Aerodynamic Performance Modeling and Optimization of Small Scale Wind Turbine Rotors

Athwel Gamarallage Thusitha Sugathapala
Senior Lecturer, Department of Mechanical Engineering
University of Moratuwa
Moratuwa 10230, Sri Lanka

Abstract—**This paper presents aerodynamic performance modeling and optimization of small scale wind turbine rotors for off-grid electricity generation applications targeting local manufacturing through simplification of blade geometry with constant chord and linear twist. The classical momentum theory and blade element theory are used to derive the governing equations, which are solved through an iterative procedure and search method for optimization. The results show that the aerodynamic performances of the simplified rotor designs are quite satisfactory, thus enhancing potential for local manufacture.**

Keywords- wind energy; rotor aerodynamics; power coefficient

I. INTRODUCTION

Development of renewable energy (RE) resources is considered to be one of the key interventions for the mitigation of energy and environment crises arisen from excessive usage of fossil fuels for energy generations. Wind energy resources are abundantly available in Sri Lanka, and the government has resolved to implement aggressive development plan to realize its target of 100% RE scenario by 2035, while improving the energy access to the entire community [1]. The emphasis is on popularizing decentralized RE energy projects that could contributed to local socio-economic development. Thus, development of small-scale wind energy conversion systems (WECSs) for rural-electrification with local manufacturing perspectives is an important step. However, the design and manufacture of the rotor, which is one of the key sub-systems in WECSs, need sophisticated tools and skills. Therefore, the overall objective of this research work is to establish a basic mathematical model for the aerodynamics performance characteristics of a wind turbine rotor and thereby develop an optimized rotor with simplified bade geometry for local manufacture.

II. AERODYNAMIC PERFORMANCE OF WIND TURBINE ROTORS

A. Governing Parameters

The function of a wind turbine rotor is to convert optimally the kinetic energy in the moving air into mechanical energy to drive the electric generator. The rotor slows down the moving air while converting kinetic energy to pressure energy, which forces the rotor blades to generate the mechanical energy. This process is basically governed by fluid dynamics principles together with characteristics of flow around aerofoil sections of the blades [2]. Thus, the governing variables of the rotor flow dynamics include the input parameters such as fluid properties (primarily, density ρ and dynamic viscosity μ), flow characteristics (primarily, air velocity U), and geometrical properties (such as rotor diameter D, number of blades B, aerofoil shape and size variations of the blades) and output parameters such as rotor power P, rotational speed ω and torque T. The system performance of a wind energy conversion facility is also determined by the performance characteristics of other sub-systems such as the mechanical transmission systems, electric generator, control systems, and electric load characteristics. The scope of this research work is limited only to the modeling of the aerodynamic performance characteristics of small-scale rotors for electricity generation applications.

B. Functional Relationships

The basic relationships between the governing parameters could be derived through dimensional analysis technique, in which the overall performance parameters of the rotor are expressed in non-dimensional forms as the Power Coefficient C_P and Torque Coefficient C_Q, defined by [3].

$$C_P = \frac{P}{P_{air}} = \frac{P}{\frac{1}{2}\rho U^3 \left(\pi R^2\right)}, \quad \square \tag{1}$$

$$C_Q = \frac{T}{\frac{1}{2}\rho U^2 \left(\pi R^3\right)} = C_P \left(\frac{U}{R\omega}\right) = \frac{C_P}{\lambda_0}, \tag{2}$$

where $R = D/2$ is the rotor radius and $\lambda_0 = R\omega/U$ is referred to as the tip speed ratio. In general, both C_P and C_Q are functions of several non-dimensional independent parameters such as Reynolds number $Re = \rho UD/\mu$, λ_0, B, rotor solidity σ and aerodynamic performance parameters of the blade profile such as lift coefficient C_L and drag coefficient C_D. In a rotor design, these parameters are selected for optimum performance for a given application. Then the rotor performances, given by C_P and C_Q, become functions of Re and λ_0, among which Re representing effects of friction on the aerodynamics loads has minor effects. As such, the overall performance characteristics of a given wind turbine rotor are represented by the variations of C_P and C_Q with λ_0.

III. Mathematical Formulation

A. Introduction

Accurate modeling of the flow field generated by the wind rotor is essential for realization of the design objectives. Even for steady, uniform and irrotational oncoming wind, the flow-field behind the rotor (the wake) is unsteady, highly turbulent and consists of complicated vortex systems. Therefore development of a model which could represent all the characteristics of the flow field around a wind turbine is a very difficult task and usually needs numerical modeling through computational fluid dynamics (CFD) techniques. CFD techniques demand for much resources and may not be feasible for small scale applications. Hence, number of simplified flow models for rotor aerodynamics has been developed, with different order of accuracy and complexity depending on the approximations employed in the analysis. These models could be categorized into three main groups: (a) Actuator Disk Theories (e.g. Axial momentum theory, Multiple-stream tube & angular-momentum analysis, General momentum theory with wake rotation, Double actuator disk analysis), (b) Strip Theories (e.g. Blade element theory, Modified blade element theories) and (c) Vortex Theories (e.g. Free wake analysis, Prescribed wake technique, Unsteady prescribed wake technique) [4], [5], [6], [7].

The present work is based on the blade element - momentum theory, which is shown to be capable of relating rotor design parameters to the aerodynamics characteristics of the flow, and simpler to implement [8]. The optimum blade design of a wind rotor (as well as performance prediction at different operating conditions of a given rotor) can be performed by considering the forces acting on a blade element, which is referred to as blade element theory. The combination of blade element theory and momentum theory leads to the required information on the variation of the chord length c and blade angle β along the blade span, as well as estimation of the effect of blade drag on the rotor performance. The following sections present such methodology, where the momentum theory is based on the multiple-stream tube technique with the inclusion of effect of the wake rotation.

B. Momentum Theory

In the momentum theory, basic conservation laws are applied to the flow field without referring to the geometry or the forces on the blades and the rotor is considered as a conceptual plane element called actuator disk (see Figure I). Actuator disk is a region of zero thickness which causes for sudden increment of pressure and momentum of the fluid.

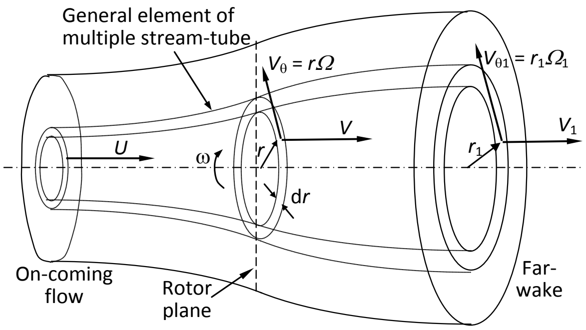

FIGURE I. MULTIPLE-STREAM TUBE MODEL FOR THE FLOW THROUGH THE ROTOR

In this approach, the fluid loading is assumed to be steady and, linear and angular momentum equations are applied to an axi-symmetric element of the stream tube of thickness dr at the radial position r to derive expressions for elementary axial force dF_A and elementary torque dT as [8]

$$dF_A = \pi\rho(1 - k^2)U^2 r dr, \quad (3)$$

$$dT = \pi\rho(h - 1)(1 + k)\omega U r^3 dr, \quad (4)$$

where Ω is the angular speed of the wake behind the rotor, V_1 is the axial velocity in the far wake, k is the axial interference factor defined by $V_1 = kU$ and h is the tangential induction factor defined by $\Omega = (h - 1)\omega$.

In order to obtain a closed solution for the problem, the flow parameters are required to relate to the characteristics of the rotor blades, which is described in the next section.

C. Blade Elemen Theory

This theory considers the forces acting on the rotor in relation to the geometry and aerodynamic characteristics of the blade profile. Consider the flow-field and forces acting on the blade element, as shown in Figure II. The velocity triangle given in the figure refers to that at the mid-chord, representing the average of the inlet and outlet velocity triangles.

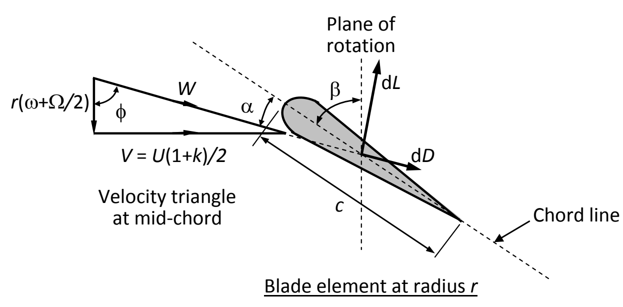

FIGURE II. VELOCITIES AND FORCES ON A BLADE ELEMENT

The following trigonometric relations can be derived from Figure II:

$$\beta = \phi - \alpha, \quad (5)$$

$$\cot\varphi = \lambda\frac{(h+1)}{(k+1)}, \qquad \lambda = \frac{r\omega}{U} = \lambda_0\frac{r}{R}. \qquad (6)$$

where λ is the local speed ratio, α is the angle of attack and ϕ is referred to as the flow angle. The elementary lift and drag forces dL and dD can be expressed in terms of C_L and C_D of the blade profile, which can be resolved to obtain expressions for elementary axial force dF_A and elementary torque dT. After some algebra, the results become [8]

$$dF_A = \frac{1}{8}\rho U^2 BcC_L(1+k)^2\frac{\cos(\varphi-\varepsilon)}{\sin^2\varphi\cos\varepsilon}dr, \qquad (7)$$

$$dT = \frac{1}{8}\rho U\omega BcC_L(1+k)(1+h)\frac{\sin(\varphi-\varepsilon)}{\sin\varphi\cos\varphi\cos\varepsilon}r^2dr, \quad (8)$$

where $\varepsilon = \tan^{-1}(C_D/C_L)$. Note that, for a given blade profile C_L and C_D are known functions of α.

D. Blade Element – Momontum Theory

The solution to the flow around wind turbine rotor can be derived by relating the results of the blade element theory given in Equations (7) and (8) to those derived from the momentum theory given in Equations (3) and (4). Firstly, the expressions for axial interference factor and tangential induction factor are obtained as

$$\frac{(1-k)}{(1+k)} = \left(\frac{\sigma C_L}{4}\right)\frac{\cos(\varphi-\varepsilon)}{\sin^2\varphi\cos\varepsilon}, \qquad (9)$$

$$\frac{(h-1)}{(h+1)} = \left(\frac{\sigma C_L}{4}\right)\frac{\sin(\varphi-\varepsilon)}{\sin\varphi\cos\varphi\cos\varepsilon}, \qquad (10)$$

where $\sigma = Bc/(2\pi r)$ is the local rotor solidity.

Now, the energy conversion performance of the rotor at radius r is represented by the local power coefficient C_{Pr} as

$$C_{\mathrm{Pr}} = \frac{\mathrm{d}P}{\mathrm{d}P_{air}} = \frac{\omega dT}{\frac{1}{2}\rho U^3(2\pi r dr)} = \lambda^2(1+k)(h-1). \qquad (11)$$

The power coefficient of the rotor is obtained by integrating the local power coefficient, as

$$C_P = \frac{\int \mathrm{d}P}{P_{air}} = \frac{2}{R^2}\int_{r_0}^{R} C_{\mathrm{Pr}}rdr, \qquad (12)$$

where r_0 is the hub radius. Note that the above result for the rotor performance includes only the effects of wake rotation and blade drag. Another important factor, which affects the performance of a wind rotor, is the number of blades in the rotor. This is caused by the so-called tip losses. This loss is usually represented by the multiplication factor n_B given by [8]

$$n_B = \left[1 - \frac{1.386}{B}\sin\left(\frac{\varphi_0}{2}\right)\right]^2, \text{ where } \varphi_0 = \frac{2}{3}\tan^{-1}\left(\frac{1}{\lambda_o}\right). \quad (13)$$

IV. Method of Solution

For a given rotor configuration at a specified λ_0, the expressions given in Equations (5), (6), (9) and (10) can be used to determine the four unknown parameters ϕ, α, h and k. Substitution of these results in Equation (11) yields the required local power coefficient, and integration of which leads to the power coefficient of the rotor as given in Equation (12) together with Equation (13) and the torque coefficient through Equation (2).

On the other hand, for a new rotor design (for selected blade profile and λ_0), the local power coefficient can be maximized to obtain additional expressions for the estimation of optimum blade geometry (i.e. c and β variations). In order to facilitate the optimization procedure, Equations (6), (9) and (10) can be combined to eliminate ϕ and σ. This leads to the following condition:

$$\lambda^2(h^2-1) + 2\lambda\left(\frac{C_D}{C_L}\right)(h-k) - (1-k^2) = 0. \qquad (14)$$

Now, the local power coefficient given in Equation (11) can be maximized under the constraint condition given in Equation (14) by the method of Lagrange Multipliers. After some algebraic simplifications, this leads to

$$\lambda^2 h(h-1) + \lambda\left(\frac{C_D}{C_L}\right)(h+k) - k(1+k) = 0. \qquad (15)$$

For a selected aerofoil profile for the blade section, the optimum angle of attack and the corresponding minimum C_D/C_L ratio are known constants and Equations (14) and (15) can be solved to find k and h at a given radial position. The corresponding value of ϕ can be obtained from Equation (6). Equation (5) and either Equation (9) or Equation (10) give the optimum values of c and β. The corresponding optimum local power coefficient is then given by Equation (11).

V. Design of an Optimum Wind Turbine Rotor

A. Design Parameters

The basic design parameters of the rotor are D, λ_0, B and sectional geometry of the blade (aerofoil profile, β and c). D is determined by the output power requirement and the design wind speed of the selected site. The performance of a rotor is highly dependent on the blade section's C_D/C_L ratio. In highspeed wind power plants, laminar flow profiles (aerofoils) are used since optimum C_D/C_L ratio of these profiles are very low. The selection of λ_0 and B is influenced by the type of application. For electricity generation applications, lower number of blades (2 or 3) with higher tip speed ratio ($\lambda_0 > 5$) is selected. Table I summarizes the main design parameters selected in the present study.

TABLE I. DESIGN PARAMETERS OF THE WIND TURBINE ROTOR

Parameter	Quantity	Remarks
Rated power	200 W	Small scale battery charging application (for a household)
Design wind speed	6 m/s	Relevant to a moderate wind potential site
Rated wind speed	9 m/s	
Rotor diameter	2 m	Selected based on rated power and rated wind speed
Hub diameter	0.6 m	To facilitate the mounting of the generator and rotor blades
Design tip speed ratio	6	Electricity generation application
Number of blades	2	To reduce the cost of the rotor blades
Blade profile	K2	Thin cambered section with low C_D/C_L ratio

The aerofoil section selected for the present study is a new profile designated as K2, which is aerodynamically more efficient than conventional profiles and considered to be more suitable for a small scale two-bladed rotor. The shape of the K2 aerofoil (drawn to scale) is presented in Figure III and its aerodynamic characteristics (at Re = 8×10^4) are presented in Table II [9].

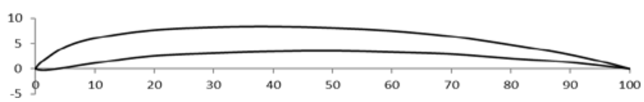

FIGURE III. K2 AEROFOIL PROFILE

TABLE II. AERODYNAMIC CHARACTERISTICS OF K2 PROFILE

Angle of attach (deg)	C_L	C_D	C_D/C_L
0.00	0.350	0.033	0.0943
1.25	0.550	0.026	0.0473
2.50	0.750	0.022	0.0293
3.75	0.900	0.020	0.0222
5.00[a]	1.020	0.020	0.0196
6.25	1.120	0.024	0.0214
7.50	1.200	0.028	0.0233
8.75	1.280	0.030	0.0234
10.00	1.330	0.030	0.0226
11.25	1.350	0.030	0.0222
12.50	1.350	0.030	0.0222
13.75	1.330	0.030	0.0226
15.00	1.300	0.030	0.0231
16.25	1.250	0.028	0.0224

a. Optimum angle of attack (minimum C_D/C_L ratio)

B. *Results of Optimum Rotor Design*

Since the governing equations are in complex non-linear forms and the condition for maximum local power coefficient cannot be expressed in analytical form, direct analytical solution is not possible. The method of solution in the present study is therefore based on an iterative procedure together with a search method to locate the point of optimum local power coefficient. The results for the optimum rotor are presented in Table III. The tip-loss factor n_B, given by Equation (13), becomes 0.923 and thus the optimum power coefficient becomes 0.426, representing an efficient rotor. The corresponding torque coefficient is 0.071.

According to these results, the optimum chord length varies from 74 mm at the tip to 215 mm at the hub and blade angle from 1.5° to 14.6°. Such strong nonlinearity yields difficulties in manufacture and fabrication. Therefore, the chord length variation is linearized from 100 mm to 170 mm, together with a lower twist, for the simplicity, and the results are presented in the same table for comparison. The optimum power coefficient and the corresponding torque coefficient of the linearized geometry with the tip losses are estimated to be 0.40 and 0.066; only about 7% reduction in comparison to the optimum rotor.

TABLE III. OPTIMUM ROTOR PARAMETERS

***r* (m)**	**λ**	**Optimum Rotor**			**Linearized Chord**		
		β (deg.)	**c (m)**	C_{Pr}	**β (deg.)**	**c (m)**	C_{Pr}
0.3	1.8	14.6	0.215	0.523	9.0	0.170	0.557
0.4	2.4	10.3	0.173	0.528	9.0	0.160	0.570
0.5	3.0	7.5	0.144	0.525	8.0	0.150	0.564
0.6	3.6	5.6	0.122	0.518	7.0	0.140	0.544
0.7	4.2	4.1	0.106	0.510	6.0	0.130	0.516
0.8	4.8	3.1	0.093	0.502	5.0	0.120	0.479
0.9	5.4	2.2	0.083	0.492	4.0	0.110	0.421
1.0	6.0	1.5	0.074	0.483	4.0	0.100	0.342
C_P (without tip-loss)				0.461			0.430
C_Q (without tip-loss)				0.077			0.072

The aerodynamics performance characteristics of wind turbine rotors are usually represented by the variations of C_P and C_Q with λ_0, and the two curves for the optimum rotor with linearized chord are presented in Figure IV and Figure V.

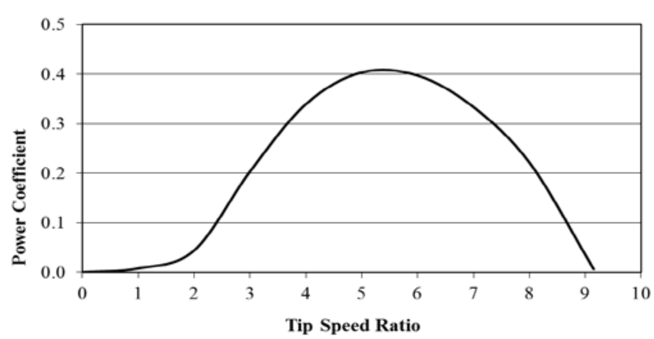

FIGURE IV. VARIATION OF POWER COEFFICIENT

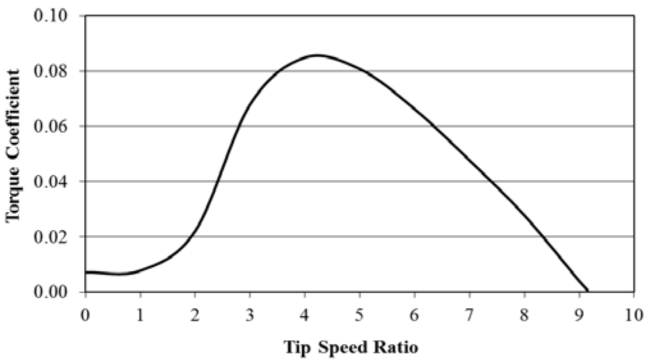

FIGURE V. VARIATION OF TORQUE COEFFIENT

Further simplification in the rotor design is introduced by employing blades with a constant chord. For the present study, five different values, viz 90 mm, 100 mm, 110 mm, 120 mm, and 130 mm, are selected and the effects on the aerodynamic performances are investigated at different blade angle variations (including optimum, linearized and a set of constant angles of 2°, 3°, 4°, 5° and 6°). Further increase in the chord results in considerable reduction in power coefficient and therefore not considered in the present analysis. The effects of these variations on the power coefficient of the rotor are illustrated in Figure VI.

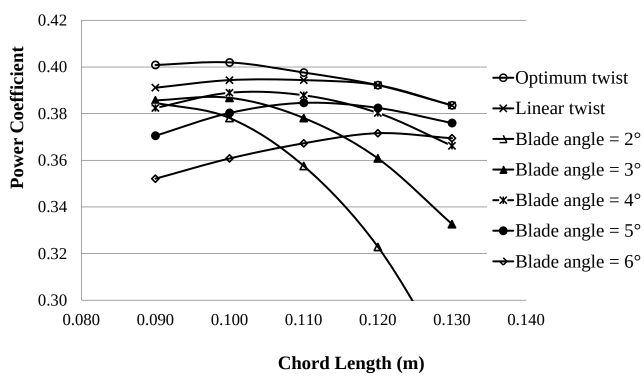

FIGURE VI. POWER COEFFICIENTS AT DIFFERENT BLADE ANGLE CONDITIONS FOR ROTORS WITH CONSTANT CHORD LENGTHS

The effects of linearization of the blade angle variation on the overall power coefficient of the rotor is negligible, in particular at higher chords. The reduction of the power coefficient is higher for smaller chords, for example 2.6% and 1.7% for 90 mm and 100 m chord, respectively.

Effects of employing constant blade angle for constant chord blades show more complicated behavior. For low solidity rotors, reduction of the power coefficient is smaller at small blade angles (in particular 2°, 3° and 4°), but becomes significant at higher blade angles (i.e. > 4°). However, higher solidity rotors show the opposite behavior, where the reduction is considerable at small blade angles but become negligible at higher blade angles (4°, 5° and 6°) considered in the analysis. Rotors with 100 mm and 110 mm chords employing constant blade angle of 4° show the best performance.

VI. CONCLUSIONS

Design of a new rotor and its aerodynamic performance prediction could be carried out effectively by the theoretical approach based on momentum theory and blade element theory. However, the resulting governing equations include a set of complicated non-linear equations and numerical methods have to be used to solve them.

The use of constant chord blade geometry with K2 aerofoil profile has not resulted significant effect on the performance, especially with linearized blade twist. Based on the aerodynamic performance, the two bladed rotor with 120 mm or 130 mm constant chord employing linearized blade twist could be recommended for small scale wind turbine systems. If the manufacturing process restricts the use of blade twist, the recommended configuration is 110 mm chord and 4° blade angle.

ACKNOWLEDGMENT

The author wishes to acknowledge the Practical Action Consulting (PAC) for providing resources to undertake this research study in the initial stages and Mr. Sunith Fernando for providing guidance and aerofoil data required to perform the flow modeling of wind turbine rotors.

REFERENCES

[1] MoPE, Sri Lanka Energy Sector Development Plan for a Knowledge-based Economy, Ministry of Power & Energy, Government of Sri Lanka, 2015. Available [online]: http://powermin.gov.lk/english/.

[2] M. O. L. Hansen, Aerodynamics of Wind Turbines, 2nd ed., Chapter 4, Earthscan, 2008, pp. 27-40.

[3] J. F. Manwell, J. G. McGowan and A. L. Rogers, Wind Energy Explained – Theory, Design and Application, Chapter 3, John Wiley, 2002, pp. 83-138.

[4] H. Snel, "Review of Aerodynamics for Wind Turbines", Wind Energy, Vol. 6 Issue 3, John Wiley, 2003, pp. 203-211.

[5] M. O. L. Hansen and H. A. Madsen, "Review Paper on Wind Turbine Aerodynamics", ASME J. Fluids Engineering, Technology Review, Vol. 133, Issue 11, 114001, November 2011.

[6] K. K. M. N. P. Samaraweeera, Prediction of Flow Field of a Ceiling Fan by Flow Singularity Modelling, M.Sc. Thesis, Department of Mechanical Engineering, University of Moratuwa, October 2013.

[7] N. Karthikeyan, K. K. Murugavel, S. A. Kumar and S. Rajakumar, "Review of Aerodynamic Developments on Small Horizontal Axis Wind Turbine Blade", Renewable and Sustainable Energy Reviews, Vol 42, Elsevier, 2015, pp. 801–822.

[8] W. R. G. A. Wijesundara, A Study on Wind Turbines Working on the Principle of Magnus Effect, M.Phil Thesis, Department of Mechanical Engineering, Open University of Sri Lanka, October 2000.

[9] R. A. Attalage, K. K. C. K. Perera and A. G. T. Sugathapala, Evaluation of Small Wind System for Battery Charging in Sooriyawewa, Hambantota District, Sri Lanka. Project report submitted to Intermediate Technology Development Group (ITDG) South Asia, September 2001, unpublished.