

Adaptive Control And Multiple Parameters Identification With Single Differential Equation

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Abstract. How to identify multiple parameters quickly and accurately is a key or problem of control system optimization. Based on Terminal attractor and Sigmoid function, an adaptive control law is designed, and the result of parameter identification is analyzed. In the end, a conclusion can be made that theoretical analysis is correct and parameter identification method is effective by numerical simulation.

Introduction

Parameter identification is a complex problem that attracted many researchers in recent years^[1-4]. But it is still a complex problem especially for high order systems with uncertainty^[5-9]. Sometimes it is even more difficult to identify the unknown parameter than to design a controller to make the system stable. So in this paper the parameter identification problem for first order system with one differential equation is discussed with terminal control method^[10-11]. Terminal control has robustness and is not sensitive to parameter variation. Sigmoid function is one of the most transfer function in artificial neural networks, it is used in artificial neural networks[1-3] at the earliest, it has continuity, smooth, differentiability, boundedness. In this paper, Terminal attractor and Sigmoid function will be used in adaptive control to solve the problem of multiple parameter identification.

Problem Description

One order system can be written as:

$$\dot{x} = \sum_{i=1}^m a_i g_i(x) + u \quad (\text{Eq.1})$$

where a is unknown constant parameter, the goal is designing a controller such that the system state x can trace the expected value x^d .

Design Adaptive Identification Controller

An ordinary adaptive control method is used as follows, define a error variable as $z_1 = x_1 - x_1^d$, then

$$\dot{z}_1 = \dot{x}_1 - \dot{x}_1^d = \sum_{i=1}^m a_i g_i(x) + u \quad (\text{Eq.2})$$

Design state feedback control law as:

$$u = -\sum_{i=1}^m \hat{a}_i g_i(x) - \sum_{i=1}^n k_i f_i(z_1) \quad (\text{Eq.3})$$

where $n = 5$,

$$f_1(z_1) = z_1, \quad f_2(z_1) = z_1^3, \quad f_3(z_1) = z_1^{1/3} \quad (\text{Eq.4})$$

$$f_4(z_1) = \frac{z_1}{|z_1| + \varepsilon}, \quad \varepsilon = 0.2, \quad (\text{Eq.5})$$

$$f_5(z_1) = \frac{1 - e^{-\tau z_1}}{1 + e^{-\tau z_1}}, \quad \tau = 0.5 \quad (\text{Eq.6})$$

where $f_3(z_1)$ is Terminal attractor, and $f_5(z_1)$ is Sigmoid function, $f_4(z_1)$ and $f_5(z_1)$ both have boundedness, Obviously, $f_i(z_1)$ meet $z_1 f_i(z_1) \geq 0$, then

$$\dot{z}_1 = \sum_{i=1}^m \tilde{a}_i g_i(x) - \sum_{i=1}^n k_i f_i(z_1) \quad (\text{Eq.7})$$

where the error variable \tilde{a} can be defined as:

$$\tilde{a}_i = a_i - \hat{a}_i, \quad (\text{Eq.8})$$

design regulating law:

$$\dot{\hat{a}}_i = \Gamma_i z_1 g_i(x) \quad (\text{Eq.9})$$

where \hat{a} is unknown estimated parameter value, choose initial value $\hat{a}(0) = 0$, then

$$\dot{\tilde{a}}_i = -\dot{\hat{a}}_i \quad (\text{Eq.10})$$

choose Lyapunov function:

$$V = \frac{1}{2} z_1^2 + \sum_{i=1}^m \frac{1}{2\Gamma_i} \tilde{a}_i^2 \quad (\text{Eq.11})$$

then

$$\dot{V} = z_1 \dot{z}_1 + \sum_{i=1}^m \frac{1}{\Gamma_i} \tilde{a}_i \dot{\tilde{a}}_i \quad (\text{Eq.12})$$

then

$$\dot{V} = \sum_{i=1}^m z_1 \tilde{a}_i g_i(x) - \sum_{i=1}^n k_i z_1 f_i(z_1) - \sum_{i=1}^m \frac{1}{\Gamma_i} \tilde{a}_i \Gamma_i z_1 g_i(x) = -\sum_{i=1}^n k_i z_1 f_i(z_1) \leq 0 \quad (\text{Eq.13})$$

So $z_1 \rightarrow 0$.

Parameter Identification Result Analysis

When $z_1 \rightarrow 0$, where $u = -\sum_{i=1}^m \hat{a}_i g_i(x)$, then

$$\dot{z}_1 = \sum_{i=1}^m \tilde{a}_i g_i(x) \quad (\text{Eq.14})$$

So when $z_1 \rightarrow 0$, where $\dot{z}_1 \rightarrow 0$, $\dot{z}_1 = \sum_{i=1}^m \tilde{a}_i g_i(x) = 0$. If $g_i(x)$ are uncorrelated, then $\tilde{a}_i \rightarrow 0$.

Numerical Simulation

Choose $n = 2$, $a_1 = 5$, $a_2 = -5$, $g_1(x) = x$, $g_2(x) = \sin(x)$, $x_1^d = 2$, program can be written as follows:

```
clc;clear;a1=5;a2=-5;x1d=2;a1g=0;a2g=0;x=0;u=0;tf=15;dt=0.001;
```

```
for i=1:tf/dt
```

```
t=i*dt;g1x=x;g2x=x^3;
```

```
dx=a1*g1x+a2*g2x+u; x=x+dx*dt;
```

```
k1=5;k2=5;k3=1;k4=1;k5=1;esten=0.2;tao=0.5;
```

```

k1=5;k2=5;k3=5;k4=5;k5=5;
z1=x-x1d; f1=z1; f2=z1^3; f3=z1^(1/3);f4=z1/(abs(z1)+esten);
f5=(1-exp(-tao*z1))/(1+exp(-tao*z1));
ta1=5;ta2=5;da1g=ta1*g1x*z1;da2g=ta2*g2x*z1;
a1g=a1g+da1g*dt;a2g=a2g+da2g*dt;
u=-a1g*g1x-a2g*g2x-k1*f1-k2*f2-k3*f3-k4*f4-k5*f5;
tp(i)=t;xp(i)=x;a1gp(i)=a1g; a2gp(i)=a2g;
end
figure(1);plot(tp,xp,'k');xlabel('t/s');ylabel('state x');
figure(2);plot(tp,a1gp,'k');xlabel('t/s');ylabel('state a1g');
figure(3);plot(tp,a2gp,'k');xlabel('t/s');ylabel('state a2g');
And the simulation results are as follows:

```

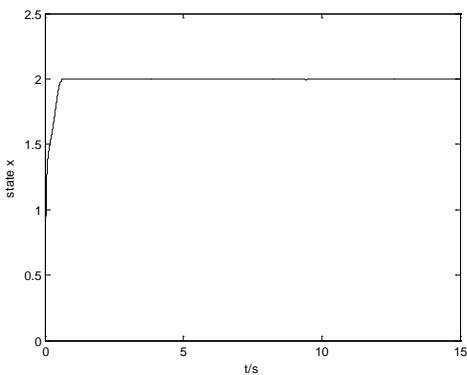


Fig.1 state x

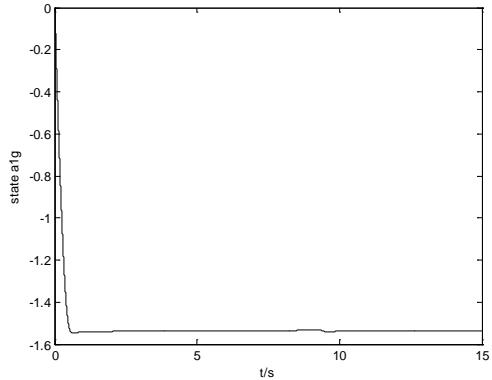


fig.2 state a1

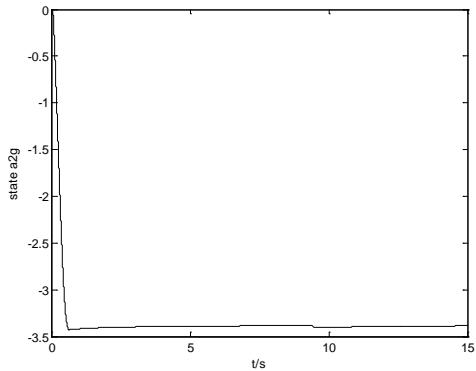


Fig.3 state a2

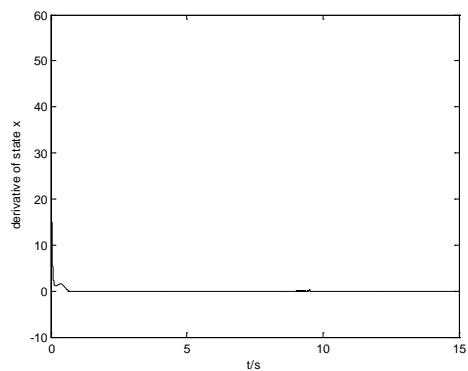


fig.4 state x

From the simulation results, we can know that system state can track expected state, but parameter identification can not be achieved. The main reason is that estimated parameter error can balance at optional position when final state x is constant.

Consider input is periodic signal, as $u = \sin(t)$, program can be written as follows:

```

clc;clear;a1=5;a2=-5;x1d=2;a1g=0;a2g=0;x=0;u=0;tf=150;dt=0.001;
for i=1:tf/dt

```

```

    t=i*dt;g1x=x;g2x=x^3; x1d=sin(t);
    dx=a1*g1x+a2*g2x+u; x=x+dx*dt;
    k1=5;k2=5;k3=1;k4=1;k5=1;esten=0.2;tao=0.5;
    k1=5;k2=5;k3=5;k4=5;k5=5;
    z1=x-x1d; f1=z1; f2=z1^3; f3=z1^(1/3);f4=z1/(abs(z1)+esten);
    f5=(1-exp(-tao*z1))/(1+exp(-tao*z1));
    ta1=5; ta2=5; da1g=ta1*g1x*z1; da2g=ta2*g2x*z1; a1g=a1g+da1g*dt;a2g=a2g+da2g*dt;
    u=-a1g*g1x-a2g*g2x-k1*f1-k2*f2-k3*f3-k4*f4-k5*f5;

```

```

tp(i)=t;xp(i)=x;a1gp(i)=a1g; a2gp(i)=a2g;      dxp(i)=dx; x1dp(i)=x1d;z1p(i)=z1;
end
figure(1);plot(tp,xp,'k');xlabel('t/s');ylabel('state x');
figure(2);plot(tp,a1gp,'k');xlabel('t/s');ylabel('state a1g');
figure(3);plot(tp,a2gp,'k');xlabel('t/s');ylabel('state a2g');
figure(4);plot(tp,dxp,'k');xlabel('t/s');ylabel('derivative of state x');
figure(5);plot(tp,z1p,'k');xlabel('t/s');ylabel('z1');

```

And the simulation results are as follows:

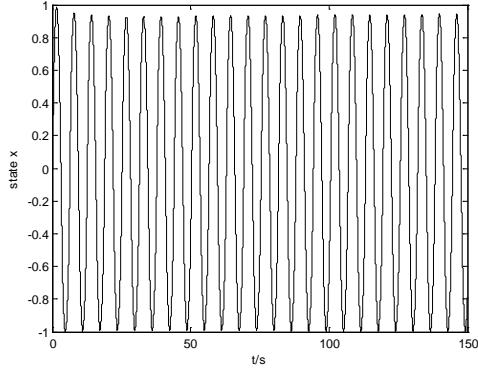


Fig.5 state x

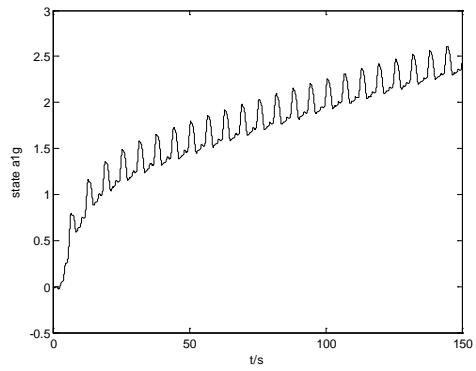


fig.6 state a1

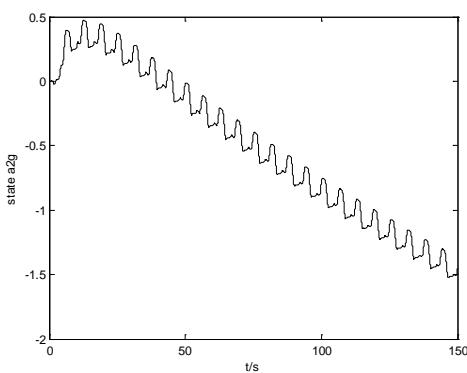


Fig.7 state a2

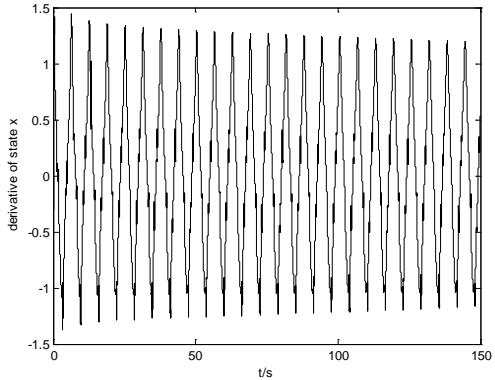


fig.8 state x

From the simulation results, we can see that the trend of unknown parameter identification is correct. According to the simulation results, when expected signal is a constant, system error can converge to zero quickly, but the result of parameter identification is not ideal, the main reason is that multiple parameter identification offset.

Conclusion

By analyzing theory and the simulation results, we can make a conclusion that the method of combining Terminal and Sigmoid function in this paper is effective to parameter identification. When the control gain is matched with identification gain, unknown parameter identification can success easily. But multiple parameter identification offset, the result is not ideal. The input signal should be considered.

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