5\\6&0\\0&2\\2&0\\0&1\end{bmatrix},\quad D=\begin{bmatrix}0&0\\0&0\\3&0\\0&4\\1&0\\0&2\end{bmatrix},\quad B=\begin{bmatrix}0\\1\\0\\0\\0\\1\end{bmatrix},\quad C=\begin{bmatrix}2&0&3&0&0&1\\0&1&0&2&-1&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\1&0&1&0&0&1\\0&1&0&1&1&0\end{bmatrix},\quad F_s=\begin{bmatrix}3&0\\0&2\\0&0\\0&0\\1&0\\0&1\end{bmatrix},$$

$d(t)=\begin{bmatrix}d_1(t) & d_2(t)\end{bmatrix}^T$, $f_a(t)=\begin{bmatrix}f_{a1}(t) & f_{a2}(t)\end{bmatrix}^T$, and $f_s(t)=\begin{bmatrix}f_{s1}(t) & f_{s2}(t)\end{bmatrix}^T$.

The transformational matrix T

$$T=\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&-3&0\\0&0&0&1&0&-2\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix}$$

then

$$TE=\begin{bmatrix}1&0\\0&1.5\\0&0\\0&0\\2&0\\0&1\end{bmatrix},\quad TD=\begin{bmatrix}0&0\\0&0\\0&0\\0&0\\1&0\\0&2\end{bmatrix},\quad TB=\begin{bmatrix}0\\1\\0\\-2\\0\\1\end{bmatrix}$$

Transformation Matrices S0 and S are designed as follows:

$$S_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & -3 & 0 \\ 0 & 1 & 0 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad S = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Design the matrices as $P_1 = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$, $P_2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, $F_1 = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix}$, $F_2 = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$, $A_s = \begin{bmatrix} -4 & 0 \\ 0 & -1 \end{bmatrix}$, $B_s = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, $L_1 = \begin{bmatrix} -1 & -3 \\ 0 & 2 \\ 5 & 1 \\ 20 & 10 \end{bmatrix}$ and $L_2 = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$.

The reconstruction arithmetic of actuator fault fa(t) could be obtained

$$\hat{f}_{a1}(t) = \rho_{11} \frac{2e_1}{|2e_1| + \sigma_1}$$

$$\hat{f}_{a2}(t) = \rho_{12} \frac{3e_2}{|3e_2| + \sigma_1}$$

From (53), the reconstruction arithmetic of sensor fault fs(t) could be obtained

$$\hat{f}_{s1}(t) = 0.5\rho_{31} \frac{e_{a1}}{|e_{a1}| + \sigma_3}$$

$$\hat{f}_{s2}(t) = 0.5\rho_{32} \frac{e_{a2}}{|e_{a2}| + \sigma_3}$$

From (55), the estimation arithmetic of unknown input disturbance could be obtained

$$\hat{d}_1(t) = 2(\rho_{21} \frac{4e_5}{|4e_5| + \sigma_2} - \rho_{11} \frac{2e_1}{|2e_1| + \sigma_1})$$

$$\hat{d}_2(t) = 0.5(\rho_{22} \frac{2e_6}{|2e_6| + \sigma_2} - \rho_{12} \frac{3e_2}{|3e_2| + \sigma_1})$$

The observed parameters are as follows: $\rho_1 = \begin{bmatrix} \rho_{11} & 0 \\ 0 & \rho_{12} \end{bmatrix} = \begin{bmatrix} 20 & 0 \\ 0 & 40 \end{bmatrix}$, $\rho_2 = \begin{bmatrix} \rho_{21} & 0 \\ 0 & \rho_{22} \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 0 & 60 \end{bmatrix}$, $\rho_3 = \begin{bmatrix} \rho_{31} & 0 \\ 0 & \rho_{32} \end{bmatrix} = \begin{bmatrix} 40 & 0 \\ 0 & 30 \end{bmatrix}$, $\sigma_1 = \sigma_2 = \sigma_3 = 0.01$.

Two overlapped sine signals were used to simulate the incipient fault for actuator fault $f_a(t)$, where $f_{a1} = 2\sin 40t + 2\sin 5t$, $f_{a2}(t) = 8\sin(20t)$ .The sensor fault $f_{s1}(t) = 5\sin(40t)$, $f_{s2}(t)$ is simulated by a white noise with sample period of 0.02 s, which is the combination of abrupt fault and intermittent fault. The unknown input disturbances $d_1(t) = 4\sin 15t$ and $d_2(t) = 6\sin(20t)$. The initial value of state variable x in the simulation example is −2. The simulation results are as follows:

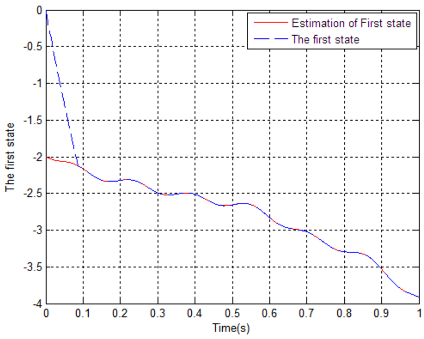


Fig. 1. The first state $x_1$ and its estimation $\hat{x}_1$

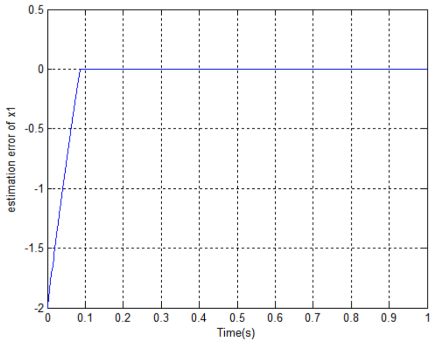


Fig. 2. The estimation error $e_1$

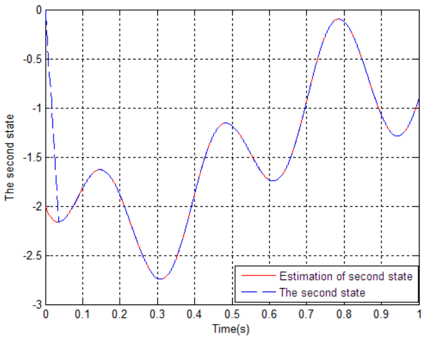


Fig.3. The second state x2 and its estimation $\hat{x}$ 2

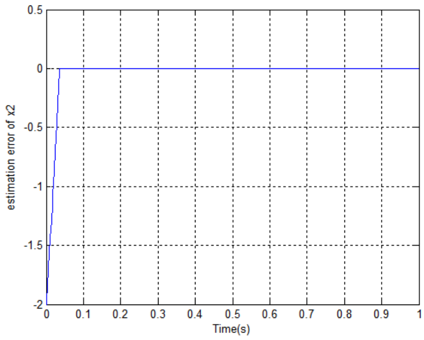


Fig. 4. The estimation error e2

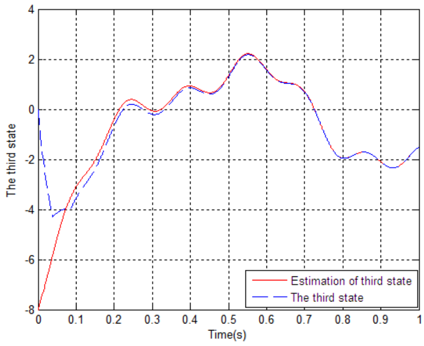


Fig. 5. The third state x3 and its estimation $\hat{x}$ 3

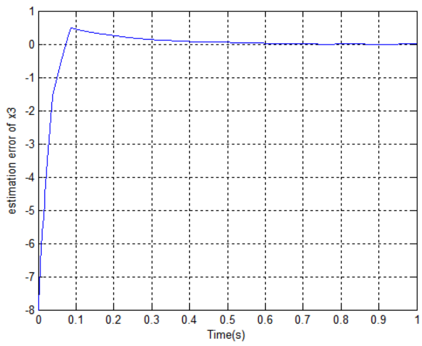


Fig. 6. The estimation error e3

Figures 1 to 6 show the estimation results of the six state vectors and the corresponding estimation errors. The above results imply that the observers converge quickly, which sets a foundation for the reconstruction of the multi-dimensional fault and unknown input disturbance in the context below.

## 6. Conclusions

In this paper, a type of nonlinear system with actuator faults, sensor faults, and unknown input disturbances is studied. A fault reconstruction method of nonlinear system is presented. First, a primary transformation is made on the output equation of the system to obtain two output sub-equations.

## References:

[1] B. Iskander, S. Anis, and B.H. Faycal,"Robust sensor faults reconstruction for a class of uncertain linear systems using a sliding mode observer:an LMI approach," 2nd Mediterranean Conference on Intelligent Systems and Automation,2009.

[2] A.B. Brahim, S. Dhahri, F.B. Hmida, and A. Sellami,"Robust and simultaneous

reconstruction of actuator and sensor faults via sliding mode observer,"International Conference on

Electrical Engineering and Software Applications , 2013.

[3] C.P. Tan and C. Edwards,"Sliding mode observers for detection andreconstruction of sensor faults,"Automatica,vol.38,pp.1815-1821,2002.

[4] X.J. Meng and G.H. Yang,"Nonlinear fault tolerant control for polynomial systems with input saturation," The Control and Decision Conference, Guiyang, China, pp.3918-3923,2013.

[5] H.G. Zhang, Z.S. Wang, and D.R. Liu,"A Comprehensive Review of Stability Analysis of Continuous-Time Recurrent Neural Networks," IEEE Transactions on Neural Networks and Learning Systems,vol.25, pp.1229-1262,2014.

[6] X.S. Si and D.H. Zhou,"A Generalized Result for Degradation Model-Based Reliability Estimation," IEEE Transactions on Automation Science and Engineering, vol.11,pp.632-637,2013.