Target tracking based on amendatory Sage-Husa adaptive Kalman Filtering

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***Abstract* —Aiming at the boundedness of target tracking inaccuracy, even the lost of tracking caused by the error accumulation and state mutation in the aspect of target tracking using Kalman filter. The paper proposes a novel algorithm which is based on the modified Sage-Husa adaptive Kalman filter. This algorithm adjusts the predicted value of Sage-Husa adaptive Kalman filter in time by setting judgment and amendment rules, which can inhibit accumulation of error on target tracking and improve the filter precision in steady state and strengthen the stability and adaptability of filter. The experimental results show that the effectiveness of the novel algorithm.**

***Keywords* —Sage-Husa adaptive Kalman filter, Target tracking, Kalman filter, Amendment**

I. INTRODUCTION

Accurate and reliable target tracking is a major purpose and the main difficulty in the design of target tracking system. Tracking algorithm should not only to meet the requirements of real-time processing system, but also ensure a high tracking accuracy[1]. The Kalman filter algorithm[2] is at present the most widely used tracking algorithm in target tracking.

When the number of measurement k is increasing, the mean square error matrix estimated according to the filtering equation calculating tends to zero or a steady state value. But the deviation between estimate value and actual value becomes bigger and bigger, which makes the filter gradually lose the estimated effect. The phenomenon is known as the divergence of filter[3]. One important reason that leads to this phenomenon is the mutation of motion state. The robustness of Kalman filter is poor so that tracking accuracy declines, or is even lost.

Aiming at the divergence problem of standard Kalman filter, scholars put forward some adaptive Kalman filter. Such as adaptive fading Kalman filter based on innovation covariance[4], adaptive Kalman filter based on neural network and based on fuzzy logic[5][6], adaptive algorithm for adjusting observation noises based on double-Kalman filter[7], methods of adaptive filter to integrated navigation system of autonomous underwater vehicle[8], etc.In the mentioned literature[8] , Sage –Husa adaptive Kalman filter is most widely applied. But, when the state of motion mutates, the model takes a long time to reach a steady state, and the accuracy of model is low.

As to this point, this paper puts forward a new way of thinking that combines the ideas of amendatory Kalman filter[9] with Sage-Husa adaptive Kalman filter algorithm. The algorithm sets threshold criterion and amendment rules by the rates $\boldsymbol{Y}_k$ and measurement noise covariance $\boldsymbol{R}$, and amends the predicted value of adaptive Kalman filter in time so as to improve the accuracy of the algorithm and the performance of the algorithm.

II. SAGE-HUSA ADAPTIVE KALMAN FILTER

While Sage-Husa adaptive Kalman filter algorithm are conducting data recursion , it can simultaneously estimate and update the statistical properties of process noise covariance matrix Q and measurement noise covariance matrix R of the system in real time through varying noise statistics estimators, thus to achieve the goal of reducing model errors, restraining filtering divergence and improving the filtering accuracy.

State equation and measurement equation of discrete system is described as follows

$$\boldsymbol{X}_k = \boldsymbol{\Phi}_{k,k-1}\boldsymbol{X}_{k-1} + \boldsymbol{W}_{k-1} \tag{1}$$

$$\boldsymbol{Z}_k = \boldsymbol{H}_k\boldsymbol{X}_k + \boldsymbol{V}_k \tag{2}$$

In the formula, $\boldsymbol{X}_k \in \boldsymbol{R}^{n\times 1}$ is state vector. $\boldsymbol{\Phi}_{k,k-1} \in \boldsymbol{R}^{n\times n}$ is state –transition matrix form at t_{k-1} moment to at t_k moment. $\boldsymbol{Z}_k \in \boldsymbol{R}^{m\times 1}$ is measurement vector at t_k moment; $\boldsymbol{H}_k \in \boldsymbol{R}^{m\times n}$ is measurement matrix at t_k moment; $\boldsymbol{W}_{k-1} \in \boldsymbol{R}^{n\times 1}$ is measurement noise at t_{k-1} moment, $E(\boldsymbol{W}_{k-1}) = 0$, $E(\boldsymbol{W}_k\boldsymbol{W}_j^T) = \boldsymbol{Q}_k\delta_{kj}$; $\boldsymbol{V}_k \in \boldsymbol{R}^{m\times 1}$ is observation noise, $E(\boldsymbol{V}_k) = 0$, $E(\boldsymbol{V}_k\boldsymbol{V}_j^T) = \boldsymbol{R}_k\delta_{kj}$,while is independent of $\boldsymbol{W}_k$

Sage-Husa adaptive Kalman filter is described as follows:

$$\boldsymbol{X}_{k/k-1} = \boldsymbol{\Phi}_{k,k-1}\boldsymbol{X}_{k-1} + \boldsymbol{q}_k \tag{3}$$

$$\boldsymbol{Y}_k = \boldsymbol{Z}_k - \boldsymbol{H}_k\boldsymbol{X}_{k/k-1} - \boldsymbol{r}_k \tag{4}$$

$$\boldsymbol{X}_k = \boldsymbol{X}_{k/k-1} + \boldsymbol{K}_k\boldsymbol{Y}_k \tag{5}$$

$$\boldsymbol{P}_{k/k-1} = \boldsymbol{\Phi}_{k,k-1}\boldsymbol{P}_{k-1}\boldsymbol{\Phi}_{k,k-1}^T + \boldsymbol{Q}_k \tag{6}$$

$$\boldsymbol{K}_k = \boldsymbol{P}_{k/k-1}\boldsymbol{H}_k^T(\boldsymbol{H}_k\boldsymbol{P}_{k/k-1}\boldsymbol{H}_k^T + \boldsymbol{R}_k)^{-1} \tag{7}$$

$$\boldsymbol{P}_k = (\boldsymbol{I} - \boldsymbol{K}_k\boldsymbol{H}_k)\boldsymbol{P}_{k/k-1} \tag{8}$$

In the formula, q_k, Q_k, r_k, R_k respectively are mathematical expectation, variance of the process noise and mathematical expectation, variance of the measurement noise, which are determinate by time-varying noise estimator.

Time-varying noise estimator can be represented as follows:

$$R_k = (1-d_k)R_{k-1} + d_k(Y_kY_k^T - H_kP_{k/k-1}H_k^T) \quad (9)$$

$$r_k = (1-d_k)r_{k-1} + d_k(Z_k - H_kX_{k/k-1}) \quad (10)$$

$$Q_k = (1-d_k)Q_{k-1} + d_k(K_kY_kY_k^TK_k^T + P_k - \Phi_{k,k-1}P_k\Phi_{k,k-1}^T) \quad (11)$$

$$q_k = (1-d_k)q_{k-1} + d_k(X_k - \Phi_{k,k-1}X_{k-1}) \quad (12)$$

In formulas: $d_k = (1-b)/(1-b^{k+1}), 0 < b < 1$ b is forgetting factor. In the paper $b = 0.7$

In above, from (1) to (12) constitute Sage-Husa kalam filtering algorithm. The algorithm are conducting data recursion by measurement data, it can simultaneously estimate and amend the statistical properties of process noise and measurement noise, thus to achieve the purpose of the adaptive filter.

III. AMENDATORY SAGE-HUSA ADAPTIVE KALMAN FILTER

Based on the improvement of Sage-Husa adaptive Kalman filter by the idea of amendatory predictive value, Sage-Husa adaptive Kalman filter based on amendament is proposed. As can be seen from the formula (3) and (5), predicted value at t_k moment is determined by the state estimation of t_{k-1} moment, and predicted value at t_k moment effects the state estimated value at t_k moment. Therefore, in order to improve the precision of state estimated at t_k moment, the precision of predicted value should be improved at t_k moment.

Y_k reflects the degree of deviation from the predicted value to the measured value. When the motion state of the target mutates, the accuracy of predicted value is low. Thus, in order to improve the predicated accuracy, the trust on measured value should be increased. If the error Y_k between the predicated value and measured value at t_k moment is too much large, the error of estimated value is big at t_k moment. So, modification should be made for X_{k-1} .The predicted value $X_{k/k-1}$ at t_k moment is recalculated through amendatory X_{k-1}, so as to improve the accuracy of estimated value X_k at t_k moment and reach the goal of real-time correction. The flowchart of amendatory Predicted value is shown in figure

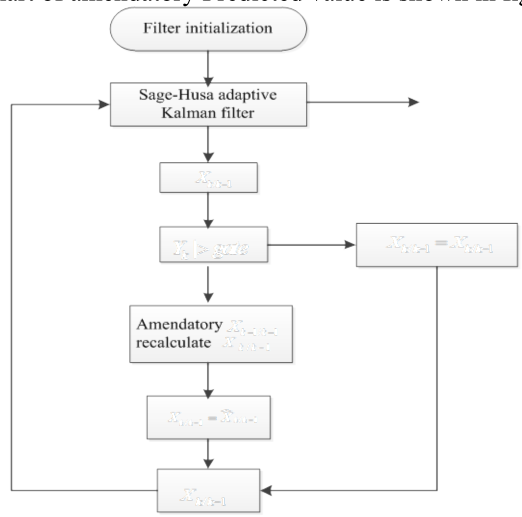


Figure 1 Flow chat of amendatory predicted value

To solve the above problems, first of all, we should determine the judgement rules of big error between predicted value and measured value. Secondly, we should formulate amendment rules for X_{k-1}.

Without taking into consideration of noise and interference, the measured value is uniquely determined at each time. If a circle is made with the measured value set as the center and the maximum error of measured values (mean square value measurement error covariance) as a radius, for a simple description, it is called measured circle. It is sure that there is real value within measured circle. If the predicted value at current time is within measured circle, we can determine the state of target without mutation. Otherwise, it means that the previous time of estimated error is too big or the state of target mutates so that the error of predicted value at current time increases.

Therefore, by amended predicted value, the amended predicted value is made to fall into the measured circle, and then we use Sage-Husa adaptive Kalman to estimate, which can further reduce the series of problems caused the mutation of target state and the accumulation error. Amendatory principle is shown in figure 2:

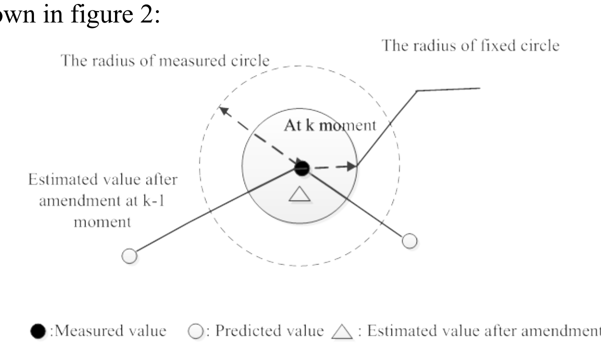


Figure 2 Schematic of amendatory predicted value

A. The set of judgement rules

Y_k reflects the degree of error between the predicted value and the measured value. Because the measured value itself also carries error, in order to get more close to the real value, the initial condition of amended rules should be based on that the error between the predicted value and the measured value should be greater than a certain threshold. $Y_k^{'} = \sqrt{R_{k(1,1)}}$, $R_{k(1,1)}$ reflects the position of measured error at t_k moment (i.e. the value in the first row and the first column in R_k).

If $|Y_k| > gate$, $\boldsymbol{X}_{k/k-1}$ is not accurate. By amending $\boldsymbol{X}_k$, $\boldsymbol{X}_{k/k-1}$ is being recalculated based on formula (3);In order to reduce the influence of cumulative error, and at the same time increase the trust of measured value, the threshold is set as: $gate = 0.7Y_k^{'}$ (The coefficient 0.7 is not fixed. In order to balance measured value and predicted value's impact on amendatory predicted value, its value should be within the interval). A circle with measured value as the center and gate as radius can be called fixed circle. Amendatory rules should guarantee the point out of the fixed circle that falls into the fixed circle by amendatory rules.

B. The set of amendment rules

Specific fixed rules are as follows：

1) Amendatory position estimated value: according to the error of Y_k , amendatory position estimated value is amended by compensation.
2) Amendatory velocity estimated value: according to the error of Y_k , amendatory velocity estimated value is amended by compensation.
3) Amendatory acceleration estimated value: according to the change value of velocity estimation before and after amendment, amendatory acceleration estimated value is amended by compensation.

Order $F = Y_k^{'} / R_{k(1,1)}$, specific amendatory formula is as follows:

$$\hat{x}_{k-1} = x_{k-1} + Y_k.c_1 \tag{13}$$

$$\hat{x}^{'}_{k-1} = x^{'}_{k-1} + Y_k.c_2 / T \tag{14}$$

$$\hat{x}^{''}_{k-1} = x^{''}_{k-1} + (\hat{x}_{k-1} - x_{k-1}).c_3 / T^2 \tag{15}$$

x_{k-1} , $x^{'}_{k-1}$, $x^{''}_{k-1}$ are respectively position , velocity and acceleration before amendment . $\hat{x}_{k-1}$, $\hat{x}^{'}_{k-1}$, $\hat{x}^{''}_{k-1}$ are respectively position, velocity and acceleration after amentdment. In formulas $c_1 = 0.7F$, $c_2 = 0.1F$, $c_3 = 0.3F$ 。

c_1, c_2, c_3 are chosen respectively correlation with velocity, position and acceleration, that is to say $s = v_0 t + 0.5at^2$. c_1, c_2, c_3 are respectively the formula the position, velocity and acceleration.

If we want to amend velocity or acceleration , c_2 or c_3 should be greater than 0.5 in target tracking process. The rest of the two parameters, a value is less than 0.5, the other one is calculated by the formula. The paper mainly amends position, so c_1 is bigger.

Thus it can be seen that compared with Sage-Husa adaptive Kalman Filtering, the amendatory Sage-Husa adaptive Kalman Filtering algorithm's each iteration increases one comparison and one amendment but without much calculated quantity increased. Sage-Husa adaptive Kalman filter after increasing the amendatory predicted value can better adapt to the state of mutation, improve the accuracy of the algorithm, improve the performance of the algorithm, and inhibit the accumulation of errors.

IV. SIMULATIONS AND EXPERIMENTAL ANALYSES

In order to validate the effectiveness of the amendatory Sage-Husa adaptive Kalman, we use 1000 Monte-Carlof to simulate experiments, setting the position root mean square error (RMSE) of the target as evaluation index. The RMSE is defined as:

$$\mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{j=1}^{N}(x(k) - x^{j}(k\,|\,k))^2} \tag{16}$$

In the above formulate, N is Monte-carlof simulation number, J is the jth simulation. $x(k)$ and $x^{j}(k\,|\,k)$ respectively represent the real value of tracking target state and the filter estimate value at k moment.

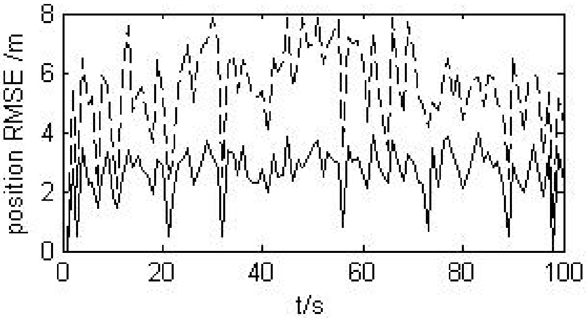
A. Target tracking of uniform linear motion

According to the literature [9] model, if the target does uniformly linear motion in the two-dimensional coordinate, the initial value of state $\boldsymbol{X} = [0,2,0,20]^T$. The target state lasts 100s, and the sampling frequency $T = 1$ s. The initial value of system noise covariance $\boldsymbol{Q} = diag[0.5,1,0.5,1]$ and observance noise covariance $\boldsymbol{R} = diag[100,100]$. Figure 3 represents the position RMSE tracked on the target by Sage-Husa adaptive Target tracking of uniformly accelerated linear motion

According to the literature [9] model, suppose the target do uniformly accelerated linear motion in the one-dimensional coordinate $\boldsymbol{X} = [0,2,0.5]^T$ .It can respectively represent position, velocity and acceleration.

The target state lasts 100s, and the sampling frequency $T = 1$ s.The initial value of system noise covariance *Q*=diag[1,1,1], and measurement noise covariance matrix *R*=diag[100,100,100]. The measured initial value is equal to the state initial value. Figure 3 represent position RMSE tracked on the target by Sage-Husa adaptive Kalman filter proposed in literature [8] and amendatory Sage-Husa adaptive Kalman filter algorithm proposed in the paper respectively.

$$\Phi = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \qquad H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



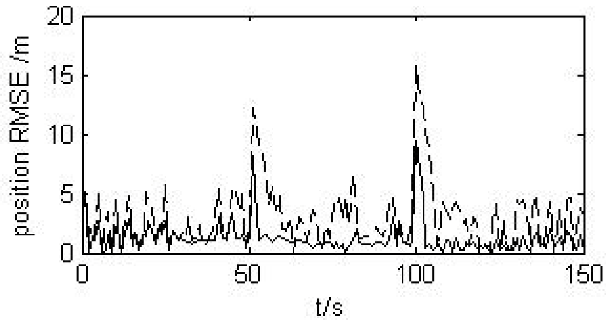
----- Sage-Husa adaptive Kalman filter
—— amendatory Sage-Husa adaptive Kalman filter

Figure 3 position RMSE of uniformly accelerated linear motion

As can be seen from the figure 3, the algorithm proposed in the paper which can better adapt to the slow changing of state of the tracking target , and is higher and better than Sage-Husa adaptive Kalman in accuracy and adaptability aspects.

B. Target tracking of mutative motion state

According to the literature [9] model, suppose that the initial value of state $\boldsymbol{X} = [0,2,0]^T$. The target does uniformly linear motion from 0s to 50s and does uniformly accelerated linear motion from 50s to 100s with the speed of 1m/s^2, then the uniformly linear motion from 100s to 150 s. The sampling frequency $T = 1$ s.The initial value of system noise covariance ***Q***=diag[1,1,1]; the measurement noise covariance ***R***=diag[100,100,100].The initial value of measured speed is the initial state value, while the initial value of measured position is the first measured value. Figure 4 represent position RMSE tracked on the target by Sage-Husa adaptive Kalman proposed in literature [8] and amendatory Sage-Husa adaptive Kalman proposed in the paper respectively.



----- Sage-Husa adaptive Kalman filter
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Figure 4position RMSE of mutation motion stat

From figure 4, position RMSE at both 50s and 100s mutate significantly. As can be seen from the position RMSE, the algorithm the paper proposes can reach a steady state quickly after the mutation, and the error is obviously lower than Sage-Husa adaptive Kalman filter, so it can better adapt to the mutant motion state.

V. CONCLUSION

The algorithm proposed by this paper not only estimates and amends the statistical properties of the system process noise and measurement noise in real time, but also amends the real-time predicted value of the system , which further improves the precision of the algorithm.

So, amendatory Sage-Husa adaptive Kalman Filter proposed by the paper can not only better adapt to the state of mutation, but also inherit the stability and adaptability of Sage-Husa adaptive Kalman Filtering, which further improves its accuracy in target tracking model.

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