Free vibration of laminated composite plates by the various shear deformation theories

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Keywords: Vibration; Laminated composite plates; Shear deformation theories; Natural frequency.

Abstract. In this paper, the analysis of various higher-order shear deformation theories for the free vibration of laminated composite plates is presented. A Navier-type analytical method was used to solve the governing differential equations. Natural frequencies of simply supported laminated composite plates are calculated. The present results are compared with the available published results.

Introduction

The use of laminated composite has generally increased in weight sensitive applications such as aerospace and automotive structures due to their low maintenance cost and high strength-to-weight ratio. The vibration problem of laminated composite structures has attracted the attention of many researchers.

Sahoo and Singh [1] proposed a new trigonometric zigzag theory for the analysis of laminated composite and sandwich plates. Ngo-Cong et al. [2] presented a new effective radial basis function (RBF) collocation technique for the free vibration analysis of laminated composite plates using the first order shear deformation theory (FSDT).

In this paper, the theories of Touratier [3], Mantari [4], Karama [5], Levinson [6] are used to study the free vibration behavior of laminated composite plates. The governing differential equations are solved by a navier-type analytical method. The present results are compared with the available published results.

Governing equations and boundary conditions

Considering a plate of uniform thickness h. According to the higher order shear deformation theory, the displacement field is given as:

$$\begin{aligned} U &= u(x,y) - z\frac{\partial w(x,y)}{\partial x} + f(z)f_x(x,y) \\ V &= v(x,y) - z\frac{\partial w(x,y)}{\partial y} + f(z)f_y(x,y) \\ W &= w(x,y) \end{aligned} \tag{1}$$

The transverse shear function in Touratier [3] is:

$$f(z) = \frac{h}{p}\sin(\frac{pz}{h}) \tag{2}$$

The transverse shear function in Mantari [4] is:

$$f(z) = \sin(\frac{pz}{h})e^{\frac{1}{2}\cos(\frac{pz}{h})} + \frac{p}{2h}z \tag{3}$$

The transverse shear function in Karama [5] is:

$$f(z) = ze^{-2(z/h)^2} \tag{4}$$

The transverse shear function in Levinson [6] is:

$$f(z) = z(1 - \frac{4z^2}{3h^2}) \tag{5}$$

By substituting the displacement field into the strain–displacement relationships, the following strain components can be write as:

$$\begin{aligned}
e_x &= \frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2} + f(z)\frac{\partial f_x}{\partial x} \\
e_y &= \frac{\partial v}{\partial y} - z\frac{\partial^2 w}{\partial y^2} + f(z)\frac{\partial f_y}{\partial y} \\
g_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - 2z\frac{\partial^2 w}{\partial x \partial y} + f(z)(\frac{\partial f_x}{\partial y} + \frac{\partial f_y}{\partial x}) \\
g_{yz} &= \frac{df(z)}{dz} f_y \\
g_{xz} &= \frac{df(z)}{dz} f_x
\end{aligned} \tag{6}$$

By the principle of virtual displacements, the Euler–Lagrange equations can be:

$$\begin{aligned}
&\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_1 u_{,tt} + I_4 f_{x,tt} - I_2 \frac{\partial w}{\partial x}_{,tt} \\
&\frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = I_1 v_{,tt} + I_4 f_{y,tt} - I_2 \frac{\partial w}{\partial y}_{,tt} \\
&\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_y}{\partial y^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} = I_2 \frac{\partial u}{\partial x}_{,tt} + I_2 \frac{\partial v}{\partial y}_{,tt} + I_1 w_{,tt} - I_3 \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)_{,tt} + I_5 \left(\frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} \right)_{,tt} \\
&\frac{\partial M_x^f}{\partial x} + \frac{\partial M_{xy}^f}{\partial y} - Q_x^f = I_4 u_{,tt} + I_6 f_{x,tt} - I_5 \frac{\partial w}{\partial x}_{,tt} \\
&\frac{\partial M_y^f}{\partial y} + \frac{\partial M_{xy}^f}{\partial x} - Q_y^f = I_4 v_{,tt} + I_6 f_{y,tt} - I_5 \frac{\partial w}{\partial y}_{,tt}
\end{aligned} \tag{7}$$

Navier methods

The simply supported boundary conditions and the governing equations are satisfied by the following displacement functions.

$$\begin{aligned}
u &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} U_{mn} \cos(a\,x)\sin(b\,y)e^{iwt} \\
v &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} V_{mn} \sin(a\,x)\cos(b\,y)e^{iwt} \\
w &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn} \sin(a\,x)\sin(b\,y)e^{iwt} \\
f_x &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} X_{mn} \cos(a\,x)\sin(b\,y)e^{iwt} \\
f_y &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Y_{mn} \sin(a\,x)\cos(b\,y)e^{iwt}
\end{aligned} \tag{8}$$

where $\alpha = \frac{m\pi}{a}$, $\beta = \frac{n\pi}{b}$, w is the natural circular frequency. Substituting Eq. (8) into Eq. (7) and collecting the coefficients, we can obtain the following equation:

$$[K - w^2 M]\{\Delta\} = \{0\}, \{\Delta\}^T = \{U_{mn}, V_{mn}, W_{mn}, X_{mn}, Y_{mn}\} \tag{9}$$

The natural circular frequency w can be obtained by solving the eigenvalue equations (9).

Numerical examples

Here all layers of the laminated plates are the same material properties, the lamina properties are assumed to be:

$$\frac{E_1}{E_2} = 10, 20, 30, 40; G_{12} = G_{13} = 0.6E_2; G_{23} = 0.5E_2; v_{12} = 0.25; \rho = 1$$

The dimensionless natural frequencies are given by

$$\bar{w} = \frac{wa^2}{h}\sqrt{\frac{\rho}{E_2}} \tag{10}$$

Table 1 lists the non-dimensional fundamental frequency of the simply supported laminate plate of various modulus ratios of E_1/E_2 and side-to-thickness ratio a/h. It is found that the results are in very close agreement with the values of Reddy [7] and Liew [8] based on the FSDT.

Figs. 1 show the non-dimensional fundamental frequency of the simply supported square plate .

Table 1 The non-dimensional fundamental frequency of the simply supported square plate (0°/90°/90°/0°)

a/h	Method	E_1/E_2			
		10	20	30	40
5	Liew[7]	8.299	9.568	10.327	10.855
	P. Touratier	8.274	9.530	10.277	10.793
	P. Mantari	8.326	9.617	10.387	10.920
	P. Karama	8.281	9.542	10.292	10.811
	P. Levinson	8.272	9.526	10.272	10.787
10	Reddy[8]	9.853	12.383	13.892	15.143
	P. Touratier	9.830	12.221	13.868	15.113
	P. Mantari	9.864	12.270	13.945	15.214
	P. Karama	9.845	12.228	13.879	15.128
	P. Levinson	9.844	12.218	13.864	15.107

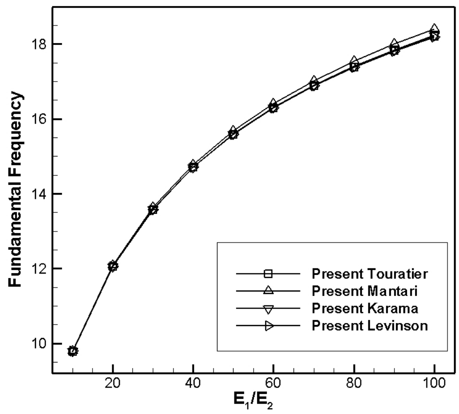

Fig.1 The non-dimensional fundamental frequency of the simply supported pate

Conclusions

In this paper, the analysis of various higher-order shear deformation theories for the free vibration of laminated composite plates is presented. A Navier-type analytical method was used to solve the governing differential equations. Natural frequencies of simply supported laminated composite plates are calculated. The results show that Mantari model produces the biggest results.

Acknowledgement

The Project is sponsored by “Liaoning BaiQianWan talents Program”(No.2014921048)

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