Evaluation on Influential Factors of Hydraulic Gear Pumps Wear Life Using FAHP

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Abstract. Considering that there are plenty of risk factors affecting hydraulic gear pumps wear life, multiple-factors assessment approach based on *FAHP* is proposed. Fuzzy consistent matrix is constructed by applying fuzzy theory into conventional *AHP* to improve the precision of judgments of decision-makers. The relative importance of the gear pump wear-life factors has been effectively analyzed by establishing the gear pump wear life evaluation model.

Introduction

The gear pump acts as one of the most critical power components of the hydraulic system, whose operation reliability and safety directly affect the service capability and steady operation of the entire system. Also, it determines the system health and remaining lifetime. With the development of the hydraulic transmission technology, gear pumps are progressing towards high-speed, high-pressure and high-reliability. However, the service life of gear pump has become the bottleneck to restrict its development, which depends largely on its wear life [1].

As the leakage of the hydraulic system will inevitably lead to the hydraulic oil contamination, the oil pollution has such a prominent influence on the hydraulic gear pump wear life that it cannot be ignored. Because of the complexity and stage of the abrasion mechanism, the influential factors of the gear pump wear life show polymorphism. Performing a reasonable and valid assessment on hydraulic gear pumps wear-life factors and their impacts has important theoretical and practical value for carrying out a contamination wear life test or researching the gear pump abrasion mechanism. In this paper, the fuzzy evaluation model for hydraulic gear pump wear life factors has been established by applying the Fuzzy Analytic Hierarchy Process (*FAHP*) with the uncertainty and vagueness of judgments fully considered. Through the decision-making analysis, we can calculate the relative impacts of different factors on the hydraulic gear pump performance.

Fuzzy Analytic Hierarchy Process Theory Research

Analytic Hierarchy Process (*AHP*) was developed by Saaty in 1981, which is the most common method of multiple-criteria decision making assessments and this method makes complex system assessment problems more hierarchical, simplistic and principled. However, due to the uncertainty and vagueness on judgments of the decision-makers, the pair wise comparison in the conventional *AHP* seems to be imprecise and insufficient to capture the right judgments of decision-makers. That is the reason why the fuzzy theory is introduced in the pair wise comparison of *AHP* to make up for this deficiency in the conventional *AHP*. Briefly, fuzzy theory and *AHP* are combined, referred to as fuzzy *AHP*.

Generally, *FAHP* is mainly divided into two forms: one based on the form of the membership functions and the other on a fuzzy consistent matrix. In *FAHP* theoretical analysis, Jijun Zhang [2] introduced the definition and properties of the fuzzy consistent matrix by analyzing the deficiency of traditional *AHP* method, employed the matrix as the pairwise comparison judgment matrix and finally discussed the principles and steps of *FAHP*. Yuejin Wu [3] proved some important properties of the fuzzy consistent matrix, obtained a concise priority formula of fuzzy complementary judgment matrix and improved the ordering principle of *FAHP*. In *FAHP* application, Ayag.Z and Ozdemir R G[4] chose the triangular fuzzy numbers function as membership function. According to the principle of productivity, flexibility, reliability, precision and other attributes, the scientific purchase decision on the choice of the machine tool was made by applying *FAHP* assessment method to the evaluation of machine tool alternatives. Huadong Yang and Hui Su [5] applied the *FAHP* method based on triangular membership function into the reducer design scheme to select the best scheme among many feasible ones, which showed that the method is valid for multi-object decision making. Xiaohu Chen, Yao Ding et al. [6] chose normal distribution function as membership function to establish the evaluation matrix, determined the weight of the eigenvector through the least squares method, and then made a health status assessment of hydraulic pump. Orlando Durán and José Aguiloa [7] proposed an analytic hierarchical process based on fuzzy numbers multi-attribute method to achieve the justification and evaluation of an advanced manufacturing system. An example of machine tool selection was introduced to illustrate the approach. Weiguo Li, Qian Yu et al.[8] put forward the transformer risk assessment method using *FAHP* and artificial neural network to increase the speed and accuracy of the assessment and provided feasible decision basis for transformer risk management and maintenance decisions as well. In this paper, *FAHP* method based on fuzzy consistent matrix is applied to evaluating the influential factors of hydraulic gear pumps wear life. The priorities of the impacts of these factors on gear pump contamination wear life are finally acquired through a quantity disposal.

Fuzzy judgment matrix. Whether *FAHP* can be applied into evaluating the research object effectively depends on the establishment of the fuzzy consistent matrix. Fuzzy judgment matrix R is used to compare the relative importance of the pairwise comparison between elements $a_1, a_2, \cdots, a_n$, namely

$$R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nn} \end{bmatrix} \tag{1}$$

Fuzzy judgment matrix is defined as follows

Definition 1[9].If the judgment matrix $R = \left(r_{ij}\right)_{n\times n}$, where $0 \le r_{ij} \le 1 (i = 1, 2, \cdots, n; j = 1, 2, \cdots, n)$, then it is called fuzzy matrix;

Definition 2[9].If the fuzzy matrix $R = \left(r_{ij}\right)_{n\times n}$, where $r_{ij} + r_{ji} = 1\left(i = 1, 2, \cdots, n; j = 1, 2, \cdots, n\right)$, then it is called fuzzy complementary matrix;

Definition 3[9].If the fuzzy complementary matrix $R = \left(r_{ij}\right)_{n\times n}$, where $\forall i, j, k$, $r_{ij} = r_{ik} - r_{jk} + 0.5$, then it is called fuzzy consistent matrix.

Before the establishment of the fuzzy consistent matrix, fuzzy complementary matrix based on actual assessment requirements is usually constructed, and then its consistency should be verified.

Firstly, normalizing rank aggregation method is applied to obtain the priorities of the fuzzy complementary matrix, referred to as the vector $\boldsymbol{\omega} = \left(w_1, w_2, \cdots, w_n\right)^T$, where

$$w_i = \frac{\sum_{j}^{n} a_{ij} + \frac{n}{2} - 1}{n\left(n-1\right)}, i = 1, 2, \cdots, n \tag{2}$$

Secondly, the consistency of the fuzzy complementary matrix should be verified.

Definition 4[10].The weight vector of the fuzzy complementary matrix R is

$\boldsymbol{\omega} = \left(w_1, w_2, \cdots, w_n\right)^T$, $\sum_{i=1}^{n} w_i = 1, w_i \ge 0$,let

$$w_{ij}=\frac{w_i}{w_i+w_j}\left(\forall i,j=1,2,\mathbf{L},n\right) \tag{3}$$

then matrix $w^*=\left(w_{ij}\right)_{n\times n}$ is called the eigenmatrix of the matrix R.

Definition 5[10].The fuzzy judgment matrix $A=\left(a_{ij}\right)_{n\times n}$ and $B=\left(a_{ij}\right)_{n\times n}$, let

$$I\left(A,B\right)=\frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|a_{ij}-b_{ij}\right| \tag{4}$$

which acts as the compatibility index between matrix A and B.

Concerning compatibility index, if $I\left(R,w\right)\le a$, the fuzzy complementary matrix R is considered to be consistent. In addition, the value of a depends on the decider's assessment attitude. The smaller a is, the stricter requirement the fuzzy judgment matrix consistency has. Usually the judgments can be considered acceptable if $I\left(R,w\right)\le 0.1$.

Establishment of the fuzzy complementary matrix. When evaluating decision-makings for a case, a judgment matrix must be constituted by comparing the relative importance of the factors between the pairwise factors. On the basis of the definition of the fuzzy matrix, to quantify the judgment, 0.1-0.9 scaling method shown in Table 1 is applied to perform the pairwise comparison. The numerical scales can be provided to quantitatively describe the membership.

Table1 Fuzzy numerical scales

Scale	Definition	Description
0.5	Equal important	Two factors are equally important compared with each other
0.6	Little important	Factor a_i is Little important compared with factor a_j
0.7	Obvious important	Factor a_i is obvious important compared with factor a_j
0.8	Intensively important	Factor a_i is intensively important compared with factor a_j
0.9	Extremely important	Factor a_i is extremely important compared with factor a_j
0.1,0.2,0.3,0.4	Converse comparison	If r_{ij} is obtained by comparing factor a_i and factor a_j,then r_{ji} obtained by comparing factor a_j and factor a_i is $1-r_{ij}$

Evaluation Model of hydraulic gear pump wear life

There are an increasing number of factors affecting the hydraulic gear pump wear life. Evaluating wear life factors is a multiple-criteria decision making problem with the presence of many quantitative and qualitative attributes. In order to quantify the gear pump wear life effectively, special researches on the physical mechanism of its contamination wear life should be conducted. According to the engineering practice and the expert opinions, the evaluation model of hydraulic gear pump wear life is created, as shown in Fig. 1. After constructing the hierarchy model, factors or indicators can be compared at a given level on a pair wise basis to estimate their relative importance in relation to the indicator at the immediate proceeding level, and Fig. 2 shows the process of *FAHP* assessment on the gear pump wear life factors.

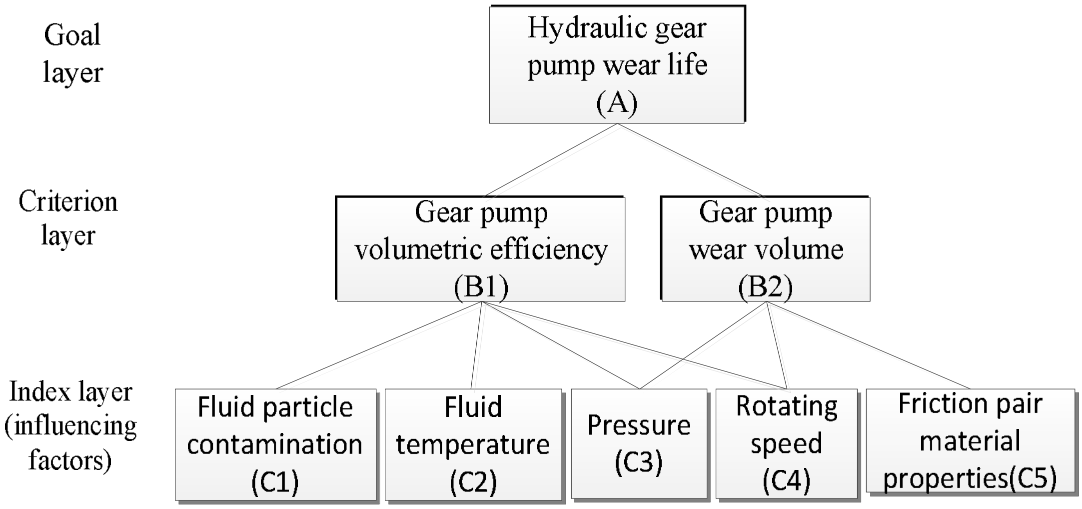

Fig 1 *AHP* evaluation model of gear pump wear life factors

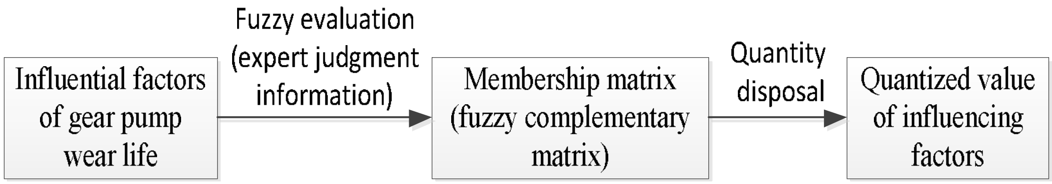

Fig 2 Process of gear pump wear factors assessment based on *FAHP*

FAHP-applied gear pump wear life factors assessment analysis

On the basis of the evaluation model given in Fig. 1 and the Fuzzy numerical scales method listed in Table 1, experienced experts were asked to compare the relative importance of the pairwise in relation to the indicator at the upper level. According to the different export opinions on the gear pump wear factors assessment case, the fuzzy comparison matrixes of pairwise comparisons for the factors or indicators are given in Table 2, Table 3, and Table 4.

Table 2 Relative importance of pairwise comparison at the index level relative to the indicator B_1

Experts	B_1-C	Fluid particle contamination	Fluid Temperature	Pressure	Rotating speed
Expert 1	Fluid particle contamination	0.5	0.7	0.6	0.8
	Fluid Temperature	0.3	0.5	0.4	0.6
	Pressure	0.4	0.6	0.5	0.7
	Rotating speed	0.2	0.4	0.3	0.5
Expert 2	Fluid particle contamination	0.5	0.8	0.7	0.8
	Fluid Temperature	0.2	0.5	0.4	0.6
	Pressure	0.3	0.6	0.5	0.6
	Rotating speed	0.2	0.4	0.4	0.5

Table 3 Relative importance of pairwise comparison at the index level relative to the indicator B_2

Experts	B_2-C	Pressure	Rotating speed	Friction pair material properties
Expert 1	Pressure	0.5	0.5	0.7
	Rotating speed	0.5	0.5	0.6
	Friction pair material properties	0.3	0.4	0.5
Expert 2	Pressure	0.5	0.6	0.7
	Rotating speed	0.4	0.5	0.6
	Friction pair material properties	0.3	0.4	0.5

Table 4 Relative importance of pairwise comparison at the criterion level relative to the indicator A

Experts	A-B	Volumetric efficiency	Wear volume
Expert 1	Volumetric efficiency	0.5	0.7
	Wear volume	0.3	0.5
Expert 2	Volumetric efficiency	0.5	0.8
	Wear volume	0.2	0.5

Taken in account the experts judgments on the pairwise comparison at the index level in relation to the gear pump wear life indicator of the volumetric efficiency, the fuzzy comparison matrixes of the wear-life factors in relation to the indicator B_1 are presented in matrix $\boldsymbol{R}_1$ and matrix $\boldsymbol{R}_2$.

$$R_1 = \begin{bmatrix} 0.5 & 0.7 & 0.6 & 0.8 \\ 0.3 & 0.5 & 0.4 & 0.6 \\ 0.4 & 0.6 & 0.5 & 0.7 \\ 0.2 & 0.4 & 0.3 & 0.5 \end{bmatrix}, \text{ and } R_2 = \begin{bmatrix} 0.5 & 0.8 & 0.7 & 0.8 \\ 0.2 & 0.5 & 0.4 & 0.6 \\ 0.3 & 0.6 & 0.5 & 0.6 \\ 0.2 & 0.4 & 0.4 & 0.5 \end{bmatrix}$$

For the fuzzy complementary judgment matrix R_1, the weight vector of which, as follow, can be obtained by Eq. 2.

$$\boldsymbol{\omega}_1 = (0.300, 0.233, 0.267, 0.200)$$

And then Eq. 4 is applied to establishing the characteristic matrix of the matrix R_1, which is given by

$$w_1^* = \begin{bmatrix} 0.500 & 0.563 & 0.529 & 0.600 \\ 0.437 & 0.500 & 0.466 & 0.538 \\ 0.471 & 0.534 & 0.500 & 0.572 \\ 0.400 & 0.462 & 0.428 & 0.500 \end{bmatrix}$$

Finally, we can get the compatibility index between matrixes R_1 and w_1^* according to Eq. 3, and such $I(R_1, w_1^*) = 0.0375 < 0.1$ .Therefore, the consistence of the fuzzy judgment matrix R_1 is considered acceptable, and the weight distribution of the matrix reasonable.

Similarly, for the fuzzy complementary judgment matrix R_2 based on the expert 2, its weight $\boldsymbol{\omega}_2$ and characteristic matrix w_2^* can be obtained as follows.

$$\boldsymbol{\omega}_2 = (0.317, 0.225, 0.250, 0.208)$$

$$w_2^* = \begin{bmatrix} 0.500 & 0.585 & 0.559 & 0.604 \\ 0.415 & 0.500 & 0.474 & 0.520 \\ 0.441 & 0.526 & 0.500 & 0.546 \\ 0.396 & 0.480 & 0.454 & 0.500 \end{bmatrix}$$

Because of the compatibility index $I\left(R_2,w_2^*\right)=0.095<0.1$ and $I\left(R_1,R_2\right)=0.0375<0.1$, it is concluded the fuzzy judgment matrix R_2 is consistent and that its weight distribution is feasible. In addition, the Fuzzy judgment matrixes R_1 and R_2 are considered to meet the requirement of the satisfactory compatibility. For real case analysis, the pairwise comparison judgment matrixes relative to the same indicator are given by different experts. If the compatibility of these matrixes and between them can be all verified, the mean value of weight sets can be taken as the finial comprehensive weight. Thus the respective weight of hydraulic gear pump wear-life factors relative to the index of the pump volumetric efficiency B_1, based on the two experts' judgment opinions, can be given by $\boldsymbol{\omega}_{B_1}=\left(0.3085,0.2290,0.2585,0.2040\right)$.

Using the same method above, the respective weight of hydraulic gear pump wear-life factors relative to the index of the pump wear volume B_2 is $\boldsymbol{\omega}_{B_2}=\left(0.3750,0.3415,0.2830\right)$, and the respective weight of hydraulic gear pump index level relative to the index of the hydraulic pump wear life A is $\boldsymbol{\omega}_A=\left(0.625,0.375\right)$.All the matrix can be verified to be compatible, and meet the consistency requirements. Finally, the absolute weight of the index layer (influencing factors) in relation to the target layer can be obtained by multiplying the weights of the corresponding level, which are listed in table 5.From the table 5, the absolute weight of hydraulic gear pump wear-life factors can be shown in Fig.3.

Table 5 Weight index of hydraulic gear pumps wear-life factors

Goal layer	Criterion layer	Respective weight of criterion layer	Index layer	Respective weight of the index layer	Absolute weight
Gear pump wear-life (A)	Volumetric efficiency (B1)	0.6250	Fluid particle contamination(C1)	0.3085	0.1928
			Fluid Temperature(C2)	0.2290	0.1431
			Pressure(C3)	0.2585	0.1616
			Rotating speed(C4)	0.2040	0.1275
	Wear volume (B2)	0.3750	Pressure(C3)	0.3750	0.1406
			Rotating speed(C4)	0.3415	0.1281
			Friction pair material properties(C5)	0.2830	0.1061

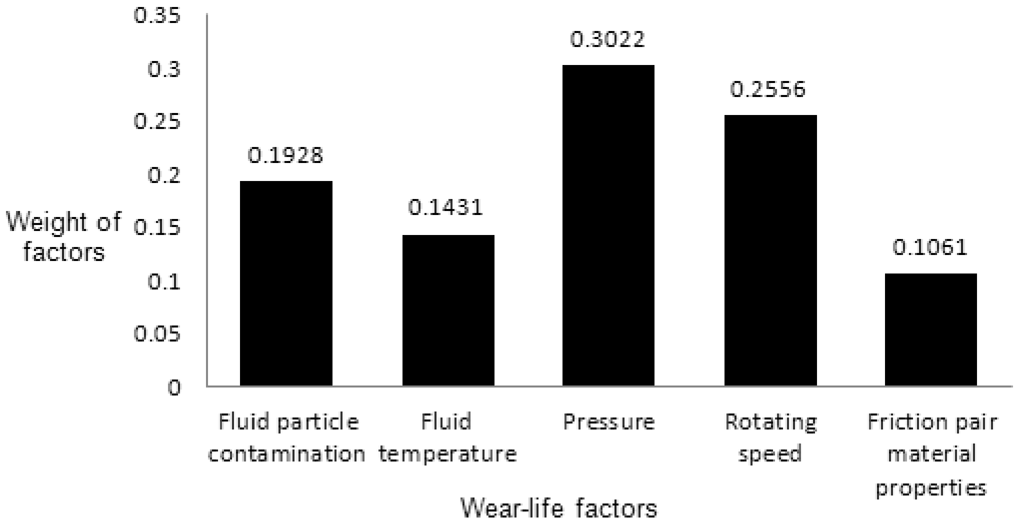

Fig.3 Weight of hydraulic gear pump wear-life factors

Conclusions

In this paper, a fuzzy *AHP* approach to evaluate the influential factors of hydraulic gear pump wear life is presented. In this approach, the fuzzy theory is introduced into the conventional *AHP* in order to improve the degree of judgments of decision-makers. Taking into account the uncertainty and vagueness of decision-makers, the relative importance of those factors is obtained by evaluating gear pump wear-life factors qualitatively and quantitatively based on *FAHP*. In other words, the order of the factors priority can be arranged. Fig 3 shows that the operating pressure has most significant impact on gear pump wear life, followed by its rotating speed and fluid particle contamination. Through effective judgments and feasible analysis, we can grasp the main factors leading to the deterioration or failure of the hydraulic gear pump to provide a scientific basis for the improvement of hydraulic pump life, and furthermore to enhance the reliability and service capability of the hydraulic system.

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