Application of Multiple-Scale Analysis In Eco-hydrodynamic Simulation

Combination and comparison between different mathematic models

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***Abstract*—In the simulation of ecological formation process, as the complexity of the environment increases, only one model is usually not enough to correctly predict the change. In this program, we introduced another way--- the multi-scale analysis to simulate the formation process by combining two models together. It focuses on the formation process of the hypoxic zone which leads to the death and even extinction of many plant and fish species in the Gulf of Mexico. This project combines differential equation model and cellular automaton and applied multi –scale method to simulate the formation of the hypoxic zone in the Gulf of Mexico.**

Keywords-multiple-scale analysis; cellular automata; differential equation; hypoxic zone

I. Introduction

The goal of our program is to simulate the occurrence of the hypoxic zone by building a Cellular Automaton, combining with another model using differential equation and difference equation.

A cellular automaton is a discrete model studied in complexity theory, which consists of a regular grid of cells, each in one of a finite number of states, such as on and off (in our research it's a certain number representing the number of species). The grid can be in any finite number of dimensions. For each cell, a set of cells called its neighborhood is defined relative to the specified cell. A initial state (time t=0) is selected by assigning a state for each cell. A new generation is created (advancing t by 1), according to some fixed rule (generally mathematical functions) that determines the new state of each cells in terms of the current state of the cell and the states of the cells in the neighborhood.[1]

Application of differential and difference equation is very extensive and it's also an essential part in the simulation. Although the research on them is focused on different areas, in this case we mainly focus on the solution to these equations.

The innovation of our research is that we create the method of multi-scale analysis in the simulation, which can be applied to many different cases with different complexity, and, at the same time, increase the efficiency of the simulation.

The model we research on is based on the Gulf of Mexico, which is located between the south-east coast of the USA and east of Mexico. Due to the increasing demand of fresh water, energy, and food supply, the Gulf of Mexico eco-system is deteriorating in a unprecedented speed. The lethal damage to the environment is the application of modern agriculture techniques, which leads to the outbreak of red tide and the death of fish living in the Gulf of Mexico. In order to resolve this serious ecological damage, we need to know specifically how and where the hypoxic zone occurs.

We use five different variables to simulate the process. The ultimate goal of this project is to determine the possible location and shape of future hypoxic zone. Therefore, we divide this project into three different parts: building a digital map, building a mathematic model, and building a program of Cellular Automaton to stimulate the formation of the hypoxic zone.

II. Materials and Method

A. *Create a Mathematical Model*

Different from traditional mathematical models, our research will focus on the change of the density of the fish. We define 5 variables in our mathematical model:

1. Fish, which is represented by "F", consumes phytoplankton (plant) and oxygen;

2. Plant, which is represented by “P”, consumes oxygen, but produces more oxygen by photosynthesis. It also consumes nutritive salt and provides food for the fish and microorganism;

3. Oxygen, which is represented by “O”, is consumed by all organisms;

4. Microorganism, which is represented by “M”, consumes oxygen and decomposes plants, thus producing nutritive salt.

5. Nutritive salt, which is represented by “NU”, promotes the growth of the plants. The main elements in the nutritive salt are nitrogen and phosphorous.[2]

6. We create a picture to clearly describe the relationship among the five variables mentioned above.(A)

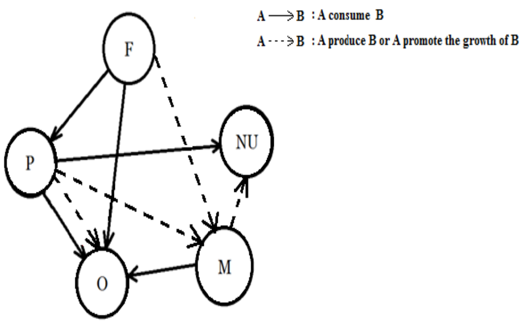

FIGURE I. THE RELATIONSHIP AMONG THE VARIABLES

Actually the influence on the ocean species is not solely determined by these five variables. The reason that we choose only these five is that they can best represent the interaction between ocean species and, in the matter of ecosystem dynamics, to pursue the integrity of a system by blindly adding variables will significantly reduce the liability of the model.

B. Relationship among the Five Variables

We assume that in a typical area in the ocean, the five variables influence one another along the change of time in a particular way. Thus, we can deduce the approximate changing condition of the organisms in the whole marine system by considering the relationship among these five variables in this particular region.

Therefore, we start to look for the relationship among the five variables and attempt to demonstrate it with mathematical expressions.

A correct mathematical equation should be able to describe these complicated organic processes in a simplest form. In a biological model, many variables have a nonlinear relationship so that it is difficult to find the accurate mathematical formula which describes these processes. However, in most cases, these empirical equations are based on the fitting results that are observed in some marine region. Strictly speaking, they are only suitable for some particular marine regions and time periods. When the official documents are not available, choosing any of the empirical equations blindly may also affect the reliability of the model. The principle is: If there exists several mathematical equations, we should choose the easiest expression with the least coefficients [3]

C. Derivation Process

1) Fish: First, we consider the main research subject -fish.

We assume that the growth rate of fish is proportional to its feeding——the consumption of fish is equal to its reproduction (baby fish). However, of course, it’s impossible for the fish to absorb all the nutrition of the plants; hence, we define a constant “α” to represent the absorptivity of the nutrition. Also, fish cannot be fed limitlessly. The plants that are fed will reach a saturation value and the amount of food must be limited by the total amount of plants, thus we use such an expression to represent this kind of relationship: The amount of food $=\alpha*[1-e^\wedge(\lambda P)]*F$ (which is also called the Ivlev equation). λis the semi-saturation coefficient of the herbivorous fish. P is the amount of plants and F is the amount of fish. At the same time, fish will also die, thus we assume that some part of the fish population will naturally die after a unit of time. Assume the death rate as β. We use a differential equation to express the natural growth rate of fish, which is represented by the equation below[3].

$$\frac{dF}{dt}=\alpha*[1-e^\wedge(\lambda P)]*F-\beta F \quad (1)$$

At the same time, we assume the amount of fish is proportional to the amount of oxygen. More generally, we assume all the organisms in the marine system will consume certain amount of oxygen that is proportional to the amount of the organisms. So that how the amount of oxygen will change relative to the previous unit of time is expressed below:

$$\frac{dF}{dt}=\alpha*[1-e^\wedge(\lambda P)]*F-\beta F+\tau\frac{O-O_c}{O_c}*F \quad (2)$$

2) Plants: First we consider how the fish is fed on plants. We only need to subtract the amount of food calculated before:

$$\frac{dP}{dt}=-\alpha*[1-e^\wedge(\lambda P)]*F \quad (3)$$

Similarly, add the effect of the amount of oxygen to the amount of plants:

$$\frac{dP}{dt}=-\alpha*[1-e^\wedge(\lambda P)]*F+\tau\frac{O-O_c}{O}*P \quad (4)$$

The growth of plants is decided by the effect of both photosynthesis and nutritive salt, thus we use the Michaelis-Menten equation to describe the nutritive salt-autotrophic phytoplankton model in this biological system. In the equation, f(I) is the function of light intensity to control the photosynthesis process. Vm is the maximum growth rate of the phytoplankton. K_n is the semi-saturation concentration in the process of assimilation of nutritive salt.

$$\frac{dP}{dt} = -\alpha * [1 - e^{\wedge}(\lambda P)] * F + \tau \frac{O - O_c}{O} * P + f(I) * \frac{V_m N}{K_N + N} P \tag{5}$$

Finally we subtract the amount of natural death of plants. δis the natural death ratio of the plants:

$$\frac{dP}{dt} = -\alpha * [1 - e^{\wedge}(\lambda P)] * F + \tau \frac{O - O_c}{O} * P + f(I) * \frac{V_m N}{K_N + N} P - \delta P \tag{6}$$

3) Oxygen: Marine organisms consume oxygen.

$$\frac{dO}{dt} = -\varepsilon_f F - \varepsilon_p P - \varepsilon_m M + \gamma P \tag{7}$$

In the equation, ε_f,ε_p,ε_m represent the speed of oxygen consumption of fish, plants and microorganisms, respectively. γrepresents the speed of production of oxygen by photosynthesis.

4) Microorganism: Microorganism is released mainly by the death of plankton and the physiological reaction by secretion. Now we assume the source of microorganism is mainly the three below:

1. The death of fish;

2. The death of plants;

3. The remain of the food of fish that they don't absorb completely.

Based on the above we derive the formula:

$$\frac{dM}{dt} = \mu\{\beta F + \delta P + (1 - \alpha) * [1 - e^{\wedge}(\lambda P)]\} \tag{8}$$

In the equation, μ is the ratio of change from fragment to microorganism. The way that oxygen affects the process is the same as above:

$$\frac{dM}{dt} = \mu\{\beta F + \delta P + (1 - \alpha) * [1 - e^{\wedge}(\lambda P)]\} + \tau \frac{O - O_c}{O} * M \tag{9}$$

5) Nutritive salt: Similarly, we use Michaelis-Menten equation to describe the nutritive salt-autotrophic phytoplankton model in this biological system. However, microorganisms can also decompose to produce nutritive salt. [2]

$$\frac{dN}{dt} = -f(I) * \frac{V_m N}{K_N + N} P + \omega M \tag{10}$$

In the equation, ω represents the ratio of decomposition of microorganisms.

After we finish the mathematical model, the next step is to find out the parameter and simulate the specific changing process by the program.

D. The Adjustment of Parameter

We have searched for some papers on the Internet and synthesized the parameters in these papers. Because there are not enough experiments to investigate the correct parameters, the parameters are not absolutely correct but they are the parameters most suitable for the Mexico Gulf.

TABLE I. ADJUSTMENT OF PARAMETER

Symbol	**Meaning**	**Value**
α	Fish's absorptivity of nutrition	0.6
λ	Semi-saturation coefficient of fish	0.035
β	Natural death rate of the fish	0.005
F(I)	Coefficient of light intensity	0.8
V_m	Maximum growth rate of phytoplankton	0.2
K_N	Semi-saturation concentration in assimilation of nutritive salt	0.05
δ	Natural death rate of the plants	0.005
ε_f	The speed oxygen consumption of fish	0.1
ε_p	The speed of oxygen consumption of plant	0.1
ε_m	The speed of oxygen consumption of microorganism	0.1
γ	Production speed of oxygen by plants	0.15
μ	The rate of change of fragments that change into microorganism	0.15
ω	The rate of decomposition of microorganism	0.8
τ	The influence rate of oxygen on organisms	0.1
O_c	Standard constant of oxygen	1.0

E. Multi-Scale Numerical Simulation

What we simulate is the changing number of species in a particular marine region. We know that in the marine system, different regions are affected by different factors, thus the condition of the species is also different. With that in mind, we use the cellular automaton system, which distributed the cells in the uniform gridding to consist of the derivation of the dynamic system through simple interaction. Therefore, the process of simulation is mainly divided into two parts: 1. the simulation of species in each cell internally; 2. the interaction among the cells.

We use the floating point map which is made in Arcgis and use different colors to represent the points in Delphi. The map is cut into units of 140*40, and use 1,2,3,4 to represent land, ocean, river and origin of river, respectively. In a Programming Software, we first use memo to guide into the map in txt format, and then compile a program to cut the character string in each line. Finally we represent each type of character by different colors, which will be revealed in figure B.

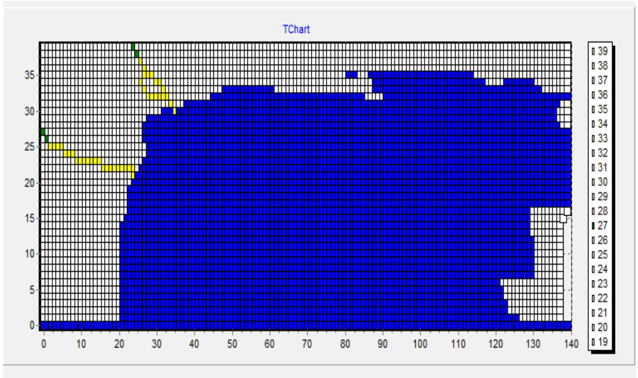

FIGURE II. MAP WITH DIFFERENT COLORS REPRESENTING DIFFERENT LANDSCAPES

Here is our simulation method:

1) *The simulation of density change inside the cell (application of the mathematical model using differential and difference equation):* We divide the whole gulf area into units of 140*40, and each of these small areas represents a cell. Inside the cell are five species interacting with each other. The way they interact is represented by the mathematical model mentioned before. In the program we first assign initial values to each species, then, according to their rate of change, construct a mathematical model.

2) *The interaction between cells (application of cellular automata):* Due to the irregular motion of the molecules, the distributions of marine species have gone on an irreversible process from uniformity to non-uniformity, which is our basic principle in the simulation process. To stabilize the result of the simulation, we assume that there is enough time for marine species to go through this process. As the simulation begins, we assume that the amounts of variables in all the cells are equal. Firstly we choose a cell randomly, and then, randomly choose another cell around the cell (upward, downward, leftward and rightward) that we initially chose and distribute the amount of species in the two cells at a particular ratio, which we need to do experiments to figure out. We use graph C to show the interaction process.

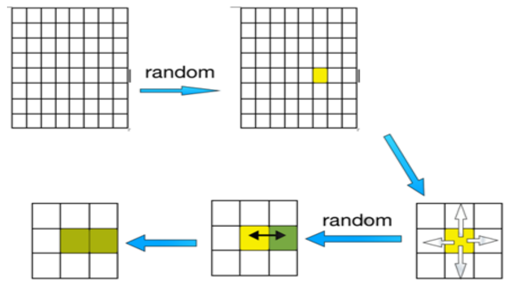

FIGURE III. INTERACTION PROCESS BETWEEN CELLS

Repeating the process mentioned above, that is, randomly choose a cell and interact with the cell around it (the neighborhood), we can successfully simulate the change of marine species.

III. Result and Discussion

We set the initiative values of the five variables and equations including the particular parameters to show the density distribution of the fish. Figure D and E are the density distribution diagrams after running the program.

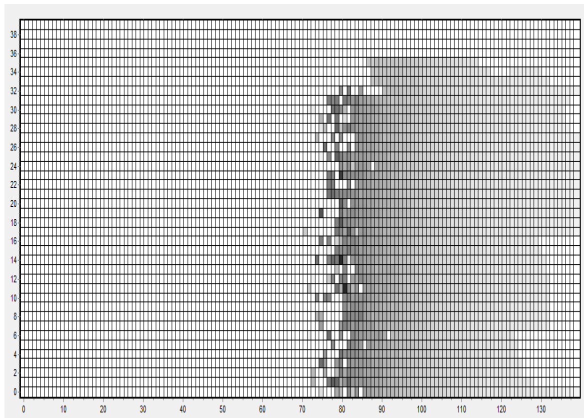

FIGURE IV. DISTRIBUTION OF DEAD FISH WHEN THE NUTRITIVE SALT IN THE RIVER IS 250

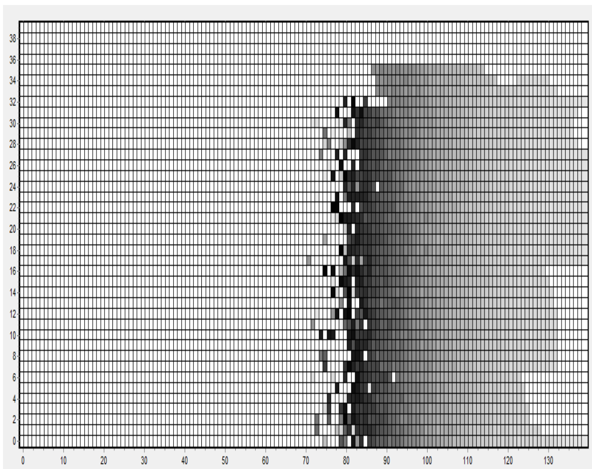

FIGURE V. DISTRIBUTION OF FISH WHEN THE NUTRITIVE SALT IS 500

In the two graphs, the darker color means the bigger number of fish that die of hypoxic in that area. From the graphs we can clearly see that as the density of the nutritive salt increase, the hypoxic area enlarges, thus causing the increasing death of fish.

Finally, we successfully simulate the process of formation of the hypoxic zone.

A. Advantages of the Multi-Scale Analysis Method

1. It enables us to combine different types of variables and different shapes of areas and reveal a favorable and clear experimental result.

2. It is easy to comprehend and make adjustments on either mathematical model or cellular automata.

3. One can increase the accuracy of the simulation result by adding the time (number of diffusions) in the model of cellular automata.

B. Error Analysis

The error of this model is mainly caused by:

The land form is divided into 140*40 cells, which is not accurate enough to establish models and simulate the changes.

Some parameters are not accurate enough; some other parameters are derived from empirical facts. When we simulate the changes and try to tweak the empirical parameters, it seems that almost all parameters can result in a ideal simulation graph. Therefore, the empirical parameters need further examination.

C. Comparison between the Simulation Methods

TABLE II. SIMULATION METHODS

Method	Limitation	Simulation Speed
Differential and difference equation	The area must be regular	Fast
Cellular automaton	Flexible with no limitation	Slow
Multi-scale analysis	Flexible with no limitation	Medium

IV. CONCLUSION

In this project, we have successfully researched the factors that cause the hypoxic areas in the Mexico Gulf using the multistage analysis.

Multi-scale analysis has both the flexibility of cellular automaton and the high simulation speed of the method of differential equation and difference equation. Therefore, it has the potential to be widely adopted.

Throughout research, we conclude that the excessive emission of nutritive salt is the major factor that causes the hypoxic areas in the Mexico Gulf.

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