Hybrid PSO-SQP Algorithm for Solving System Reliability Allocation Optimization

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Abstract: System reliability allocation is an important ingredient in system reliability design, and it is also a decision-making issue of reliability engineering. To achieve the optimization of system reliability allocation, an optimization model for system reliability allocation, which takes the system cost as the objective function, is constructed through the general cost function. In order to overcome the shortcomings of particle swarm optimization (PSO) appearing in reliability allocation optimization, the premature and/or slow speed of convergence in later period, the sequential quadratic programming (SQP) was introduced to improve the PSO algorithm. The algorithm uses PSO as the global optimizer while the SQP is employed for accelerating the local search. Thus, the particles are able to search the whole space while searching for local optimization fast, which not only assures the convergence of the algorithm, but also increases the probability of obtaining the global optimum. Applied the algorithm to the problem of system reliability allocation, the simulation results show that it has excellent global search capability and provides rational optimization results compared to the existing approaches.

Introduction

System reliability allocation, which allocates the quantitative requirement of the system reliability to components in accordance with the given standard, aims to bring the integral and partial reliability requirements into line through allocation [1]. This kind of activity enables designers at all levels to clear the reliability design requirements of products, to estimate the techniques, labors, time and resources on the basis of the reliability design requirements, and to study the possibility and resolutions to meet the demands and achieve the tasks. System reliability allocation, optimizing the limited resources from an overall perspective, is an issue which takes the optimization into consideration.

The reliability of a system is dependent on by the reliability of the components. If the values of component reliability are reasonably allocated, the highest reliability of the system can be achieved under a system cost limitation. However, it is difficult to gain optimal reliability allocation of the components. All of the traditional optimization methods are based on the gradient so that it is difficult to solve the optimal allocation of system reliability. Recently, with the advent of artificial intelligence technologies, several meta-heuristics have been proposed and successfully applied for improving the quality of system reliability allocation. For example, Li et al. [2] proposed a genetic algorithm (GA) for solving the problem of complex systems reliability optimization. Pan et al. [3] presented a neural network approach for solving the reliability optimization problems of series-parallel systems with multiple constraints. Yuan [4] used the improved PSO (IPSO) to solve the problem of system reliability allocation, which gets better results comparing with the GA. At present, most of the attention to this issue has been given to the study of a single algorithm. In fact, the efficiency is different when different algorithms are used, so there is no need to compare what kind of algorithm is more excellent. Reliability allocation algorithm should have improved the efficiency of allocation synthesizing multiple algorithms, however, there are few studies aiming at the issue up to now [5].

In this work, a hybrid algorithm called PSO-SQP algorithm is presented to solve system reliability allocation, which takes the advantages of both PSO and sequential quadratic programming (SQP). This algorithm uses PSO as the global optimizer and accelerates the local search by employing the SQP. Thus, the particles are able to search the whole space while searching for local optimization fast, which not only assures the convergence of the algorithm, but also increases the probability of obtaining the global optimum. Applied the algorithm to the problem of system reliability allocation, the simulation results show that it has excellent global search capability and provides rational optimization results compared to the existing approaches.

System reliability allocation model

In practical applications, when a highly reliable system needs to be designed, two methods are often used to improve the system reliability: adding redundant components and increasing components' reliabilities. However, no matter what method is used, the reliability of the system must be greater than the system reliability index, and it can be formulated as follows:

$$f(R_1, R_2, ..., R_i, ..., R_n) \geq R_s \ , \tag{1}$$

where R_s is the system reliability goal, R_i is the component reliability for the system, f is the functional relationship between component and system reliability.

Consider a system consisting of n components. The objective is to allocate reliability to all or some of the components in system, in order to meet goal with a minimum cost. The problem is formulated as a nonlinear programming problem as follows [6]:

$$\begin{aligned} &\min C = \sum_{i=1}^{n} C_i \ (R_i) \\ &s.t. \ f(R_1, R_2, ..., R_i, ..., R_n) \geq R_s \ , \\ &R_{i,\min} \leq R_i \leq R_{i\max}, i = 1, 2, ..., n \end{aligned} \tag{2}$$

where C is the total system cost, C_i is the cost of component, $R_{i,min}$ is the minimum reliability of component, $R_{i,max}$ is the maximum achievable reliability of component. This formulation is used to achieve a minimum total system cost, subject to R_s, a lower limit on the system reliability.

The next step is to obtain a relationship for the cost of each component as a function of its reliability. An empirical relationship can be derived from past experiences and/or data for similar components. In many cases however, such data is not available. In order to overcome this problem, a general behavior of the cost function is proposed, as follows:

$$C_i(R_i; f_i, R_{i,\min}, R_{i,\max}) = \exp[(1 - f_i)(R_i - R_{i,\min})/(R_{i,\max} - R_i)] \ . \tag{3}$$

This is an exponential behavior, where f_i is the feasibility of increasing a component's reliability, and it assumes values between 0 and 1. It can be seen that the cost function is easy to implement, with only two required inputs (in addition to the failure distribution of the component), namely the feasibility and the maximum achievable reliability.

Hybrid PSO-SQP algorithm

Overview of PSO and SQP algorithms

PSO is one of population-based evolutionary optimization methods inspired by natural concepts such as fish schooling, bird flocking and human social relations [7]. In PSO, each particle has its own position and velocity, and a potential solution to an optimization problem is represented by the position of one particle. The velocity of each particle is updated according to the following two best position every iteration. The first one is obtained so far by itself, which can be denoted as $P_i=(p_{i1},p_{i2},\ldots,p_{id},\ldots,p_{iD})$ for the i-th particle in the D-dimensional search space. The second one is obtained so far by any particle in the whole swarm, which can be represented by $P_g=(p_{g1},p_{g2},\ldots, p_{gd}, \ldots, p_{gD})$. Assume that the position vector and velocity vector of the i-th particle can be denoted as $X_i=(x_{i1},x_{i2},\ldots,x_{id},\ldots,x_{iD})$ and $V_i=(v_{i1},v_{i2},\ldots,v_{id},\ldots,v_{iD})$, respectively. The velocity and position of each particle modifying rule are given below:

$$v_{id}^{t+1} = \omega v_{id}^{t} + c_1 r_1 (p_{id}^{t} - x_{id}^{t}) + c_2 r_2 (p_{gd}^{t} - x_{id}^{t}), \tag{4}$$

$$x_{id}^{t+1} = x_{id}^{t} + v_{id}^{t+1}, \tag{5}$$

where t is the iteration number, c_1 and c_2 are constants called acceleration coefficients, r_1 and r_2 are two independent random numbers uniformly distributed in the range of [0, 1], ω is called the inertia factor.

Recently, PSO has been found to be a promising technique for optimization problems [8]. Compared to GA, PSO takes less time for each function evaluation as it does not use any of GA operators like mutation, crossover and selection operator. Although PSO has shown some advances by providing high speed of convergence in specific problems, however, it does exhibit some shortages. First, it may convergence to a local optimum when facing with complex optimization problems. Second, the convergence rate decrease considerably in the later period of evolution; when reaching a near optimal solution, the algorithm stops optimizing, and thus the achieved accuracy of the algorithm is limited.

The SQP method is considered to be the best nonlinear programming method for constrained optimization [9]. It outperforms other nonlinear programming methods in terms of efficiency, accuracy, and percentage when making successful solutions to a large number of testing problems. The method resembles closely to Newton's method for constrained optimization just as is done for unconstrained optimization. At each iteration, an approximation is made of the Hessian of the Lagrangian function using a Broyden–Fletcher–Goldfarb–Shanno (BFGS) quasi-Newton updating method. This is then used to generate a quadratic programming (QP) sub-problem whose solution is used to form a search direction for a line search procedure. Although this optimizing method is less time consuming than the population-based search algorithms, it is highly dependent on the initial estimate of the solution.

The hybrid of PSO and SQP

In this paper, a hybrid algorithm of PSO-SQP algorithm is presented to improve the convergence ability of PSO. The PSO algorithm is a global algorithm, which has a strong ability to find global optimistic result. However, it has a disadvantage that the search around global optimum is very slow. The SQP algorithm, on the contrary, has a strong ability to find local optimistic result, but its ability to find the global optimistic result is weak. Although it has advantages in terms of computational robustness and their usefulness for practical problems, it is usually difficult to choose appropriate initial solutions. Therefore, the PSO-SQP algorithm can not only inherit the excellent diversity maintenance of PSO, but also supplement the higher convergence speed and better accuracy by SQP. The basic idea can be summarized as: First, PSO algorithm is run to search the global best position in the solution space. Then SQP algorithm is used to search around the global optimum. The procedure for this PSO–SQP algorithm can be summarized as follows:

Step 1: Initialize the positions and velocities of a group of particles randomly.

Step 2: Perform the PSO algorithm, evaluate each initialized particle's fitness value.

Step 3: The best particle of the current particles is stored. If the change of the current best particle fitness value is smaller than a predefined value, go to step 5, else continues.

Step 4: The positions and velocities of all the particles are updated, and then a group of new particles is generated, go to step 2.

Step 5: Use SQP algorithm to search around global best, which is found by PSO to find finer solutions. In this case, the best solution obtained by PSO is considered as the initial guess for SQP algorithm.

Step 6: The termination is done when there is no improvement in the solution for a specified number of iterations.

Numerical simulation

In this section, we give five reliability allocation experiments to verify the effectiveness of PSO-SQP algorithm. In order to make a comparison, all examples are chosen from literature [4]. Details of the implementation are described as follows: make the following assumptions. 1) The system's reliability function can be obtained. 2) All systems consist of s-independent components. 3) The system and its components have two states, failure and safety. 4) The overall system cost is the summation of individual component costs. The initial reliability of all components is 0.8, and the system reliability of R_s is greater than 0.90.

For the four units of simple tandem system calculate system component assigns value under the different feasibilities and the maximum reliability conditions. The equation for the system reliability is given by, $R_s = R_1 * R_2 * R_3 * R_4$. $Cost = C_1 + C_2 + C_3 + C_4$.

Cases 1: $R_{i,max}$ =0.99, f_i =0.2 (i=1, 2, 3, 4).

Cases 2: $R_{i,max}$ =0.99 (i =1,2,3,4), f_1=0.1, f_2=0.4, f_3=0.6, f_4=0.9.

Cases 3: $R_{1,max}$=0.999, $R_{2,max}$=0.995, $R_{3,max}$=0.99, $R_{4,max}$=0.985; f_i =0.9 (i =1, 2, 3, 4).

Consider the system shown in Figure 1. The equation for the system reliability is given by,

$$R_s = 1 - \{[(1-R_1)(1-R_4)]^2 R_3 + [1 - R_2 + R_2(1-R_1)(1-R_4)]^2 (1-R_3)\} \quad . \quad Cost = 2C_1 + 2C_2 + C_3 + 2C_4 \, .$$

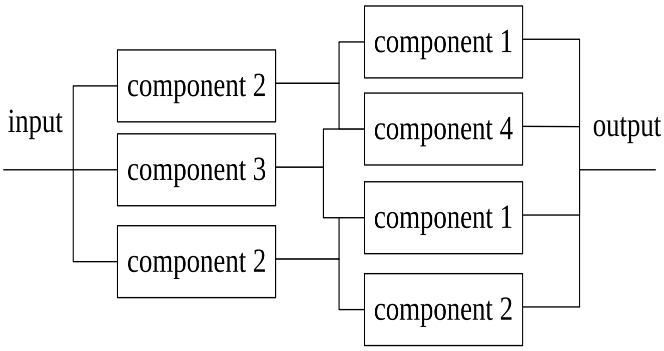

Fig 1 A complex system structure

Cases 4: $R_{i,max}$=0.9(i =1, 2, 3, 4), f_1=0.1, f_2=0.4, f_3=0.6, f_4=0.9.

Cases 5: f_i =0.5(i =1, 2, 3, 4), $R_{1,max}$=0.99, $R_{2,max}$=0.95, $R_{3,max}$=0.90, $R_{4,max}$= 0.85.

Using PSO-SQP algorithm to calculate the reliability allocation, the results for cases 1 through 5 are summarized in Table 1. In order to compare with the results of the IPSO, the results are put in parentheses [4].

Table 1 Summary table for the 5 cases.

	Case1	Case2	Case 3	Case 4	Case 5
R_1	0.9740 (0.9739)	0.9636 (0.9640)	0.9802 (0.9784)	0.5000 (0.5026)	0.6261 (0.6446)
R_2	0.9740 (0.9736)	0.9708 (0.9707)	0.9765 (0.9771)	0.5641 (0.5655)	0.6393 (0.6212)
R_3	0.9740 (0.9740)	0.9762 (0.9764)	0.9720 (0.9727)	0.6674 (0.6690)	0.6445 (0.6699)
R_4	0.9740 (0.9745)	0.9856 (0.9851)	0.9674 (0.9678)	0.7266 (0.7305)	0.5267 (0.5111)
$Cost$	24065 (24194)	701.97 (713.17)	472.33 (483.98)	7.8555 (7.8719)	8.2921 (8.3223)

It is clear from Table 1 that the total cost of system obtained by the hybrid algorithm is comparatively less compared to the IPSO. At the same time, calculated the reliability allocation problems in literature [2], the simulation results show that it has excellent global search capability and provides more rational optimization results in comparison to the GA. Therefore, it is feasible and effective to solve the reliability allocation problem of complex systems by PSO-SQP algorithm.

Conclusions

In this paper, in order to overcome the shortcomings of PSO appearing in reliability allocation optimization, the premature and/or slow speed of convergence in later period, the hybrid PSO-SQP algorithm is presented. The hybrid algorithm has the advantages of both PSO and SQP algorithm while does not inherent their drawbacks. Applying the algorithm to the issue of system reliability allocation, the simulation results show that it has excellent global search capability and provides more rational optimization results comparing with the IPSO. We may draw the conclusion that it is feasible and effective to solve the problem of system reliability allocation by PSO-SQP algorithm.

Acknowledgments

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