Parametric analysis and design methods of Torsion thrust measurement system

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Abstract. Parametric analysis and of system design is the basic issue of torsion thrust measurement system. Based on parametric analysis of thruster size, weight and maximum thrust, as well as torsion damping ratio, vibration frequency and the torsional stiffness coefficient, the thesis proposes a method for parametric analysis and system design. Due to consideration of the key issues of thruster parameters, system parameter calibration and measurement time, the method proposed in this thesis is a practical method for engineering analysis and design and provides the basis for torsion system analysis and design.

Introduction

At present, international communities have developed a variety of new concepts space-borne micro-t hrusters, including advanced air-conditioning, plasma, Hall, field emission electric propulsion, colloids , plasma, laser ablation and other kind micro-thrusters, and the thrust they generates ranges from micr onewton to hundreds of millinewtons, and the micro impulse they generates ranges from micronewto n per second to millinewton per second[1,2].

When it comes to measurement and evaluation of microthrust and tiny impulse that micro thrusters ge nerated, torsion system is the system commonly used. In order to simulate the real space environment , micro-thrusters are usually installed on torsion system in a vacuum. For example, on condition of th ermal shock of high and low temperature, various operations and measurements are completed in a va cuum.

Within the parametric analysis and design of torsion thrust measurement, thrusters' size, weight, maxi mum thrust, the calibration convenience of system parameters and the time required in thrust measure ment process, are all needed to taken into consideration [3].

Parametric analysis of thrusters and torsion system

Figure 1 shows the physical map of torsion system. Figure 2 shows the schematic diagram of torsion system. The thruster is mounted on a horizontal torsion pendulum, and additional counterweight is mounted on the other side to maintain balance. When thrusters generate thrust, the torsion pendulum will pivot about the torsion axis. By measuring the torsion angle over time, i.e., measuring the response of the torsion system, the thrust is calculated by certain methods.

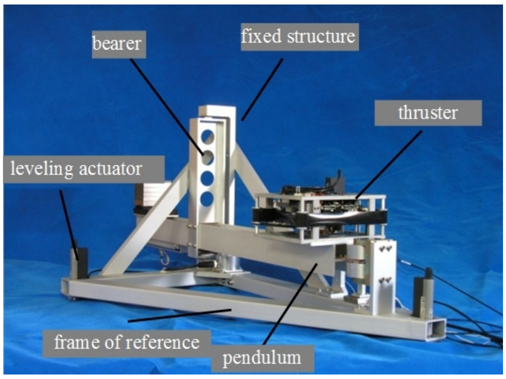

Fig. 1 the torsion thrust measurement system

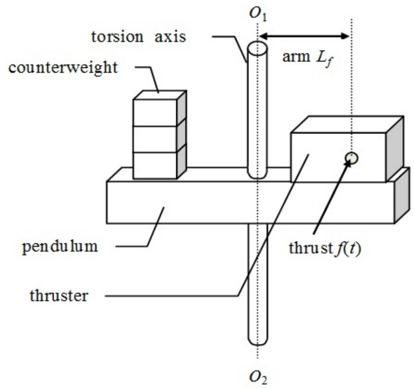

Fig. 2 schematic diagram of torsion thrust measurement system

The torsion system parameters include damping ratio, vibration frequency, torsional stiffness and moment of inertia, etc., and parameters of measured thrusters are as follows:

(1) Size and weight of thrusters. They impact thrusters' center gravity position, counterweight form, and structural rigidity selection of the pendulum, thus affecting the selection and design of inertia pendulum system.

(2) The magnitude of the measured thrust. The larger the measured thrust is, the greater is the torsion angle. In order to achieve high-precision torsion angle, torsion angle needs to be limited to a certain size range, thus affecting the selection and design of torsional stiffness in torsion system.

The torsion system parameters are within the following relationship:

(1) The undamped natural frequency depends on the torsional stiffness coefficient and moment of inertia. Set torsional stiffness coefficient as k and moment of inertia as J, then the undamped natural frequency of pendulum system $w_n = \sqrt{k/J}$.

(2) The damped vibration frequency depends on the damping ratio z , torsional stiffness and moment of inertia: $w_d = \sqrt{1-z^2}w_n = \sqrt{1-z^2}\sqrt{k/J}$.

(3) The damping ratio depends on damping coefficient, torsional stiffness and moment of inertia. Set the damping coefficient damping as c, then $z = c/(2\sqrt{Jk})$.

In engineering, z 、 w_d and k can be obtained according to system calibration response, and J can be converted by the equation $J = k/w_n^2$.

Analysis of system response and system parameters

Thrust is measured by achieved system response. System parameters determine the characteristics of the system response, thus affecting the thrust measurement.

Extreme torsion angle and system parameters

When external force $f(t)(0 \le t \le T_0)$ (T_0 is the reaction time) works, the torque torsion withstood is $M(t) = f(t)L_f$ (L_f is the moment arm). The torsional vibration equation is given by Equation (1).

$$\ddot{q} + 2zw_n\dot{q} + w_n^2 q = M(t)/J \quad 0 \le t \le T_0 \tag{1}$$

In the initial condition $q(0)=0$ and $\dot{q}(0)=0$, Equation (2) shows torsion angle changing over time (torsion system response), where， $f(0)=0$ and $q(0)=0$.

$$q(t) = \frac{L_f}{Jw_d}\int_0^t f(t)\mathrm{e}^{-zw_n(t-t)}\sin w_d(t-t)\mathrm{d}t \tag{2}$$

Step response method is commonly adopted method in torsion system parameter calibration. With the action of step torque $M(t) = A$ (constant), the response of torsion system is given below..

$$q(t) = \frac{A}{Jw_n^2} - \frac{A}{Jw_d w_n}e^{-zw_n t}\sin(w_d t + a) \text{ ,} \quad a = \arctan\frac{w_d}{zw_n} = \arctan\frac{\sqrt{1-z^2}}{z}$$

$$\frac{\mathrm{d}q(t)}{\mathrm{d}t}=\frac{A}{Jw_d w_n}e^{-zw_n t}[zw_n\sin(w_d t+a)-w_d\cos(w_d t+a)]=\frac{A}{Jw_d}e^{-zw_n t}\sin(w_d t)$$

where, $Jw_n^2=k$, obviously when $t\to\infty$, the steady torsion angle is $q(\infty)=A/k$.

when $\mathrm{d}q(t)/\mathrm{d}t=0$, the torsion angle get extreme values, then $\sin(w_d t)=0$. Equation (3) shows the time corresponding to extreme points.

$$t_{Mk}=n\pi/w_d \quad (n=0,1,2,\mathbf{L}) \tag{3}$$

Extreme points corresponding to the torsion angle are given as below.

$$q(t_{Mn})=\frac{A}{Jw_n^2}-\frac{A}{Jw_d w_n}e^{-n\pi z w_n/w_d}\sin(n\pi+a)$$

where, when $t_{M0}=0$, $q(t_{M0})=0$ is not an extreme point. When $n\geq 1$, The extreme torsion angle is given by Equation (4).

$$q(t_{Mn})=\frac{A}{Jw_n^2}-\frac{A}{Jw_d w_n}e^{-n\pi z w_n/w_d}(-1)^n\sin a=q(\infty)-(-1)^n q(\infty)e^{-\frac{n\pi z}{\sqrt{1-z^2}}} \quad (n=1,2,\mathbf{L}) \tag{4}$$

By measuring the extreme torsion angle value and time and using statistical mathematical methods, the vibration frequency can be calibrated according to the Equation (3) and damping ratio can be calibrated according to the Equation (4).

Steady torsion angles and system parameters

In the step response method, the torsional stiffness $k=A/q(\infty)$ is obtained on basis of measuring steady torsion angles, adopting statistical mathematical methods and known accurate calibration torque A. Therefore, the key issue is measuring the steady-state torsion angles.

Equation (5) shows torsion angles changing over time.

$$q(t)=q(\infty)-q(\infty)\frac{w_n}{w_d}e^{-zw_n t}\sin(w_d t+a) \tag{5}$$

Because

$$\int_{T_0}^{T_1}e^{-zw_n t}\sin(w_d t+a)\mathrm{d}t=-\frac{e^{-zw_n T_1}}{w_n}\sin(w_d T_1+2a)+\frac{e^{-zw_n T_0}}{w_n}\sin(w_d T_0+2a)$$

In time interval $[T_0,T_1]$, the mean torsion angles are given by Equation (6).

$$\begin{aligned}\bar{q}(t)&=\frac{1}{T_1-T_0}\int_{T_0}^{T_1}q(t)\mathrm{d}t=\\&=q(\infty)+q(\infty)\frac{e^{-zw_n T_1}}{w_d(T_1-T_0)}\sin(w_d T_1+2a)-q(\infty)\frac{e^{-zw_n T_0}}{w_d(T_1-T_0)}\sin(w_d T_0+2a)\end{aligned} \tag{6}$$

When $zw_n T_0>>1$ and $w_d(T_1-T_0)\geq 1$, $\bar{q}(t)\to q(\infty)$, we can equal the mean torsion angle with the steady torsion angle. Therefore, the estimated steady-state torsional angle value is shown in Equation (7), Where $q(t_i)$ is sampled n times in the time interval $[T_0,T_1]$.

$$\hat{q}(\infty)=\frac{1}{n}\sum_{i=1}^{n}q(t_i) \tag{7}$$

The design method of torsion system parameters

In torsion system parameters designing, it is necessary to consider the impact on thruster parameters, but also consider the impact on system response, including the impact on system parameter calibration and measurement.

Analysis of System Parameters Influencing Factors

Designing torsion system parameters are from two main factors.

(1)Thruster parameters. Thruster size and weight impacts the selection of moment of inertia, where moment of inertia can be preliminary estimated according to the torsion pendulum layout of thruster weight and counterweight.

(2)System parameter calibration. Before measuring thrust, the torsion system parameters need to be calibrated, including damping ratio, rotation frequency, and torsional stiffness, especially at high and

low temperatures in vacuum, for system parameters usually changing in different temperature.
Figure 3 shows the system response under a step force, which can be divided into two stages: 1) fluctuation stage, where the torsion angle needs to have several extreme points in order to calibrate the vibration frequency and damping ratio; 2) steady stage, where he torsion angle needs to be as smooth possible in order to accurately calibrate the torsional stiffness coefficient.

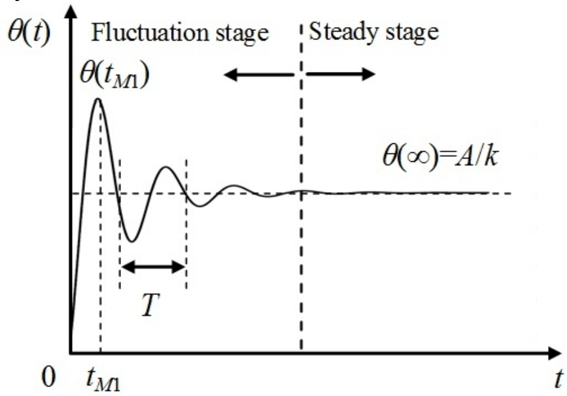

Fig. 3 system system response under a step force

Analysis and design of system parameters

Analysis and design of system parameters involves many factors about thruster parameters and system parameter calibration, which needs a comprehensive analysis and design.

Analysis of the damping ratio and vibration frequency calibration

According to the relationship between the extreme points of torsion angle and time, the following equation is get.

$$\left|\frac{q(t_{Mn})-q(\infty)}{q(\infty)}\right| = e^{-\frac{n\pi z}{\sqrt{1-z^2}}} \quad (n=1,2,\mathbf{L})$$

When the above value reaches 5%, the torsion angle basically reaches steady stage, Equation (8) is drawn, where the actual extreme torsion angle numbers prior to the steady stage are $n-1$ 。

$$z = \frac{(\ln 20/\pi)}{\sqrt{n^2+(\ln 20/\pi)^2}}, \quad n = \frac{\ln 20\sqrt{1-z^2}}{\pi z} \qquad (8)$$

Analysis of torsional stiffness

Torsional stiffness calibration requires $zw_nT_0 >> 1$ and $w_d(T_1-T_0) \geq 1$. Define $T_0 = K_0(2\pi/w_d)$, $T_1 = K_1(2\pi/w_d)$ (i.e. several times of vibration cycles) and $zw_nT_0 = 5$ （ $e^{-zw_nT_0} = 0.006738$ ）, then Equation (9) is drawn.

$$K_0 = \frac{5\sqrt{1-z^2}}{2\pi z}, \quad K_1 \geq K_0 + \frac{1}{2\pi} \qquad (9)$$

Figure 4 shows the relationship between numbers of extreme points prior to steady stage and damping ratio, where numbers of extreme points before steady stage decreases as damping ratio increases. Numbers of extreme points should be at least three and the damping ratio is constrained as $z \leq 0.2$ in order to calibrate the damping ratio and vibration frequencies in a single step response test.
Figure 5 shows the relationship between K_0 , K_1 and the damping ratio, where K_0 and K_1 all decreases with the damping ratio increases, which namely means that the torsional stiffness calibration consumed time prior to steady stage reduces. The damping ratio is limited to $z \geq 0.1$, when the measurement time is about eight times the vibration period, in order to calibrate the torsional stiffness and vibration frequencies in a single step response test [4].

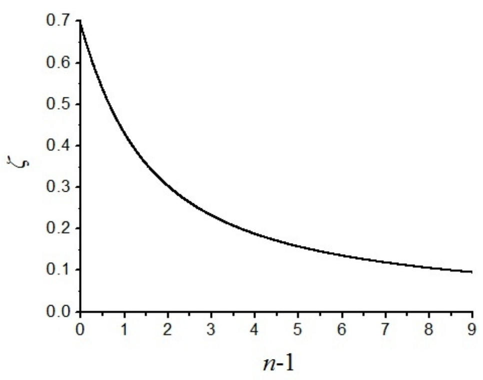

Fig. 4 relationship between numbers of extreme points ratio

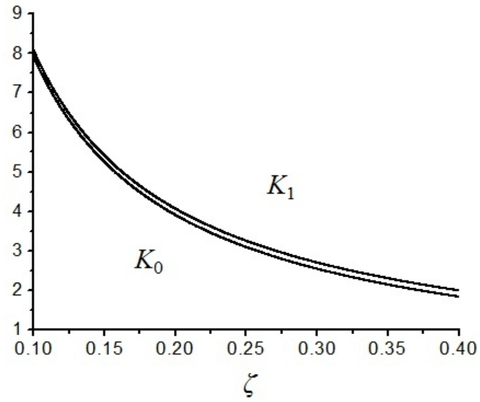

Fig. 5 relationship between K_0, K_1 and the damping

Obviously when designing system parameters, large damping ratio is not conducive to the calibration of damping ratio and vibration frequency and meanwhile small damping ratio is not conducive to the calibration of the torsional stiffness coefficient (coefficient of torsional rigidity), thus damping ratio is perfectly chosen in the range of $0.1 \le z \le 0.2$.

Analysis of thruster parameters and sensor measuring

Define the maximum thrust as $f_{\max}$, maximum range of displacement sensor as $q_{\max}$, thus the first extreme torsion angle (maximum) should meet

$$q(t_{M1}) = \frac{f_{\max} L_f}{k} \left(1 + e^{-\frac{\pi z}{\sqrt{1-z^2}}} \right) \le q_{\max}$$

Equation (10) can be drawn.

$$k \ge K_{k\max} \frac{f_{\max} L_f}{q_{\max}}, \quad K_{k\max} = 1 + e^{-\frac{\pi z}{\sqrt{1-z^2}}} \qquad (10)$$

If the damping ratio range chooses $0.1 \le z \le 0.2$, $K_{k\max} = 2.371276 \sim 2.898900$.

Analysis of system response speed

System response speed (response speed level) is usually described by the peak time, which is the time response curve required to reach the first peak. In step response method, the peak time is the $t_{M1} = \pi / w_d$, where w_d is given by Equation(11).

$$w_d = \sqrt{1-z^2} w_n = \sqrt{1-z^2} \sqrt{k / J} \qquad (11)$$

With the increase in torsional stiffness, vibration frequency increases and the peak time reduces, and the system response speed increases, which reduces the calibration time of system parameters $T_1 = K_1 (2\pi / w_d)$.

However, excessive torsion stiffness coefficient results in damping ratio decrease, departing from the scope $0.1 \le z \le 0.2$, which needs a comprehensive trade-offs [5].

Design methods of system parameters

Based on previous comprehensive analysis, the design method of torsion system parameters is as follows:

(1)According to the size and weight of thrusters as well as layout of thrusters and counterweight on pendulum, the moment of inertia J and arm L_f can be preliminary estimated.

(2) According to thrusters' maximum thrust and sensors' maximum range, the torsional stiffness coefficient range is determined as below.

$$k \ge 2.371276 \frac{f_{\max} L_f}{q_{\max}}$$

(3) Set up a desktop measurement system, within the torsional stiffness coefficient range, Select a flexure pivot its torsional stiffness coefficient is k (a part to provide torsional stiffness), calibrate damping ratio as z .

(4) If $z < 0.1$, increase k; If $z > 0.2$, decrease k until $0.1 \le z \le 0.2$.

Application examples

A thruster's maximum thrust, size and weight are all known: $f_{\max} = 10\text{mN}$, $50\text{cm}\times 50\text{cm}$, 15kg. According to the layout of thrusters and counterweight on pendulum, the moment of inertia is preliminary estimated as $J = 3\text{kg}\cdot\text{m}^2$. The arm is $L_f = 0.5\text{m}$. The sensor's maximum range is $q_{\max} = 0.5^{\text{o}}$. According to $f_{\max}$ and $q_{\max}$, the torsional stiffness coefficient range is determined as below.

$$k \ge 2.371276 \times \frac{(10\times 10^{-3})\times 0.5}{0.5\times(\pi/180)} \approx 1.36$$

The vibration frequency $w_d \approx w_n = \sqrt{k/J}$, and when $k = J$, the vibration frequency is approximately 1 and vibration cycle is approximately 2π s, which is in a acceptable level. Consequently, a flexure pivot $k = 3\text{N}\cdot\text{m/rad}$ is chosen.

After torsion system set up and system parameters calibration, as a result, the damping ratio is $z > 0.2$, which indicates k is relatively high, so a new flexure pivot $k = 2.6\text{N}\cdot\text{m/rad}$ is chosen.

After re-calibration of system parameters, the result shows $k = (2.6178 \pm 0.0627)\text{N}\cdot\text{m/rad}$, ω_d= （0.9206±0.0044）rad/s, and ζ=（0.1974±0.0042）, which indicates z meets the $0.1 \le z \le 0.2$ condition and system parameters design is reasonable.

Summary

By analysis of the relationship between torsion system parameters, thruster parameters and system response, the thesis proposes analysis and design methods of torsion system parameters, which have the following characteristics:

(1)Based on analysis of calibration requirement form the damping ratio, vibration frequency and the torsional stiffness coefficient, the damping ratio range $0.1 \le z \le 0.2$ is determined.

(2)Based on analysis of thrusters' size, weight and maximum thrust as well as sensors' maximum range, the design method of torsional stiffness coefficient is determined.

(3)Due to comprehensive consideration of thruster parameters and system parameters calibration problem, the design method of system parameters is a comprehensive optimization design.5 Summary

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