Recovery of Initial Conditions of Time-Varying CML with Symbolic Dynamics

Minfen Shen[1, a], Qiong Zhang[2, b], Lisha Sun[2]

[1]Shantou Polytechnic, Shantou, Guangdong, 515078, P. R. China

[2]Department of Electronic Engineering, Shantou University, 515063, P. R. China

[a]email: mfshen@stu.edu.cn, [b]email: qiongzhang@stu.edu.cn

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Abstract. A novel computationally efficient algorithm in terms of time-varying symbolic dynamics method is proposed to estimate the unknown initial conditions of coupled map lattices (CML). The presented method combines symbolic dynamics with time-varying control parameter to develop a time-varying scheme for estimating the initial condition of multi-dimensional spatial-temporal chaotic signals. The performance of the presented time-varying estimator is analyzed and compared with the common time-invariant estimator. Several simulations are carried out to show that the proposed method provides an efficient estimation of the initial condition of each lattice in the coupled system.

Introduction

Chaotic signals, generated from chaotic system have received much attention recently and been widely used in many engineering contexts such as physiology, weather forecast, neural network, economics and information processing [1-6]. Recent studies mainly focus on the limit of forward direction of chaotic maps while the initial process of iteration is always ignored.

Chaos provides a good model for many applications, but it does not fully exploit all the available information contained in the practical signals. The reason is that chaos is basically a temporal phenomenon which assumes that the spatial structure keeps unchanged. However, the spatial effects in nearby area cannot be ignored since many signals are generated from the spatial-temporal dynamics system and correlated with each other. When a system is spatially extended with effectively many degrees of freedom, spatial temporal chaos should be considered [3, 14, 16]. To investigate the spatial–temporal nonlinear dynamical behavior, we often use CML to model real systems. This problem has been of great interest in recent years [1, 3, 5, 14, and 16]. Consequently, the signal processing is always reduced to the problem of estimating the dynamical information of CML. If the map function of CML is known, the problem of signal estimation is equivalent to estimate the initial condition of the CML system in principle. Once it is available, the whole information embedded in the system can be estimated. Therefore, the estimation of the unknown initial condition of CML seems to be very important for spatial–temporal signal processing. Obviously, the higher the estimation precision, the better the description of the system is.

Due to the extreme sensitive to the initial conditions and parameters of the chaos, the initial conditions which approach to each other will separate exponentially, which cause errors propagation. In order to avoid error propagation and improve the estimation precision, some backward direction iteration technology, such as the dynamical programming implementation of ML estimation, the recursive ML algorithm and the halving method have been extensively studied[7-12]. On the other hand, the development of symbolic dynamics brings us some new light. In [13], a symbolic dynamics-based method is presented to provide an asymptotically unbiased estimation and attain the CRLB with high SNR. These algorithms are all based on the 1-D chaotic map, which ignore the spatial effects among the signals. When large number of individual chaotic lattices coupled to CML, however, the effect of the spatial coupled among the signals needs to be considered and the spatial-temporal system becomes a time-varying system. Both the control parameter and the coupling strength are changed with time at each lattice. Therefore, it is rather difficult to recover the initial condition of each lattice by using the 1-D chaotic map based algorithm directly. Only a coarse estimation based on statistical property of the whole lattices can be recovered [14].

In this paper, to provide an optimal estimation of unknown initial conditions for each lattice of the CML, an algorithm based on time-varying control parameter of logistic map which fully employs the information provided by the coupling term is presented. We propose a time-varying symbolic dynamics-based efficient method for estimating the initial conditions of the CML for spatial-temporal chaotic signal processing, which employs the backward iteration algorithm. Computed results are carried out to show that the proposed method is more efficient to estimate the initial condition for CML.

Model and Method

Symbolic dynamics is considered to be a coarse grained description of trajectories of a general class of systems, which keeps both robust and statistical properties of the system invariant [15]. It provides a model for the orbits of the dynamical system through a space of sequences. Obviously, the symbolic sequences depend on the initial condition of the trajectory. Based on the connection between the symbolic sequence and the initial condition of the CML system, an algorithm for estimating the unknown initial conditions of the CML system is derived.

Given a data set $\mathbf{X}$, the symbol sequence is achieved by quantifying $\mathbf{X}$ into a symbol. Let $\mathbf{E}=\{E_0,E_1,\cdots,E_{q-1}\}$ be a finite disjoint partition of a phase space $\mathbf{X}$, i.e. $\bigcap_{i=0}^{q-1} E_i=\mathbf{\Phi}$, $\bigcup_{i=0}^{q-1} E_i=\mathbf{X}$. The time series $S=\{0,1,2,\cdots,q-1\}$ is defined as a symbol set of the given system. Consequently, if data point $x(n)$ at time n of the system trajectory is the i-th element of the partition, we can assign a symbol $s(n)=i$, $i\in S$, followed by some partition rules. Considering 1-D onto logistic map, its phase space can be divided into two disjoint partition $E_0=[0,0.5]$ and $E_1=(0.5,1]$. So the symbolic sequence is obtained according to the following relationship:

$$s(n)=\begin{cases}0, & x(n)\in E_0\\ 1, & x(n)\in E_1\end{cases}. \tag{1}$$

Thus, for the given 1-D dynamical function, any orbit starts at $x(0)$ can be encoded as a symbolic sequence $S=\{s(0),s(1),\cdots,s(q-1)\}$. Let f_i be the dynamical function of 1-D chaotic map and we assume that it is invertible, which is denoted by f_i^{-1}. Since the map function is one-to-one for generating partition, the initial condition $x(0)$ of the CML system can be determined uniquely as $n\to\infty$ [14]:

$$x(0)=\lim_{n\to\infty} f_{s_{(0)}}^{-1}\circ f_{s_{(1)}}^{-1}\circ f_{s_{(2)}}^{-1}\circ\cdots f_{s_{(n-1)}}^{-1}(x(n)). \tag{2}$$

where the symbol $\circ$ denotes the intersection of the map. If the map is ergodic, then there always exists a generating partition. With the increase of n, equivalently, successive iterations of the map will provide increasingly more information of $x(0)$.

If there is no system coupling, the initial conditions estimation for CML is the same as that for 1-D individual map based on equation (2). However, in practical applications, a large numbers of individual lattices evolved by 1-D map are usually coupled with each other to construct a CML system. When multiple 1-D maps coupled to CML, both the control parameters and coupling strength have been changed at each lattice of the system. For the CML system with multi-channel signals, it is a time-varying system and the effect of the spatial coupled among the lattices needs to be properly considered. To overcome this limitation, the time-varying characteristics of the CML need to be properly described so that an optimal algorithm can be developed to estimate the unknown initial conditions of the spatial-temporal chaotic signal.

In this study, the well-known GCM (global coupled map) is employed as a model to analyze the problem of estimating the initial condition of the CML system, which is defined as [5]:

$$x_i(n+1) = (1-\varepsilon) f[x_i(n)] + \frac{\varepsilon}{L-1} \sum_{j=1, j\neq i}^{L} f[x_j(n)], \quad \varepsilon \neq 0 \quad . \tag{3}$$

where n is a discrete time step while i denotes the site index. The logistic map $f(x) = \lambda x(1-x)$ is used in this contribution in which λ represents the control parameter and ε denotes the coupling strength of the CML. L is the lattice size of the model. Without loss of generality, it is assumed that $\varepsilon \neq 0$ for the CML system. Moreover, the periodical verge condition is assumed as $x(i+L) = x(i)$.

Let $g(n)$ denote the coupling term among the lattices of the GCM system which is described as

$$g(n) = \frac{\varepsilon}{L-1} \sum_{j=1, j\neq i}^{L} f[x_j(n)] \qquad \varepsilon \neq 0 . \tag{4}$$

Based on the symbolic dynamics analysis, the true value of the initial condition $x_i(n)_{GCM}$ of the GCM system at *i*-th lattice can be obtained as

$$\hat{x}_i(n)_{GCM} = f^{-1}_{s_i(n)}[x_i(n+1)] = \frac{1}{2}\left[1 + [2s_i(n) - 1]\sqrt{1 - 4x_i(n+1)/\lambda_i(n)}\right]. \tag{5}$$

where $s_i(n)$ denotes the symbolic time series of the system and $\lambda_i(n)$ time-varying control parameter.

Test results

Fig.1 represents the distribution of the given data and the estimated data with both time-invariant and time-varying symbolic dynamics-based methods for the GCM system in noiseless environment. It shows that the behavior of the proposed time-varying scheme is much better than the time-invariant algorithm. We observe that the estimation errors are also symmetrically distributed at 0.5 and obtain the maximum at the threshold of 0.5. For the time-varying method, even if $x(n) \to 0.5$, the absolute error is still controlled within 0.015, showing its better fitness to the given signal.

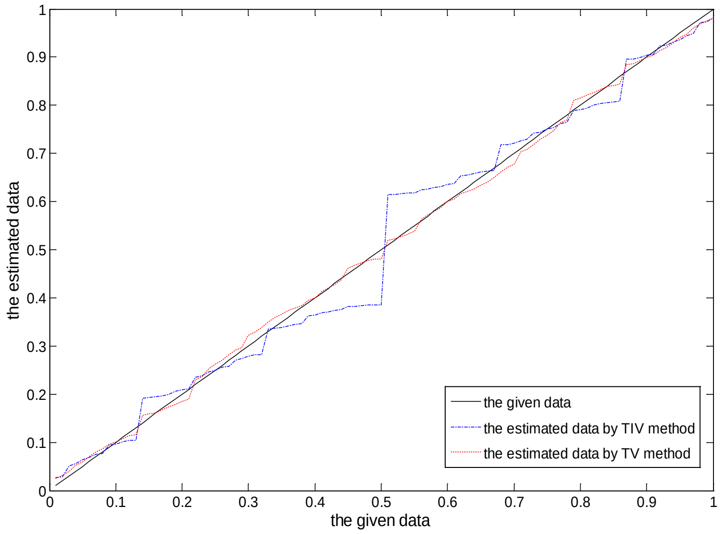

Fig.1. The estimation results using time-invariant (TIV) and time-varying (TV) methods.

Conclusion

In this paper, a novel method using time-varying control parameter is proposed to estimate the initial condition of CML generated by iterating logistic map. Based on the theory of symbolic dynamics, a computationally efficient algorithm for the purpose of estimating the unknown initial conditions of multi-dimensional spatial-temporal chaotic signal was developed. The computed results show that the proposed method can indeed give us a more accurate estimation of the initial conditions of the CML than the conventional approach. Both theoretical and experimental analyzes suggest that the proposed approach is suitable to carry out the inverse problem of spatial-temporal coupled system, which has a wide application in many engineering.

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