Calculation and simulation of a Hertzian dipole

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Abstract. As being used generally in numerous numerical calculations and models, Hertzian dipole's calculation show its importance. This paper begin with the very basic level to deduct the equations of a Hertzian dipole fed by a current in time-varying fields in spherical co-ordinates, and then a simulation in Matlab helps understanding how each variable in the equation affect the electric field.

Introduction

The Hertzian dipole is a theoretical construction [1], which can be regarded as a common used tool in radiation analysis in numerical calculation to simplify the antenna geometries models. Since the Hertzian dipole is one of the most commonly used methods in basic wireless communications and its widely used applications, it has been researched by scientists all over the world. Author starts from the derivation process of a Hertzian dipole and simulate the fields of a short piece of wire carrying a sinusoidal current which is uniform along the length of the wire as an example.

Calculation to fields

The electric potential at a point is the work done in bringing a unit positive charge from infinity to that point: $-\int_{\infty}^{r} E.dl = V$ where E is the electric field, r is the location of the point and V is a scalar field which is a directionless number associated with every point in space. [2] Conversely, $E = -\nabla V$, for static fields, there are no sources or sinks, so $\nabla \times E = 0$. If a vector field has zero curl, then that vector field is a gradient of scalar field. According to Gauss's Law for electric fields $\vec{\nabla} \circ \vec{E} = \frac{\rho}{\varepsilon}$,then Poisson's Equation comes out $\nabla^2 V = \frac{\rho}{\varepsilon}$. And in a region of space with no net charge, $\rho = 0$, and here the Laplace Equation is obtained: $\nabla^2 \vec{V} = 0$. If there exists a field Y in a vector field X such that $X = \nabla \times Y$, then $\vec{\nabla} \circ \vec{X} = 0$, Gauss's law represents magnetic fields as $\vec{\nabla} \circ \vec{B} = 0$. Define magnetic vector potential A such that B is the curl of A , to relate magnetic vector potential and current density, from Ampere-Maxwell and for a static field $\frac{\partial \vec{E}}{\partial t} = 0$, $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$, hence $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \mu_0 \vec{J}$, then $\vec{\nabla}(\vec{\nabla} \circ \vec{A}) - \vec{\nabla}^2 \vec{A} = \mu_0 \vec{J}$ .Let $\vec{\nabla} \circ \vec{A} = 0$,then $\vec{\nabla}^2 \vec{A} = -\mu_0 \vec{J}$, these can be done because of the definition of A is not unique,we can have $\vec{A}' = \vec{A} + \vec{\nabla} \circ \vec{C}$,as curl of grad is zero, the curl of this expression is independent of C,and here a suitable choice of C allows us set the div of A' to be zero. Equation for V due to a point charge is $V = \sum_i \frac{q_i}{4\pi\varepsilon_0 r_i}$, and for a space charge $V = \frac{1}{4\pi\varepsilon_0} \int_\tau \frac{\rho(x', y', z')}{r} d\tau$, so by analogy, $A = \frac{\mu}{4\pi} \int_\tau \frac{\vec{J}(x', y', z')}{r} d\tau$ is a solution to $\vec{\nabla}^2 \vec{A} = -\mu_0 \vec{J}$. If the current is time-varying, we must account for this as $A = \frac{\mu}{4\pi}\int_{\tau}\frac{\vec{J}(x',y',z';t-\frac{r}{c})}{r}d\tau$ .To find the electric and magnetic fields around the wire with a sinusoidal current, the above equation becomes $A = \frac{\mu}{4\pi}\int_{-l/2}^{l/2}\frac{\vec{I}\sin(\omega(t-r/v))}{r}dx$, if r is far more greater than l, $A = \frac{\mu\vec{I}}{4\pi r}\sin(\omega(t-r/v))\int_{-l/2}^{l/2}dx$. Converting into spherical coordinates, $A_r = \frac{\mu Il}{4\pi r}\sin(\omega(t-r/v))\cos\theta$ and $A_\theta = \frac{-\mu Il}{4\pi r}\sin(\omega(t-r/v))\sin\theta$ and $A_\phi = 0$. B is the curl of these equations, so for magnetic field, $B_r = 0$, $B_\theta = 0$, and $B_\phi = \frac{\mu Il}{4\pi r}(\frac{w}{v}\cos(\omega(t-r/v)))\sin\theta$, for electric field, $\vec{\nabla}\times\vec{B} = \mu_0(\vec{J}+\varepsilon_0\frac{\partial\vec{E}}{\partial t})$. But J is zero outside the wire, $\frac{\partial\vec{E}}{\partial t} = \frac{1}{\mu_0\varepsilon_0}\vec{\nabla}\times\vec{B}$, substituting for B and integrating with time, $E_r = \frac{Il\cos\theta}{2\pi\varepsilon_0 r}\{\frac{1}{rv}\sin\omega(t-r/v) - \frac{1}{r^2\omega}\cos\omega(t-r/v)\}$, $E_\theta = \frac{-Il\sin\theta}{4\pi\varepsilon_0 r}\{\frac{-\omega}{v^2}\cos\omega(t-r/v) - \frac{1}{rv}\sin\omega(t-r/v) + \frac{1}{r^2\omega}\cos\omega(t-r/v)\}$ and $E_\phi = 0$.

Simulation to time-varying electric fields in Matlab

Close to the dipole, r is small, hence terms with power of r on the denominator dominate the equations.

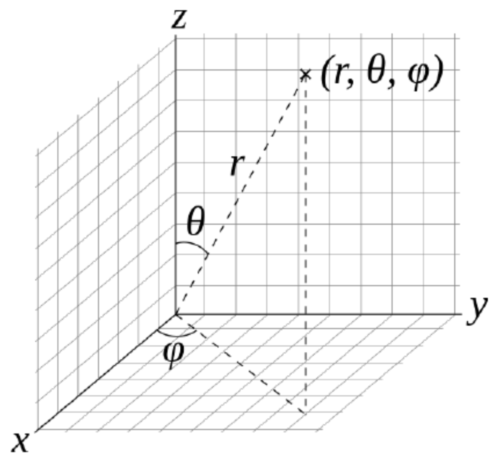

Fig. 1 Hertzian dipole in spherical co-ordinates

Near field. Field equations here become

$$B_\phi = \frac{\mu Il}{4\pi r}(\frac{w}{v}\cos(\omega(t-r/v)))\sin\theta$$

$$E_r = \frac{Il\cos\theta}{2\pi\varepsilon_0 r}\{\frac{1}{rv}\sin\omega(t-r/v) - \frac{1}{r^2\omega}\cos\omega(t-r/v)\}$$

$$E_\theta = \frac{-Il\sin\theta}{4\pi\varepsilon_0 r}\{-\frac{1}{rv}\sin\omega(t-r/v) + \frac{1}{r^2\omega}\cos\omega(t-r/v)\}$$

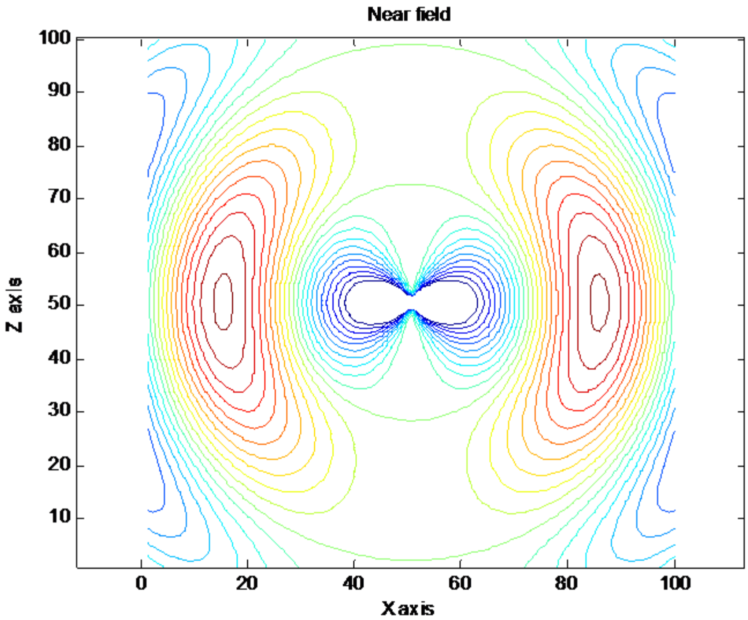

Fig. 2 Near field

Far field. Field equations become

$$E_r = 0 \qquad E_\theta = \frac{\eta I l \sin\theta}{2r\lambda}\cos(\frac{2\pi r}{\lambda} - \omega t) \qquad E_\phi = 0$$

$$H_r = 0 \qquad H_\theta = 0 \qquad H_\phi = \frac{\eta I l \sin\theta}{2r\lambda}\cos(\frac{2\pi r}{\lambda} - \omega t)$$

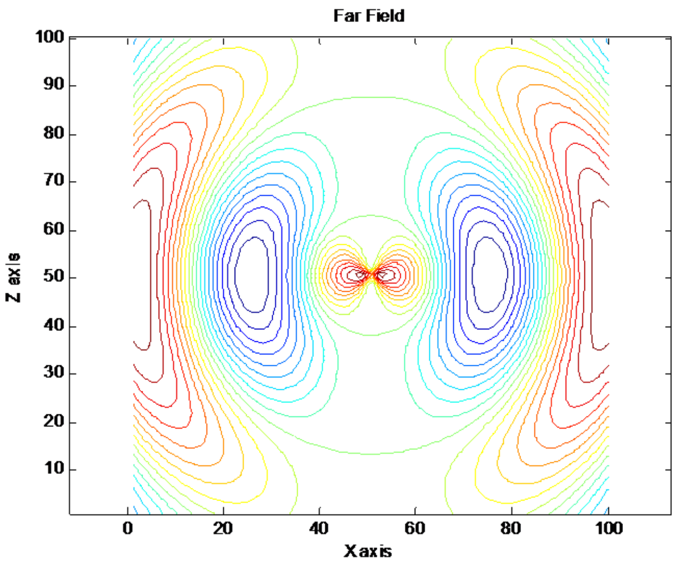

Fig. 3 Far field

Explanations to this simulation. The two graph above are the screen shots of the simulating video. The far field electric field lines are horizontal rings around the axis of the vertically oriented dipole, these rings moving and expanding from the center of the image to the two sides along x-axis horizontally, but it is different with near field which does not exist a ring at the very beginning. L and η in the equations can be regard as constant. Because of λ=2*pi*v/ω, and v and ω can be seen as constants λ can been seen as a constant too.

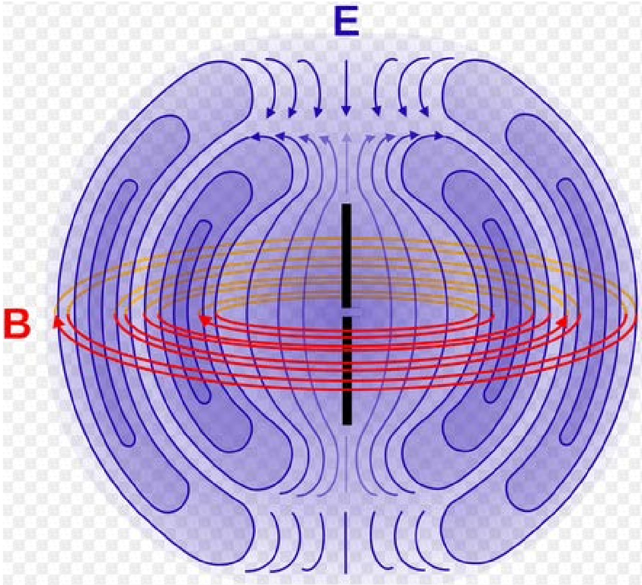

Fig 4 Electromagnetic fields due to a radiating dipole[1]

(1)For near field:

(a) Compare Biot-Savart law.

(b) Terms in r^{-2} are the induction field.

(c) Terms in r^{-3} are the electrostatic field.

(2)For far field:

(a) Depend on $\sin\theta$, E_θ and H_θ are in phase.

(b) The equation represent an electric magnetic wave propagating away from the dipole, which is recognized as spherical wave.

(c) Field zero up or down.

(d) Max at equator.

(f) It describes a radiation field which explains how attennas work. [2]

Conclusion

Hertzian dipole is generally used in numerous calculations and models, for example the radiating source is modeled by the sum of a large number of short Hertzian dipoles which allows the contributions of line-end discontinuities to be included through a VNA measurement together with a monopole approximation [3]; the Hertzian Dipole Antenna has a special differential characteristics so that broadband radiation can be realizing[4] and so on. And I believe Hertzian dipoles will be further used in various of models to make more contribution.

References

[1] Information on https://en.wikipedia.org/wiki/Dipole_antenna

[2] Tony Fagan, Wireless system, Antennas, 2015

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