# Capacity of Multi-channel Vehicular Ad Hoc Networks

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***Abstract*—capacity is one of the basic problems for wireless networks which determine its performance. With the fast development of Multi-channel technology and VANET, the capacity of Multi-channel VANET has been crucial issue needing to a in-depth solution. In the paper, the capacity of Multi-channel VANET is addressed. Furthermore, its theoretic upper and lower bounds are also obtained. Our results when the ratio of channels to interfaces grows not very fast, there is no capacity loss.**

***Keywords—capacity; Multi-channel; Vehicular Ad Hoc Networks; mobility; road topology***

## I. INTRODUCTION

As one of the most fundamental issue of wireless networks, capacity is always attracting much focus which provides in-depth view about its expandability, performance and reliability. In the landmark paper[1], Gupta and Kumar presented that when there are n nodes uniformly distributed in an unit circle, and each node randomly selects another nodes as its destination, then the capacity of each node is $\lambda(n) = \Theta\left(W / \sqrt{n \log n}\right)$. This is a disappointing result, for it shows that the large scale of ad hoc network is infeasible.From then on, many techniques are considered to improve the results [2-8] which includes mobility, base stations, antenna and multi-channel and so on.

As one kind of special Ad hoc network, Vehicular Ad hoc Networks (VANET) is an open and mobile Ad hoc network where vehicles on road can communicate with each other and with roadside access points[9]. It has similar features with other Ad hoc networks such as multi-hops, dynamic topology. However, it also has own characters. For example, vehicles usually run on road, and are impossible distributed randomly on areas; energy shortage may not be a critical issue for VANET; VANET may support multiple channels with multiple interfaces.

In Ref.[10], the throughput scaling law for VANET is addressed with single-channel single-interface support. However, due to its capability of computation and storage, multi-channel technology is a suitable to improve its capacity. For example, the newest 802.11 protocol supports tens of channels. Thus, the capacity of Multi-channel VANET(MVANET) should be further dealt with.

In the paper, the capacity of MVANET is addressed. Both the upper and lower bounds of capacity are obtained. Moreover, the impact of ratio of channels to interfaces is also solved. The rest of the paper is organized as follows: section 2 presents the analysis model and assumptions; the upper bound is obtained in section 3; section 4 constructs the lower bound; conclusions and future work are presented in the last section.

## II. MODEL

There are $n$ vehicles that are moving on a single road which is shown in Fig.1. The road topology could be a high way. Multiple parallel roads can be combined into one road whose density is given by be summation of densities of each road. A Poisson point process with density $k$ is assumed on the road[10]. In reality, the density of cars on road is limited by the physical size of cars which is assumed to be bounded positive number. At time zero, all vehicles on the same lane will choose a common speed $v \in [0, v_{\max}]$ uniformly at random. It is assumed that the vehicle do not change their speed. Thus, according to [10], the vehicular mobility does not have influence on capacity of VANET.

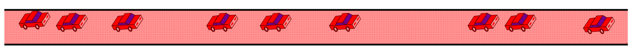

Figure 1. Road topology

There are $c$ channels, and $m$ interfaces for each vehicle node. Moreover, each interface can freely switch among channels. The total data rate is divided equally among the channels, and then the data rate supported by any one of the channels is $W / c$ bits/s.

The protocol model proposed by Gupta and Kumar[1] is adopted.

 

Suppose node $X_i$ transmits over the $m$ th channel to a node $X_j$. Then this transmission is successfully received by node $X_j$, for every other node $X_k$ simultaneously transmitting over the same channel, and the guard zone $\Delta > 0$, the following condition holds.

$$\left|X_k - X_j\right| \geq (1+\Delta)\left|X_i - X_j\right|$$

$X_i$ also denotes the location of a node.

## III. UPPER BOUND

The network capacity of Multi-channel VANET is constrained by a variety of factors, interference, bandwidth, the number of interfaces and other ones. The minimum value of upper bound caused by different factors is the upper bound of its capacity.

The network capacity of Multi-channel VANET is constrained by two factors (according to the relation of the number of channels $c$ to that of interfaces $m$ :

Constraint 1. Interference Constraint: from the protocol model, each transmission consumes a certain area of region. Thus, if there are two simultaneous transmissions on the same channel, the two regions cannot intersect. In [10], the capacity of Single-channel VANET with single road is $O\left(\frac{1}{n}\right)$ which is obtained by the interference of transmissions, and this result is also effective to Multi-channel VANET.

Constraint 2. Interface Constraint: The Capacity of Multi-channel VANET is also limited by the number of simultaneous transmissions supported by interfaces in the network. According to the channel model, each interface can transmit at a rate of $W/c$ bits/s, and each node has $m$ interfaces, so the total network capacity is bounded by $O\left(\frac{Wm}{c}\right)$ bits/s.

Combined with the above two bounds, the network capacity is bounded by $O\left(\min\left(\frac{1}{n}, \frac{Wm}{c}\right)\right)$ bits/s. Therefore, the following theorem is obtained：

Theorem 1: The upper bound of capacity of Multi-channel VANET is as follows:

When $\frac{c}{m}$ is $O(n)$, capacity of Multi-channel VANET is $O\left(\frac{1}{n}\right)$ bits/s;

When $\frac{c}{m}$ is $\Omega(n)$, capacity of Multi-channel VANET is $O\left(\frac{Wm}{c}\right)$ bits/s.

While there may be other constraints on capacity as well, the constraint we considered is sufficient to provide a tight bound.

## IV. LOWER BOUND

We adopt the following steps to construct scheduling scheme to implement the lower bound capacity of Multi-channel VANET. The network with each node equipped single interface is first dealt with, and then expanded to the general condition with $m$ interfaces following the similar steps in [8].

The transmission radius of each vehicle is $r = \Theta(n)$ which means that each vehicle node can communicate with each destination node directly with one transmission.

We first assign a channel for every node ready to transmit packets from available $c$ channels. There are $n$ nodes, and a node numbered $q$ is allowed to transmit on channel ($q \bmod c$) + 1. Then, similar to [11], we construct a conflict graph based on all transmissions in two steps. First, create a separate vertex for each transmission in the cell, and each vertex has two properties: the assigned channel and two endpoints of the transmission. Secondly, connect two vertices by an edge if they satisfy two requirements: their properties have the same channel, or have least one same endpoint. The scheduling problem thus reduces to obtaining a vertex-coloring of this graph. Thus, according to the graph vertex theory, the graph with maximum degree $e$ can be vertex-colored with at most $e$ +1colors [1]. Thus, it is evident that the graph can be vertex-colored with at most $\left\lceil\frac{n}{c}\right\rceil$ colors.

Thus, each second can be divided into $\Omega\left(1/\left\lceil\frac{n}{c}\right\rceil\right)$ equal length slots. And each transmission get a slot for transmitting. Each channel can transmit at a rate of $\frac{W}{c}$ bits/s. Therefore, capacity of Multi-channel VANET is

$$\lambda = \Omega\left(\frac{W}{c\left\lceil\frac{n}{c}\right\rceil}\right) = \Omega\left(\frac{W}{n+c}\right) \text{bits/s}$$

Due to the asymptotic order $c$, either $n$ or $c$ will dominate the denominator of $\lambda$. Hence, the capacity is equal to

$$\lambda = \Omega\left(\min\left(\frac{W}{n}, \frac{W}{c}\right)\right) \text{bits/s}$$

Based on Lemma 2 [8], We have the following theorem for Multi-channel VANET :

Theorem 2. The capacity of Multi-channel VANET is as follows:

When $\frac{c}{m}$ is $O(n)$, capacity of Multi-channel VANET is $\Omega\left(\frac{1}{n}\right)$ bits/s;

When $\frac{c}{m}$ is $\Omega(n)$, capacity of Multi-channel VANET is $\Omega\left(\frac{Wm}{c}\right)$ bits/s.

The upper and lower bounds have the same order which imply the bounds are tight..

## V. CONCLUSION

In the paper, the capacity of Multi-channel VANET is addressed. The single road topology is assumed which may be a highway. Both upper and lower bounds are obtained. According to our analysis, when $\frac{c}{m}$ is $\Omega(n)$, there are capacity loss, and when $\frac{c}{m}$ is $O(n)$, there are no capacity loss. So, it can be shown that Multi-channel technology is suitable to VANET. More complex road topology such as grid has not been dealt with, which will be our future work.


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