

Accelerating Simulated Annealing Fractional Order Derivative Kelvin Creep Model of Artificial Frozen Soil

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Abstract. It is important to master the creep properties of artificial frozen soil in engineering construction during the freezing in the shaft sinking. Given the shortage that the creep behaviors of artificial frozen soil were deemed as the materials between perfect solid and fluid, and that the components of integer order calculus constitutive relation were in great demand, the fractional order constitutive relation was used to calculate the creep character of the artificial frozen soil. Through the substitution of a multiplied dashpot by a fractional order derivative dashpot, a fractional order derivative Kelvin's creep model is developed. The series accelerated element are added to the fractional order element in Kelvin model, and the accelerated model is built. Using fractional order derivative Kelvin model to simulate the creep properties of artificial frozen soil, the model parameters was obtained by the global optimization of simulated annealing algorithm. The fractional order derivative creep model of artificial frozen soil can be reflected preferably in the 3 stages creep process.

Introduction

It is important to master the creep properties of artificial frozen soil in engineering construction during the freezing in the shaft sinking. The mechanical properties of artificial frozen soil were deemed as the materials between perfect solid and fluid. Neither do the creep properties of artificial frozen soil abide by Hooke Law, nor do it abide by Newton's Law of viscosity, but it abide by a relationship between these two laws instead^[1].

The study of fractional calculus is the arbitrary order differentiation, the properties of integral operator and its application. The rheological model theory of using fraction order derivative not only reserve advantages of the classical model theory, but also the combination of series and parallel of a few several elements can be anastomosed preferably with test results^[2-4].

The parameter identification of constitutive model is the important subject that studies on the constitutive relation of artificial frozen soil. The general idea of parameter identification of geo-material constitutive model is to select suitable combined model, according to the experimental data and then to determine the parameter of model by regression analysis with least square method etc. The error in calculation was caused in the process of hypothesis of constitutive model, therefore, there is a great error between the calculation results and the measured value. With the development of artificial intelligence, the algorithm technology of simulated annealing was applied to geo-material constitutive model and parameter identification by many scholars. And, they have obtained plentiful achievements^[5-6].

Given the shortage that the creep behaviors of artificial frozen soil were deemed as the materials between perfect solid and fluid, and that the components of integer order calculus constitutive relation were in great demand, the fractional order constitutive relation was used to calculate the creep character of the artificial frozen soil. Through the substitution of a dashpot by a fractional order derivative dashpot, a fractional order derivative Kelvin's creep model is developed. The series accelerated elements are added to the fractional order element in Kelvin model, and the accelerated model is built. Using fractional order derivative Kelvin model to simulate the creep properties of artificial frozen soil, the model parameters was obtained by the global optimization of simulated

annealing algorithm. The fractional order derivative creep model of artificial frozen soil can be reflected preferably in the 3 stages creep process. The fractional order derivative model is a new method in the calculation of the field of artificial frozen soil.

Fractional order derivative Kelvin creep model of artificial frozen soil

General Kelvin model. According to the analysis of the existing geo-material constitutive model, Kelvin model is a model which is relatively comprehensive and has a broad application. It is used to describe the relationship of stress and strain of geotechnical materials. This model can be used to consider the mechanical properties of material, such as elasticity, visco-elasticity and elastic-plastic etc. It has a wide filed of application and more mature theoretical derivation. General Kelvin model is shown in figure 1.

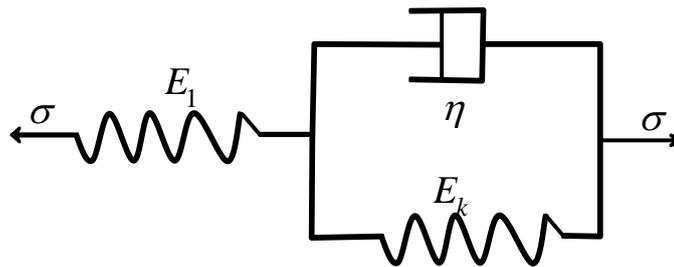


Fig.1 General Kelvin model

The constitutive equation of general Kelvin model:

$$\varepsilon(t) = \frac{\sigma}{E_k} (1 - e^{-\frac{E_k t}{\eta}}) + \frac{\sigma}{E_1} \quad (1)$$

In the formula, E_1 is the elasticity parameter of spring in general Kelvin body; E_k is the elasticity parameter of spring in series; η is the viscosity coefficient of a dashpot in general Kelvin body; t is time.

Fractional order derivative steady and accelerating Kelvin model

1) A summary of fractional order derivative

The definition of r order Riemann- Liouville fractional order derivative of function $f(x)$ ^[7]:

$$D^r [f(t)] = \frac{1}{\Gamma(1-r)} \frac{d}{dt} \int_0^t \frac{f(t-p)}{p^r} dp = I_r(t) f(0) + \int_0^t I_r(p) \frac{d}{dt} f(t-p) dp \quad (2)$$

In the formula, D^r is the operator of fractional order differentiation and the generalization of the operator D of integer order differentiation.

The definition of Abel kernel is:

$$I_r(t) = 0, \quad t \leq 0 \quad (3)$$

In the formula, $0 < r < 1$, Γ is the function of Gamma.

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt \quad (4)$$

$$\Gamma(1+z) = z\Gamma(z), \quad \text{Re}(z) > 0 \quad (5)$$

The role of fractional order derivative in function $f(t)$ equals to the general Stieltjes convolutional of Abel kernel $I_r(t)$ and function $f(t)$. The definition of constitutive relation of Abel dashpot is that the fractional order derivative of stress is proportional to the fractional order derivative of strain. The constitutive relation of Abel dashpot is:

$$\sigma = \eta D^r \varepsilon(t) = \eta I_r(t) * d\varepsilon(t) \quad (6)$$

Creep compliance:

$$D(t) = L^{-1} \frac{1}{\eta p p^r} = \frac{1}{\eta} \int_0^t \frac{t^{r-1}}{\Gamma(r)} dt = \frac{1}{\eta} \frac{t^r}{\Gamma(r+1)} \quad (7)$$

$$D^r[x(t)] = \frac{1}{\Gamma(1-r)} \frac{d}{dt} \int_0^t \frac{x(p)}{(t-p)^r} dp \quad (8)$$

Laplace transformation of $D^r f(t)$:

$$L[D^r f(t)] = L[D^m [D^{-(m-r)} f(t)]] = s^r F(s) - \sum_{k=0}^{m-1} s^{m-k-1} D^{k-m-r} f(0) \quad (m-1 < r < m, m=1,2,3,\dots) \quad (9)$$

When m equals one, let initial condition be zero, we get:

$$L[D^r f(t)] = s^r F(s) - D^{-(1-r)} f(0) = s^r F(s) \quad (10)$$

2) Nonlinear viscous dashpot

When the artificial frozen soil enters the stage of accelerating creep, it can choose strain parameter to represent whether it enters the stage of acceleration creep. And it brings an accelerating dashpot with strain trigger, which was described the deformation of each stage of the acceleration creep process of artificial frozen soil. See figure 2.

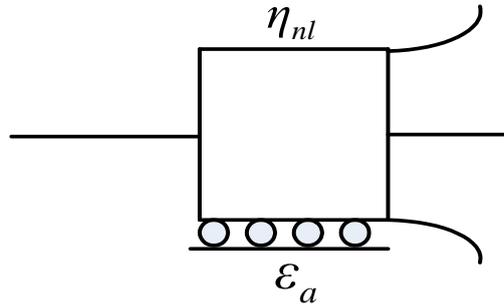


Fig.2 Accelerating viscous dashpot

When the global strain of the model is less than ε_a , the viscous dashpot is rigid body and it does not play a role; When the global strain of the model is more than ε_a , the nonlinear viscous dashpot starts triggering. The constitutive relationship of accelerating viscous dashpot:

$$\begin{aligned} \varepsilon &= \eta_{nl} e^{t^2} & (\varepsilon \geq \varepsilon_a) \\ \varepsilon_{nl} &= 0 & (\varepsilon < \varepsilon_a) \end{aligned} \quad (11)$$

In the formula, η_{nl} is the viscosity parameter of accelerating viscous dashpot.

3) The equation of fractional order derivative steady-state Kelvin creep model

Fractional order derivative general Kelvin model components are shown in figure 3.

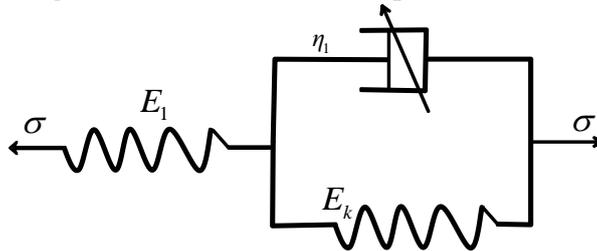


Fig.3 Fractional order derivative general Kelvin model

$$\varepsilon(t) = \frac{\sigma_c}{E_k} \left(1 + \sum_{n=1}^{\infty} (-1)^n \frac{(t/p)^m}{\Gamma(1+rn)} \right) \quad (12)$$

Accelerating fractional order derivative Kelvin creep model of artificial frozen soil as follows:

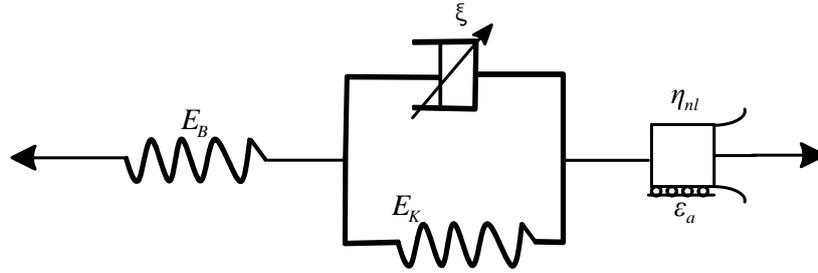


Fig.4. Accelerating fractional order derivative general Kelvin model

$$\varepsilon(t) = \frac{\sigma_c}{E_k} \left(1 + \sum_{n=1}^{\infty} (-1)^n \frac{(t/p)^m}{\Gamma(1+rn)} \right) + \frac{\sigma_c}{\eta_2} e^{t^2} \quad (13)$$

The parameters of model optimized by simulated annealing

The process of the parameters of model optimized by the simulated annealing as follows[9]:

1)The annealing temperature of initialization is T_k (let be $k = 0$), producing random initial solution set $\{x_0\}$. When the parameter is optimized:

$$\{x_0\} = [E_{K0}, \eta_{K0}, E_{B0}, \eta_{B0}] \quad (14)$$

When the parameter is optimized:

$$\{x_0\} = [E_{K0}, \eta_{K0}, E_{B0}, r_0, \xi_0] \quad (15)$$

2)When the temperature is T_k , until it reach the equilibrium state, the feasible solution x' is produced in the field of x ; the difference Δf of the objective function $f(x')$ of x' and the objective function $f(x)$ of x is calculated. According to the probability $\min\{1, \exp(-\Delta f/T_k)\} > \text{random}[0,1]$, x' was received, which $\text{random}[0,1]$ is the random number in the intervals of $[0,1]$.

3) The operation of annealing: $T_{k+1} = CT_k, k \rightarrow k+1, C \in (0,1)$. If the convergence criterion is satisfied, the annealing process will finish. Otherwise, it turns to (2). The solving process goes to the direction of optimization controlled by the annealing temperature T . And it is received the solution of inferior quality by probability $\exp(-\Delta f/T_k)$, therefore this algorithm can jump out the point of the local limit. As long as the initial temperature is high enough and the annealing process is slow enough, the algorithm can converge to the global optimal solution.

Fractional order derivative creep model of artificial frozen soil

Uniaxial creep experiment of artificial frozen soil. The water content of silt is 11% and the sampling depth is 180~200m; The water content of fine-grained sandstone is 8% and the sampling depth is 320~330m. The tests were performed in WDT-100 microcomputer control electro- hydraulic servo frozen soil uniaxial testing machine. The size of samples is $\Phi 50 \times 100\text{mm}$.

The test values of uniaxial strength of silt, fine-grained sandstone in -5°C , -10°C , -15°C are showed in table 1. The creep test is performed in -5°C , -10°C , -15°C three temperature level respectively. The test load is performed according to $0.3\sigma_s, 0.5\sigma_s, 0.7\sigma_s$ (σ_s —uniaxial strength) three load level.

Uniaxial creep fractional order derivative model of artificial frozen soil. The parameter of fractional order derivative Kelvin model and unsteady fractional order derivative generalized Kelvin mode is optimized by simulated annealing algorithm. Simulated annealing maximum internal cyclic number is 100 and cooling coefficient is 0.98. The formula of energy transformation is Boltzmann function and the energy conversion is single-step conversion. The test load of the clay, sandy clay soil is $0.3\sigma_s, 0.5\sigma_s$ in -5°C , -10°C , -15°C respectively. The parameters of fractional order derivative Kelvin creep model are shown in table 1.

Table 1 The parameters of the fractional Kelvin model

soil	Temperature	Loading coefficient	σ_c	E_K	r	ρ
Silt	-5	0.3	0.85	0.22	0.26	1.49
		0.5	1.42	0.18	0.26	2.26
	-10	0.3	1.04	0.24	0.26	1.51
		0.5	1.74	0.28	0.25	4.58
	-15	0.3	1.16	0.26	0.27	1.52
		0.5	1.93	0.22	0.24	3.45
Fine-grained sandstone	-5	0.3	0.45	0.12	0.34	1.44
		0.5	0.74	0.14	0.25	1.95
	-10	0.3	0.86	0.16	0.21	3.06
		0.5	1.44	0.22	0.29	1.29
	-15	0.3	1.41	0.21	0.26	3.93
		0.5	2.36	0.28	0.32	1.83

The silt, fine-grained sandstone are performed in -5°C , -10°C , -15°C . The test load is $0.7\sigma_s$. The parameters of fractional order derivative Kelvin creep model are shown in table 2.

Table2. The parameters of the accelerating fractional Kelvin model

soil	Temperature	σ_c	ρ	E_K	r	η_{nl}
silt	-5	1.99	1.05	0.16	0.24	59.58
	-10	2.44	4.75	0.17	0.23	40.87
	-15	2.7	6.77	0.18	0.23	6.47
Fine-grained sandstone	-5	1.04	3.50	0.13	0.22	7970.75
	-10	2.02	5.24	0.21	0.31	5117.27
	-15	3.30	1.21	0.192	0.28	3019.75

The test values of creep and the fractional order derivative Kelvin model calculation values are shown in figure 4.

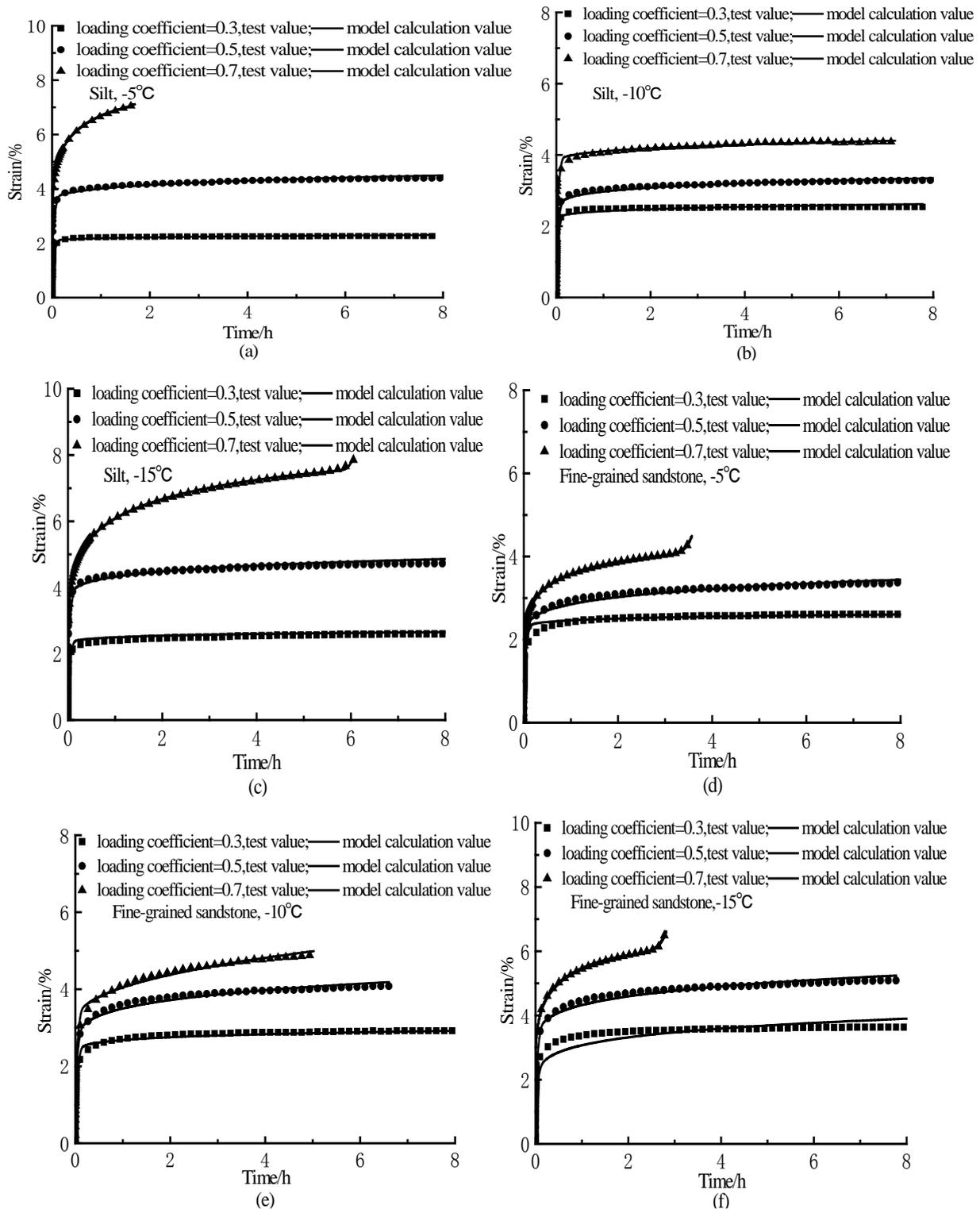


Fig.4 Creep test value and the fractional Kelvin model calculation value contrast

From figure 4, found that the creep characteristic of frozen soil can be well reflected by fractional order derivative Kelvin model which the theory of fractional calculus and integer order calculus are unified.

Conclusion

The creep properties of artificial frozen soil can be stimulated well by fractional order derivative Kelvin model and acceleration fractional order Kelvin model. The model parameters was obtained by

the global optimization of simulated annealing algorithm. The concrete achievements are listed as follows :

1) The fractional order derivative Kelvin creep model is established, which is based on fractional order derivative differential definition and integer order element model . The formula is closely deduced and the model has definite physical meaning and less model parameters.

2) By comparing the calculated value of this model with the experimental result, found that the creep curve can be simulated preferably by fractional order derivative Kelvin creep model of artificial frozen soil and acceleration fractional order derivative Kelvin creep model.

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