# Partial Linear Regression Method in the Application of the Accurate Calculation of PQDIF Harmonic Responsibility 

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#### Abstract

Quantitatively evaluating the harmonic responsibility of harmonic source at the point of common coupling (PCC) is an important task of power quality management. Considering the change of background harmonic and the fact that the current monitoring system can only provide the voltage and current statistical values, a method called partial linear regression method for accurately calculating harmonic responsibility under PQDIF is proposed. By means of measuring the information of PCC, the proposed method builds a more accurate partial linear regression model of calculation of the equivalent harmonic impedance, and then design two-stage method to calculate the unbiased estimation of the equivalent harmonic impedance and the background harmonic voltage, finally implement the accurate calculation of harmonic responsibility under PQDIF . Simulation results of IEEE 14-bus system show the effectiveness and accuracy of the proposed method.


Keywords-power quality; harmonic responsibility; power quality data interchange format; partial linear regression method

## I. Introduction

With the development of the industry, all kinds of power electronic devices and nonlinear loads are widely used in power system. The problem of harmonic pollution in distribution network is increasingly seriously. Therefore, the reasonable assessment of harmonic voltage responsibility of power system and user at the common coupling point, is the precondition of power quality management and controlling[1].

Nowadays ,methods for determination of harmonic responsibility at home and abroad is mainly about the harmonic impedance estimation of system and user, mainly divided into "intervention" [2-4] method and "nonintrusive" method[5-7]. "Intervention" method mainly through the artificial injection of harmonic current into the system to measure the harmonic impedance of system, this method may change the operation of the power system, and has large estimation error." The intervention method" mainly use the measured value of the harmonic voltage and current at PCC to determine harmonic responsibility quantitatively, such as the method of linear regression, including robust regression method [5], the binary regression method [6], partial least squares[7], etc. The essence of these methods is based on the Thevenin equivalent circuit or Norton equivalent circuit to constructs the corresponding regression equation, and divides the harmonic voltage and
current phasor at the PCC into real part and imaginary part to obtain the harmonic impedance. However, due to the background harmonic voltage is a variable in the regression equation, and the relationship between independent variable and dependent variable is not strict linear, so the linear regression will cause greater error. Besides, most of the above methods is based on the voltage and current instantaneous value, and it's not conform to the current situation that the daily power quality monitoring system mainly provide statistics such as voltage, current RMS value. Considering the actual situation that the power grid power quality monitoring data is often expressed as power quality data interchange format (PQDIF) ,so it is necessary to study the division of harmonic responsibility for PQDIF data [8]. Literature [8] just put forward a rough estimate of harmonic responsibility under PQDIF, the algorithm is simple and easy to implement, but the method cannot meet the accuracy requirement in the case of precise harmonic responsibility.

On account of the shortcomings of the above methods, this paper proposes the partial linear regression method to estimate the harmonic responsibility precisely under PQDIF data. Firstly, use the PQDIF data to construct the partial linear regression model. And then, use the Kernel Smoother method to get the system harmonic impedance and the biased estimation of background harmonic voltage. Finally, the ordinary least squares solution is used to get the real-time accurate harmonic responsibility. Experimental results prove the validity and the accuracy of this method.

## II. Definition and Calculation Formula of Harmonic Responsibility

As shown in figure 1 , busbar X is the point of common coupling (PCC).The side connected to the grid system is known as the system side; and the other side connected to several loads is known as the user side. Suppose D as a major source of nonlinear harmonic load, now we calculate the harmonic responsibility of load D at the busbar X. Set $I_{h \mathrm{D}}$ is the $h$ time harmonic current that load D inject into X, and the $h$ time harmonic voltage measured at X is $U_{\mathrm{hx}}$. So the following relation can be established:

$$
\begin{equation*}
U_{h \mathrm{X}}=Z_{h \mathrm{X}, \mathrm{D}} I_{h \mathrm{D}}+U_{h \mathrm{X}, 0} \tag{1}
\end{equation*}
$$



FIGURE I. THE PCC POINT WITH SEVERAL LOADS MODEL
Where $Z_{h \mathrm{X}, \mathrm{D}}$ is the equivalent harmonic impedance of the system and the rest of the loads except for load D. $U_{h \mathrm{X}, 0}$ is the harmonic voltage contribution at bus X of the system and the rest of the loads besides load D , that is the background harmonic voltage. The right side of equation (1) is the harmonic voltage produced by load D at X , and we recorded it as $U_{h \mathrm{X}, \mathrm{D}}$. There is:

$$
\begin{equation*}
U_{h \mathrm{X}, \mathrm{D}}=Z_{h \mathrm{X}, \mathrm{D}} I_{h \mathrm{D}} \tag{2}
\end{equation*}
$$

The vector diagram of $U_{h \mathrm{X}, \mathrm{D}}, U_{h \mathrm{X}, 0}$ and $U_{h \mathrm{X}}$ as shown in figure 2.


FIGURE II. THE VECTOR DIAGRAM OF UhX, D, UhX, 0 AND UhX
Define $\mu_{h}$ is the harmonic voltage contribution of load D at X , and $\mu_{h}$ can be written as the equation expressed by $Z_{h \mathrm{X}, \mathrm{D}}$ and $U_{h \mathrm{X}, 0}$ according to [9]:

$$
\begin{equation*}
\mu_{h}=\frac{U_{h \mathrm{X}}^{2}+\left|Z_{h \mathrm{X}, \mathrm{D}}\right|^{2} I_{h \mathrm{D}}^{2}-U_{h \mathrm{X}, 0}^{2}}{2 U_{h \mathrm{X}}^{2}} \tag{3}
\end{equation*}
$$

Therefore, the key to compute harmonic voltage responsibility under PQDIF is to use PQDIF harmonic measurement data to calculate $h$ time system equivalent harmonic impedance $Z_{h \mathrm{X}}$ and background harmonic voltage $U_{h \mathrm{X}, 0}$ accurately.

## III. Partial Linear Regression Model Applied to <br> PQDIF

## A. Calculation of Phase Angle Difference

In figure 1, we can use PQDIF harmonic measurement data to get the $h$ time harmonic active power $P_{h \mathrm{x}}$ and reactive
power $Q_{h \mathrm{x}}, h$ time harmonic voltage RMS $U_{h \mathrm{x}}$ and current RMS $I_{h \mathrm{x}}$. There are relationships between above four:

$$
\begin{align*}
& P_{h \mathrm{X}}=U_{h \mathrm{X}} I_{h \mathrm{X}} \cos \theta_{h}  \tag{4}\\
& Q_{h X}=U_{h X} I_{h X} \sin \theta_{h} \tag{5}
\end{align*}
$$

$\theta_{h}$ is the power factor angle that $U_{h \mathrm{X}}$ advances $I_{h \mathrm{X}}$, and it can be backstepped by measurement data:

$$
\begin{equation*}
\theta_{h}=\tan ^{-1}\left(Q_{h \mathrm{X}} / P_{h \mathrm{X}}\right) \tag{6}
\end{equation*}
$$

## B. Partial Linear Regression Model of System Harmonic

 Impedance CalculationIn equation (1) there is:

$$
\begin{align*}
& U_{h \mathrm{X}}=U_{\mathrm{XR}}+\mathrm{j} U_{\mathrm{XI}} \\
& Z_{h \mathrm{X}, \mathrm{D}}=Z_{\mathrm{R}}+\mathrm{j} Z_{\mathrm{X}}  \tag{7}\\
& I_{h \mathrm{D}}=I_{\mathrm{DR}}+\mathrm{j} I_{\mathrm{DI}} \\
& U_{h \mathrm{X}, 0}=U_{0 \mathrm{R}}+\mathrm{j} U_{0 \mathrm{x}}
\end{align*}
$$

So (1) can be written as:

$$
\begin{align*}
U_{\mathrm{XR}}+\mathrm{j} U_{\mathrm{XI}} & =Z_{\mathrm{R}} I_{\mathrm{DR}}-Z_{\mathrm{X}} I_{\mathrm{DI}}+U_{0 \mathrm{R}} \\
& +\mathrm{j}\left(Z_{\mathrm{R}} I_{\mathrm{DI}}+Z_{\mathrm{x}} I_{\mathrm{DR}}+U_{0 \mathrm{I}}\right) \tag{8}
\end{align*}
$$

That is:

$$
\left\{\begin{array}{l}
U_{\mathrm{XR}}=Z_{\mathrm{R}} I_{\mathrm{DR}}-Z_{\mathrm{X}} I_{\mathrm{DI}}+U_{0 \mathrm{R}}  \tag{9}\\
U_{\mathrm{XI}}=Z_{\mathrm{R}} I_{\mathrm{DI}}+Z_{\mathrm{X}} I_{\mathrm{DR}}+U_{0 \mathrm{I}}
\end{array}\right.
$$

Rewrite (9) into matrix form:

$$
\left(\begin{array}{ll}
U_{\mathrm{xR}} & U_{\mathrm{xI}}
\end{array}\right)=\left(\begin{array}{ll}
I_{\mathrm{DR}} & I_{\mathrm{DI}}
\end{array}\right)\left(\begin{array}{cc}
Z_{\mathrm{R}} & Z_{\mathrm{X}}  \tag{10}\\
-Z_{\mathrm{X}} & Z_{\mathrm{R}}
\end{array}\right)+\left(\begin{array}{ll}
U_{0 \mathrm{R}} & U_{0 \mathrm{I}}
\end{array}\right)
$$

The system harmonic impedance can be seen as a constant approximately in a certain period of time, but the background harmonic voltage changes over time, so equation (10) is a partial linear regression model. Get $m$ times sampling values in a certain period of time, there are:

$$
\begin{equation*}
y=E \omega+f \tag{11}
\end{equation*}
$$

Where:

$$
\begin{gathered}
y=\left(\begin{array}{cc}
U_{\mathrm{XR}}(1) & U_{\mathrm{XI}}(1) \\
U_{\mathrm{XR}}(2) & U_{\mathrm{XI}}(2) \\
\vdots & \vdots \\
U_{\mathrm{XR}}(m) & U_{\mathrm{XI}}(m)
\end{array}\right) E=\left(\begin{array}{cc}
I_{\mathrm{DR}}(1) & I_{\mathrm{DI}}(1) \\
I_{\mathrm{DR}}(2) & I_{\mathrm{DI}}(2) \\
\vdots & \vdots \\
I_{\mathrm{DR}}(m) & I_{\mathrm{DI}}(m)
\end{array}\right) \\
\omega=\left(\begin{array}{cc}
Z_{\mathrm{R}} & Z_{\mathrm{X}} \\
-Z_{\mathrm{X}} & Z_{\mathrm{R}}
\end{array}\right) \quad f=\left(\begin{array}{cc}
U_{0 \mathrm{R}}(1) & U_{0 \mathrm{OI}}(1) \\
U_{0 \mathrm{R}}(2) & U_{01}(2) \\
\vdots & \vdots \\
U_{0 \mathrm{R}}(m) & U_{01}(m)
\end{array}\right)
\end{gathered}
$$

In the existing power quality monitoring system, the phase angles of $h$ harmonic voltage in matrix $\boldsymbol{y}$ and the phase angles of $h$ harmonic current in matrix $\boldsymbol{E}$ is unknown, so the harmonic measurement data under PQDIF cannot be represented as (11).However, $\theta_{h}$ can be obtained by III.A. So in this paper we use phase angle difference $\theta_{h}$ to get the transformation of (11) applied to PQDIF.

Suppose at a measurement time $t$, the actual phase angle of $I_{h \mathrm{D}}$ is $\alpha_{t}$. The phase angle difference between harmonic voltage and current at bus X is $\theta_{h}$. Devide the both sides of equation (1) by $1 \angle \alpha_{t}$ at the same time we can get:

$$
\begin{equation*}
\frac{U_{h \mathrm{X}}}{1 \angle \alpha_{t}}=Z_{h \mathrm{X}, \mathrm{D}} \frac{I_{h \mathrm{D}}}{1 \angle \alpha_{t}}+\frac{U_{h \mathrm{X}, 0}}{1 \angle \alpha_{t}} \tag{12}
\end{equation*}
$$

That is:

$$
\begin{equation*}
U_{h \mathrm{X}} \angle \theta_{h}=Z_{h \mathrm{X}, \mathrm{D}} I_{h \mathrm{D}} \angle 0+\frac{U_{h \mathrm{X}, 0}}{1 \angle \alpha_{t}} \tag{13}
\end{equation*}
$$

The phase angle of harmonic current $I_{h \mathrm{D}}$ in (13) is zero, so the second column of matrix $\boldsymbol{E}$ is 0 .Then using equation (13) to calculate the parameters may produce singularity. To avoid this situation, multiply equation (13) by $1 \angle \alpha_{c}$ at the same time. $\alpha_{c}$ is the reference phase angle of $I_{h \mathrm{D}}$ and the value of $\alpha_{c}$ is without any influence on the calculation of system harmonic impedance $Z_{h \mathrm{X}, \mathrm{D}}$. For ease of calculation in this paper we set $\alpha_{c}=10^{\circ}$, there are:

$$
\begin{equation*}
U_{h \mathrm{X}} \angle\left(\theta_{h}+\alpha_{c}\right)=Z_{h \mathrm{X}, \mathrm{D}} I_{h \mathrm{D}} \angle \alpha_{c}+\frac{U_{h \mathrm{X}, 0}}{1 \angle \alpha_{t}} \angle \alpha_{c} \tag{14}
\end{equation*}
$$

Set the last item of the right side of (14) is $U_{h \mathrm{X}, 0}^{\prime}$, so the transformation of (11) expressed by $\theta_{h}$ is:

$$
\begin{equation*}
U_{h \mathrm{X}} \angle\left(\theta_{h}+\alpha_{c}\right)=Z_{h \mathrm{X}, \mathrm{D}} I_{h \mathrm{D}} \angle \alpha_{c}+U_{h \mathrm{X}, 0}^{\prime} \tag{15}
\end{equation*}
$$

Get $m$ times sampling values in a certain period of time, and the partial linear regression model applied to PQDIF fomat data is:

$$
\begin{equation*}
y=E \omega+f \tag{16}
\end{equation*}
$$

Where:

$$
\begin{gathered}
\omega=\left(\begin{array}{cc}
Z_{\mathrm{R}} & Z_{\mathrm{X}} \\
-Z_{\mathrm{X}} & Z_{\mathrm{R}}
\end{array}\right) \quad f=\left(\begin{array}{cc}
U_{0 \mathrm{R}}^{\prime}(1) & U_{010}^{\prime}(1) \\
U_{0 \mathrm{R}}^{\prime}(2) & U_{0 \mathrm{I}}^{\prime}(2) \\
\vdots & \vdots \\
U_{0 \mathrm{R}}^{\prime}(m) & U_{0 \mathrm{II}}^{\prime}(m)
\end{array}\right) \\
y=\left(\begin{array}{cc}
\operatorname{real}\left(U_{h \mathrm{X}} \angle\left(\theta_{h}+\alpha_{c}\right)(1)\right) & \operatorname{imag}\left(U_{h \mathrm{X}} \angle\left(\theta_{h}+\alpha_{c}\right)(1)\right) \\
\operatorname{real}\left(U_{h \mathrm{X}} \angle\left(\theta_{h}+\alpha_{c}\right)(2)\right) & \operatorname{imag}\left(U_{h \mathrm{X}} \angle\left(\theta_{h}+\alpha_{c}\right)(2)\right) \\
\vdots & \vdots \\
\operatorname{real}\left(U_{h \mathrm{X}} \angle\left(\theta_{h}+\alpha_{c}\right)(m)\right) & \operatorname{imag}\left(U_{h \mathrm{X}} \angle\left(\theta_{h}+\alpha_{c}\right)(m)\right)
\end{array}\right) \\
E=\left(\begin{array}{cc}
\operatorname{real}\left(I_{h \mathrm{D}} \angle \alpha_{c}(1)\right) & \operatorname{imag}\left(I_{h \mathrm{D}} \angle \alpha_{c}(1)\right) \\
\operatorname{real}\left(I_{h \mathrm{D}} \angle \alpha_{c}(2)\right) & \operatorname{imag}\left(I_{h \mathrm{D}} \angle \alpha_{c}(2)\right) \\
\vdots & \vdots \\
\operatorname{real}\left(I_{h \mathrm{D}} \angle \alpha_{c}(m)\right) & \operatorname{imag}\left(I_{h \mathrm{D}} \angle \alpha_{c}(m)\right)
\end{array}\right)
\end{gathered}
$$

IV. The Solution of the Linear Regression Model and the Accurate Calculation of Harmonic Responsibility

## A. The Principle of the Partial Linear Regression Model

The expression of the common linear regression model is:

$$
\begin{equation*}
y_{i}=a x_{i}+b+\varepsilon_{i}(1 \leq i \leq m) \tag{17}
\end{equation*}
$$

In (17), a, b is a constant, so it's easy to use least-square method to solve the value of $a, b$. However, in the equation (16), the system harmonic impedance $\omega$ is a constant, but the background harmonic voltage is a variable, so (16) is a partial linear regression model and cannot be solved by the common linear regression model.

On account of this model, this paper puts forward twostage method to solve it. In the first stage we introduce kernel smoothing estimation method to calculate the biased estimation of $\omega$. Because we hope the result is unbiased, so the second stage is to use biased value $f_{b}$ to construct common plural regression model, then use the plural least-square method to obtain the accurate unbiased value $\omega_{u b}$. Finally we can get the unbiased value $f_{u b}$ by $\omega_{u b}$ and equation(16).

## B. Use Kernel Smoothing Method to Get Biased Value

Split matrice $\boldsymbol{E}$ into two matrices as follows:

$$
\begin{equation*}
E=g+\eta \tag{18}
\end{equation*}
$$

where $E=\left(e_{1}, \ldots, e_{p}\right), g=\left(g_{1}, \ldots, g_{p}\right), \eta=\left(\eta_{1}, \ldots, \eta_{p}\right)$. And there is $E_{i}=\left(e_{1 i}, \ldots, e_{m i}\right)^{\prime}, g_{i}=\left(g_{1 i}, \ldots, g_{m i}\right)^{\prime}, \eta_{i}=\left(\eta_{1 i}, \ldots, \eta_{m i}\right)^{\prime}$.

For the partial linear regression model as (16), hypothesis 1is given:
(1) $m^{-1} \eta^{\prime} \eta \rightarrow V, \mathrm{~V}$ is a positive definite matrix;
(2) $\operatorname{tr}\left(K^{\prime} K\right)=\sum_{s=1}^{m} \sum_{t=1}^{m} K_{s t}^{2}=O\left(b^{-1}\right)$;
(3) $\left\|K \eta_{i}\right\|^{2}=O\left(b^{-1}\right)=\left\|K^{\prime} \eta_{i}\right\|^{2}(1 \leq i \leq p)$;
(4) $n^{-1} \eta^{\prime} f^{*}=O\left(n^{-1 / 2} b^{v}\right)$;
(5) For any continuous function $z(t)$, there is a probability density function $p(\mathrm{t})$ on $[0,1]$ that when $m \rightarrow \infty$ there is:

$$
\begin{equation*}
m^{-1} \sum_{i=1}^{m} z\left(t_{i}\right) \rightarrow \int_{0}^{1} z(t) p(t) d t \tag{19}
\end{equation*}
$$

Where $\boldsymbol{K}$ is a Kernel smoothing matrix. And there is theorem 1 about using the kernel smoothing method to solve the partial linear regression model.

Theorem 1: If the partial linear regression model meets the assumption 1,then when the $m \rightarrow \infty, b \rightarrow 0$ there is:

$$
\begin{align*}
E\left(\omega^{*}\right)-\omega= & b^{v} V^{-1} \int_{0}^{1} g(t) f^{(v)}(t) h_{1}(t) p(t) d t  \tag{20}\\
& +O\left(b^{v} m^{-1 / 2}\right)
\end{align*}
$$

By the theorem 1, the kernel smoothing estimation method is a biased estimate, the error of the estimate result and the real value is a constant.

So the process of using the kernel smoothing method to calculate $\omega_{\mathrm{b}}$ and $f_{\mathrm{b}}$ which are the biased estimate of $\omega$ and $f$ is:
(1) Firstly, construct a smooth nuclear matrix $\boldsymbol{K}_{m \times m}$, in which the element $\boldsymbol{K}(s, t)$ is as follows :

$$
\begin{equation*}
K(s, t)=w((s-t) / h) / \sum_{t=1}^{m} w((s-t) / h) \tag{21}
\end{equation*}
$$

Here $h$ is the window width, and the cross verification method is available for the calculation of window width, here we set $h=m^{-1 / 5}$. In (21) $w$ is the kernel function and kernel function is a limited support function, and we require kernel function has the following properties:

$$
\int_{-1}^{1} w(t) t^{\prime} d t= \begin{cases}1 & (l=0)  \tag{22}\\ 0 & (1 \leq l \leq v-1) \\ c_{1} & (l=v)\end{cases}
$$

In which $v$ is the smooth order of $f$. Many functions can be selected as the kernel function, and choosing a different kernel function makes negligible influence on the result. We choose kernel function here is:

$$
w(t)= \begin{cases}0.75\left(1-t^{2}\right) & |t| \leq 1  \tag{23}\\ 0 & |t|>1\end{cases}
$$

(2) Secondly, calculate the matrix $\boldsymbol{E}^{*}$ and $\boldsymbol{y}^{*}$ which are the residual error matrix of $\boldsymbol{E}$ and $\boldsymbol{y}$ :

$$
\begin{align*}
E^{*} & =E-K E \\
y^{*} & =y-K y \tag{24}
\end{align*}
$$

(3) Finally get the biased estimate value of $\omega$ and f :

$$
\begin{align*}
& \omega_{b}=\left(E^{*} \cdot E^{*}\right)^{-1} E^{*} \cdot y^{*}  \tag{25}\\
& f_{b}=K\left(y-E \omega_{b}\right)
\end{align*}
$$

## C. Use the Plural Least Squares Method for Unbiased Estimate

According to the theorem 1, the error of the biased estimate $f_{b}$ and the real value $f_{u b}$ is a constant, and we set the error is $\beta_{0}$, so there is :

$$
\begin{equation*}
y=E \omega+f=E \omega+f_{b}+\beta_{0} \tag{26}
\end{equation*}
$$

That is:

$$
\begin{equation*}
y-f_{b}=E \omega+\beta_{0} \tag{27}
\end{equation*}
$$

In (27), matrix $\boldsymbol{y}-\boldsymbol{f}_{\boldsymbol{b}}$ and $\boldsymbol{E}$ are known matrix, and $\omega, \beta_{0}$ is an unknown constant. Therefore equation (27) is a common linear regression model. Now we rewrite (27) into the form of the plural matrix.

$$
\begin{equation*}
r=Q \xi \tag{28}
\end{equation*}
$$

In which $r=\left(r_{1}, r_{2}, \ldots, r_{m}\right)^{\prime}, \xi=\left[\begin{array}{ll}Z_{h \mathrm{X}, \mathrm{D}} & c\end{array}\right]^{\prime}$, and $c$ is the plural corresponding to the mean value of the constant $\beta_{0}$. And there is:

$$
Q=\left[\begin{array}{cccc}
I_{h \mathrm{D}} \angle \alpha_{c}(1) & I_{h \mathrm{D}} \angle \alpha_{c}(2) & \ldots I_{h \mathrm{D}} \angle \alpha_{c}(m)  \tag{29}\\
1 & 1 & \ldots & 1
\end{array}\right]
$$

$$
\begin{align*}
& r_{i}=\left(\operatorname{real}\left(U_{h \mathrm{X}} \angle\left(\theta_{h}+\alpha_{c}\right)(i)\right)-U_{\mathrm{0R}(b)}^{\prime}(i)\right)  \tag{30}\\
& +j\left(\operatorname{imag}\left(U_{h \mathrm{X}} \angle\left(\theta_{h}+\alpha_{c}\right)(i)\right)-U_{0!(b)}^{\prime}(i)\right)
\end{align*}, i=1,2, \ldots, m
$$

$U_{0 \mathrm{R}(b)}^{\prime}$ and $U_{0 \mathrm{I}(b)}^{\prime}$ is the biased value of $U_{0 \mathrm{R}}^{\prime}$ and $U_{0 \mathrm{I}}^{\prime}$ respectively. Equation (28) is the plural linear regression model, and the plural least squares solution to (28) is:

$$
\begin{equation*}
\xi=\left(\bar{Q}^{\prime} Q\right)^{-1} \bar{Q}^{\prime} r \tag{31}
\end{equation*}
$$

The first element of $\xi$ vector is the accurate calculation result of the $h$ time system harmonic impedance $Z_{h \mathrm{X}, \mathrm{D}}$.

## D. Accurate Calculation of Harmonic Responsibility

The injection harmonic current of load $\mathrm{D} I_{h \mathrm{D}}$ and the background harmonic voltage $U_{h \mathrm{X}, 0}$ is variable as time changes. So every time the corresponding harmonic voltage responsibility value $\mu_{h}$ is also a variable. Therefore when $Z_{h \mathrm{X}, \mathrm{D}}$ is known, at the moment $i$ there is:

$$
\begin{equation*}
\mu_{h}(i)=\frac{U_{h \mathrm{X}}^{2}(i)+\left|Z_{h \mathrm{X}, \mathrm{D}}\right|^{2} I_{h \mathrm{D}}^{2}(i)-U_{h \mathrm{X}, 0}^{2}(i)}{2 U_{h \mathrm{X}}^{2}(i)} \tag{32}
\end{equation*}
$$

## V. The simulation verification

In this paper, we use the IEEE 14 bus system as figure 3 as an example with Digsilent software simulation. In the case of 5th harmonic ,we select bus 5 to bus X , and HL is the harmonic source load of concern at bus X . The background harmonic voltage at X is produced by the harmonic source load HS at bus 13 and the generators in the system. Now we calculate $Z_{5 \mathrm{X}, \mathrm{D}}$ which is the rest of the system harmonic impedance except for HL and the background harmonic voltage $U_{5 \mathrm{X}, 0}$.


FIGURE III. IEEE-14 BUS SYSTEM

(a) the real part of $5^{\text {th }}$ harmonic current

(b) the imaginary part of $5^{\text {th }}$ harmonic current

FIGURE IV. $5^{\text {th }}$ HARMONIC CURRENT OF HARMONIC SOURCE HL

(a) the real part of $5^{\text {th }}$ harmonic current

(b) the imaginary part of
$5^{\text {th }}$ harmonic current

FIGURE V. $5^{\text {th }}$ HARMONIC CURRENT OF HARMONIC SOURCE HS
We use the classic curve to simulate the injection harmonic current of load HL and HS as shown in figure 4 and 5 respectively. Besides, let the measured interval be 1 min and measured duration be 3 s , and take 3 s ' average value for each measurement result. According to the above algorithm process to measure instantaneous harmonic voltage and harmonic source branch current at bus 5 .Then we can get a set of $5^{\text {th }}$ harmonic voltage vector at bus 5 and 5th harmonic source current vector of harmonic source through FFT transformation as figure 6 and 7 respectively. And there are 380 measurement points.


FIGURE VI. $5{ }^{\text {th }}$ HARMONIC VOLTAGE MEASUREMENT VALUE AT BUS 5

(a) The amplitude of harmonic current

(b) The angle of harmonic current

$$
\begin{array}{cc}
\text { FIGURE VII. } \quad 5^{\text {th }} \text { HARMONIC CURRENT MEASUREMENT } \\
\text { VALUE OF HL }
\end{array}
$$

According to the steps in III.B to construct the partial linear regression model, and use the method in IV.B to get the biased estimation of the background harmonic voltage as figure 8. In figure 8, the solid lines and the dashed lines show the real value and the estimate value of the background harmonic voltage respectively. It can be found that the difference of the biased estimate and the actual value is a constant as theorem 1.


## FIGURE VIII. THE BIASED ESTIMATION OF THE

 BACKGROUND HARMONIC VOLTAGEThe plural regression model as (17) can be constructed by the biased value which is obtained in the first stage, so we can use the plural least-square method to obtain $\beta_{0}$ in (17) which is the difference of the biased estimation and the actual value of the background harmonic voltage. Here, we can get the unbiased value of the background harmonic voltage by adding the deviation value $\beta_{0}$ to the biased value of the background harmonic voltage $f_{b}$ as figure 9.It can be seen that the background harmonic voltage estimation curve and the actual curve is essentially the same.


FIGURE IX. THE COMPARISON OF UNBIASED ESTIMATION AND THE REAL VALUE OF THE BACKGROUND HARMONIC VOLTAGE

TABLE I. THE COMPARISON OF SYSTEM HARMONIC IMPEDANCE ESTIMATION RESULT BY THE FOUR METHODS

| Method | Amplitude | Error <br> $(\%)$ | Angle <br> $\left({ }^{\circ}\right)$ | Error <br> $(\%)$ |
| :---: | :---: | :---: | :---: | :---: |
| Real value | 1.0776 | - | 79.55 | - |
| The binary <br> linear <br> regression | 1.0755 | 0.1949 | 77.28 | 2.85 |
| Dominating <br> fluctuating <br> quantity | 1.0767 | 0.0835 | 80.04 | 0.62 |
| The <br> maximum | 1.0765 | 0.1021 | 80.11 | 0.71 |
| likelihood <br> estimation | 1.0772 | 0.0371 | 79.18 | 0.34 |
| Proposed <br> method | 18 |  |  |  |

At the same time we can calculate the estimation value of the system harmonic impedance is $1.0772 \angle 79.18^{\circ}$. As shown in table 1, by comparing the estimate result of the method in this paper with the traditional algorithm we can found that the result of the proposed method is more accurate. Besides, the simulation results show that when the background harmonic voltage fluctuates at $8 \%$, the result still can meet the precision requirement.

As shown in figure 10, we can use equation (3) to calculate the load harmonic responsibility at every measuring point on the basis of the accurate calculation of the background harmonic voltage and the system harmonic impedance.


FIGURE X. THE RESULT OF THE LOAD HARMONIC VOLTAGE RESPONSIBILITY

In figure 10 , the averaging value for each measurement point result is the load side harmonic voltage responsibility:

$$
\begin{equation*}
\mu=\left(\sum_{i=1}^{380} \mu_{i}\right) / 380 \times 100 \%=15.3 \% \tag{33}
\end{equation*}
$$

## VI. COnclusion

Considering the change of the background harmonic, this paper proposed a method called partial linear regression model for accurately calculating harmonic responsibility under PQDIF. The main conclusions are as follows:
(1) On account of the PQDIF data format harmonic information, this paper proposed a method of computing the phase angle difference absolute value between harmonic voltage and harmonic current at PCC;
(2) Using the harmonic voltage, current and the phase angle difference absolute data to construct the partial linear regression model for accurately calculating harmonic responsibility under PQDIF;
(3)On account of the partial linear regression model, this paper designed the two-stage method to calculate the unbiased estimation of the equivalent harmonic impedance and the background harmonic voltage, finally achieved the accurate calculation of harmonic responsibility under PQDIF format.

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