The Stepwise Control Laws in the Problem of the Motion Optimization of the Electric Powered Transfers in the Earth-Moon System, Including L1-L2 and L2-L1 Missions

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***Abstract*—This paper outlines the optimization of the L1-L2 and L2-L1 missions using electric propulsion. The optimization criterion is the total flight time. The optimal control laws are obtained using the Fedorenko method to estimate the derivatives, the gradient method to optimize the control laws and the Runge-Kutta method for the numerical integration of the differential equation system. The study of these transfers is based on the restricted circular three-body problem. As the results of the optimization we obtained optimal control laws, corresponding trajectories and minimal total flight times.**

Keywords-spacecraft; low thrust engine; control laws; ballistic optimization; circular orbit; libration point; earth-moon system

I. INTRODUCTION

The possibility of the usage of the electric propulsion in the space transport missions was considered in the late 1990s. The main reason was the considerable increase of the output of the spacecraft (SC) electric power supply system (up to 15 kW and higher). The electric propulsion is able to guarantee the significant fuel saving in comparison with the chemical engines due to the high specific impulse [1-2].

In accordance with the worldwide practice, the control system is considered to be the most complex and important system in the SC. The failure of this system is the most safety critical for the SC performance. Therefore, the problem of the control laws optimization of the SC transfers is quite reasonable.

The optimal interplanetary trajectories and the trajectories of flights to the Moon pass near the libration point L1 of the Earth-Moon system, as shown in works [3-4]. The usage of the electric propulsion will greatly improve the mass efficiency of such transport operations and reduce the cost of creation and maintenance of the station.

II. MATHEMATICAL MODEL OF THE TRANSFERS

Let us formulate the general statement of the optimization problem. The following parameters are considered:

$\boldsymbol{x}(t) = (\boldsymbol{r}(t), \boldsymbol{V}(t), m_f(t), \boldsymbol{r}_E(t), \boldsymbol{r}_M(t), \boldsymbol{r}_S(t))^T \in \boldsymbol{X}$ is a system state vector corresponding to boundary conditions, defined by the purpose of the transfer and possible restrictions, where X is set of admissible state area;

$\boldsymbol{u}(t) = (\delta(t), \boldsymbol{e}(t))^T \in \boldsymbol{U}$ is a vector of control functions, where U is set of admissible control parameters;

$\boldsymbol{p} = (a_0, j_{sp})^T \in \boldsymbol{P}$ is the vector of optimized design parameters. It is limited by set of admissible area of the design parameters P.

Here t is the current time, $\boldsymbol{r}(t)$ is a radius vector of the SC, $\boldsymbol{V}(t)$ is a vector of the SC velocity, $m_f(t)$ is a expended fuel mass, $\boldsymbol{r}_E(t)$, $\boldsymbol{r}_M(t)$, $\boldsymbol{r}_S(t)$ are the radius-vectors of the Earth, the Moon and the Sun, $\delta(t)$ is the function of thrust switching, $\boldsymbol{e}(t)$ is the thrusting direction unit vector, a_0 is the nominal acceleration of the SC in the initial orbit, j_{sp} is the specific impulse of propulsion system (PS).

We consider the transfers between the Lagrange's point L1 orbit and the Lagrange's point L2 orbit.

The boundary conditions of the flights are determined by:

$$\boldsymbol{x}_1 = \left(\mathbf{r}_{L1}(t_0),\quad \mathbf{V}_{L1}(t_0),\quad 0,\quad \mathbf{r}_E(t_0),\quad \mathbf{r}_M(t_0),\quad \mathbf{r}_S(t_0)\right)^T \in \mathbf{X}_{L1} \quad (1)$$

$$\boldsymbol{x}_2 = \left(\mathbf{r}_{L2}(t_1),\quad \mathbf{V}_{L2}(t_1),\quad m_f,\quad \mathbf{r}_E(t_1),\quad \mathbf{r}_M(t_1),\quad \mathbf{r}_S(t_1)\right)^T \in \mathbf{X}_{L2} \quad (2)$$

In (1-2) r_{L1} and r_{L2} are the radius-vectors corresponding to the Libration point orbits (326.36 and 449.36 thousand km respectively). The m_f is the fuel consumption during the L1-L2 transfer.

Optimizing these space transfers with low thrust we need to determine the vectors $u_{opt}(t)$ and p_{opt} (vectors of optimal control functions and optimal design parameters correspondingly) that provide the minimum duration of flight T to perform the mission purposes according to (1-2).

$$T = \min_{\boldsymbol{u}\in\mathbf{U},\, \boldsymbol{p}\in\mathbf{P}} T \Big| m = unfixed, \boldsymbol{x} \in \mathbf{X} \quad (3)$$

The BCI frame (Figure 1) combines a flat polar frame and the additional phase coordinates (the inclination i and the node longitude Ω), which describes the instantaneous position of the orbital plane relative to the averaged plane of the Moon's orbit. The line formed by intersection of the ecliptic orbit plane and the averaged plane of the Moon's orbit, defines the axis OX, which is in vernal equinox direction.

According to the BCI frame conditions the state vector of the SC is defined as follows:

$$\boldsymbol{x} = \left(r, \quad u, \quad v_r, \quad v_\varphi, \quad \Omega, \quad i\right)^T \tag{4}$$

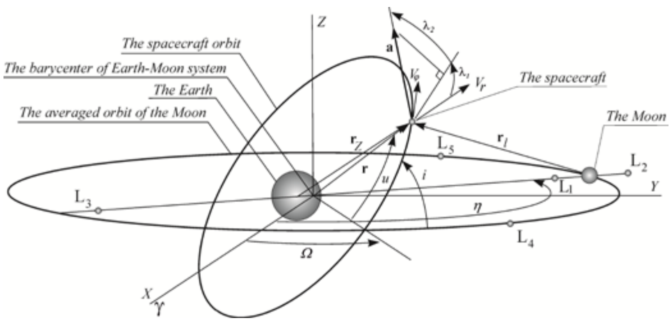

FIGURE I. ILLUSTRATION OF THE BCI FRAME COORDINATE AND THRUST STEERING ANGLES.

In the Figure 1 $\boldsymbol{V_r}$ and $\boldsymbol{V_\varphi}$ are the radial and the angular components of the vector of the SC velocity, η is the angle between the vernal equinox direction and the current position of the Moon.

The motion equations in the BCI frame are [5]:

$$\dot{\boldsymbol{x}} = \mathbf{M}(\boldsymbol{x}) \cdot \left(\frac{a_0}{1 - m_f} \chi(\boldsymbol{x}, \boldsymbol{e}) \delta \ \boldsymbol{e} + \boldsymbol{g} + \boldsymbol{f} \right) + \mathbf{D}(\boldsymbol{x}) \cdot \boldsymbol{x}$$
$$\dot{m}_f = \frac{a_0}{j_{sp}} \chi(\boldsymbol{x}, \boldsymbol{e}) \ \delta \tag{5}$$

$$\mathbf{M}(\mathbf{x}) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \dfrac{\cos i \ \sin u}{\sin i \ V_\varphi} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \dfrac{\cos u}{V_\varphi} \\ 0 & 0 & \dfrac{\sin u}{\sin i \ V_\varphi} \end{pmatrix}, \mathbf{D}(\mathbf{x}) = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{1}{r} & 0 & 0 \\ 0 & 0 & 0 & \dfrac{V_\varphi}{r} & 0 & 0 \\ 0 & 0 & -\dfrac{V_\varphi}{r} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{6}$$

Here $g\left(r, r_E, r_M, r_S\right)$ is the vector sum of the gravitational accelerations to the Earth and the Moon attraction centers, $f\left(r, r_E, r_M, r_S\right)$ is the vector of total perturbation acceleration, $\chi(x,e) = \dfrac{P\,(x,e)}{P_0}$ is the dependence of the specific fuel consumption on the state vector. The function form depends on the used models of the power plant functioning. In this study $\chi(x,e)$ is considered to be equal 1.

The gravity acceleration vectors from the Earth and the Moon in the local rotating frame are defined as

$$\mathbf{g}_E = \frac{\mu_l - 1}{\left|r_z\right|^3} \begin{pmatrix} r + R_z\left(cos(\eta - \Omega) cos\, u + sin(\eta - \Omega) sin\, u\, cos\, i\right) \\ R_z\left(-cos(\eta - \Omega) sin\, u + sin(\eta - \Omega) cos\, u\, cos\, i\right) \\ -R_z\, sin(\eta - \Omega) sin\, i \end{pmatrix} \tag{7}$$

$$\mathbf{g}_M = -\frac{\mu_l}{\left|r_l\right|^3} \begin{pmatrix} r - R_l\left(cos(\eta - \Omega) cos\, u + sin(\eta - \Omega) sin\, u\, cos\, i\right) \\ R_l\left(-cos(\eta - \Omega) sin\, u + sin(\eta - \Omega) cos\, u\, cos\, i\right) \\ R_l\, sin(\eta - \Omega) sin\, i \end{pmatrix}. \tag{8}$$

Here R_z is distance from Earth to the barycenter; R_l is distance from Moon to the barycenter; η is the Moon's longitude; $\boldsymbol{r_z}$ is the radius-vector from Earth to spacecraft, $\boldsymbol{r_l}$ is the radius-vector from Moon to spacecraft. These vectors are determined in the following manner:

$$\mathbf{r}_z = \begin{pmatrix} r\left(cos\, \Omega\, cos\, u - sin\, \Omega\, cos\, i\, sin\, u\right) - R_z\, cos\, \eta \\ r\left(sin\, \Omega\, cos\, u - cos\, \Omega\, cos\, i\, sin\, u\right) - R_z\, sin\, \eta \\ r\, sin\, i\, sin\, u \end{pmatrix}$$
$$\mathbf{r}_l = \begin{pmatrix} r\left(cos\, \Omega\, cos\, u - sin\, \Omega\, cos\, i\, sin\, u\right) - R_l\, cos\, \eta \\ r\left(sin\, \Omega\, cos\, u - cos\, \Omega\, cos\, i\, sin\, u\right) - R_l\, sin\, \eta \\ r\, sin\, i\, sin\, u \end{pmatrix} \tag{9}$$

All state variables are dimensionless. They are defined as the sum of the Earth's and the Moon's gravitational parameters and the distance from the Moon to the barycenter (R_l=1).

The following assumptions are made: the eccentricity of the Moon and the Earth orbits around barycenter is neglected; the eccentricity of the gravitational fields of the Earth, the Moon and the Sun is neglected, the SC sometimes moves in the Earth and the Moon shadow.

The thrust direction *e* is determined in the local rotating frame *Or τn*, where the axis *Or* is from the Earth to the spacecraft, the axis *On* is perpendicular to the orbital plane, and the axis *O*τ completes the Cartesian coordinate following the right-handed principle. Then unit vector ***e*** can be expressed in terms of the steering angles λ_1 and λ_2 (Figure 1).

$$e = \left(cos\, \lambda_2\, cos\, \lambda_1 \quad cos\, \lambda_2\, sin\, \lambda_1 \quad sin\, \lambda_2\right)^T. \tag{10}$$

III. STEPWISE OPTIMAL CONTROL STRUCTURE

The spacecraft characteristics were defined as follows: the initial thrust was assumed equal to *21.6 N* and the exhaust velocity as *20 km/s*. The initial SC mass including the payload block and the fuel was chosen equal *24000 kg*. In the L1-L2 and L2-L1 transfers it is supposed that the SC is inserted in the circular L1 or L2.

The transfer control program was divided into several stepwise segments of work to provide better accuracy. The three of them are introduced in the Figure 2 as an example:

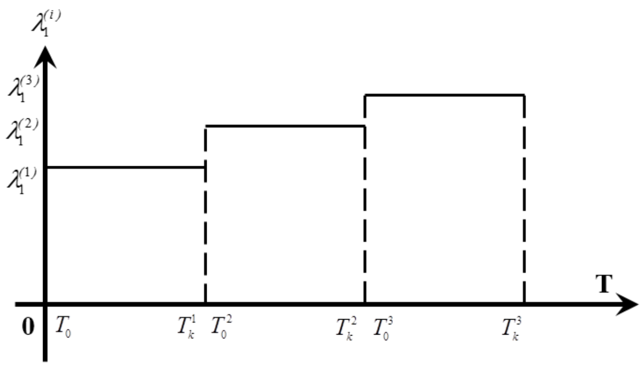

FIGURE II. THE SC CONTROL STRUCTURE DURING THE L1-L2 AND L2-L1 TRANSFERS

The thrust is directed at the angle $\lambda_1^{(i)}$ to the radius-vector of the SC. Thus, ***u*** is a piecewise continuous function which is determined by the following parameters: $\lambda_1^{(i)}$, $\partial T_0^{(i)}, \partial T_k^{(i)}$ (each $\lambda_1^{(i)}$ $\partial T_0^{(i)}$ and $\partial T_k^{(i)}$ is relevant to the corresponding segment of the trajectory).

To solve the settled problem we should find the following variables:

$$\frac{\partial I}{\partial \lambda_1^{(i)}};\frac{\partial I}{\partial T_0^{(i)}};\frac{\partial I}{\partial T_k^{(i)}};\frac{\partial I}{\partial T_0};\frac{\partial I}{\partial T};\frac{\partial I}{\partial a_0};\frac{\partial I}{\partial c_0} \qquad (11)$$

Here c_0 is the exhaust velocity.

In this work we use the Fedorenko successful linearization method [3] that accepts the limitation on composed functions that have Freshe derivatives. The method is based on making the variation optimal control problem the iteration problem of linear programming.

Thus, using the software package, designed by the authors, we get the optimal control programs for the L1-L2 and L2-L1 transfers and corresponding trajectories (Figure 3, 4). In the Figure 3, 4 red steps correspond to the active segments of the trajectory, and the green lines correspond to the passive segments, where the engine is turned off.

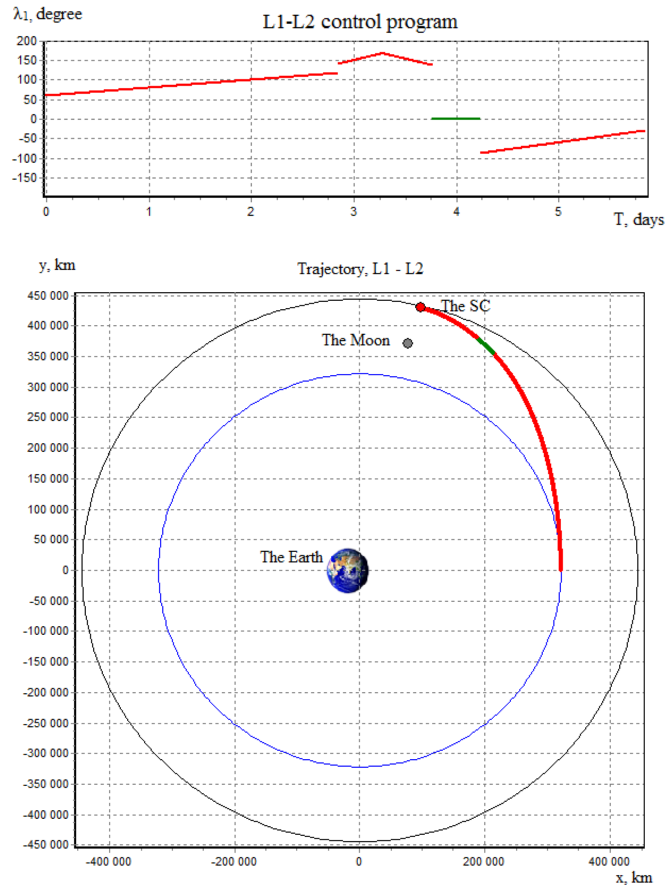

FIGURE III. THE OPTIMAL CONTROL PROGRAM FOR THE L1-L2 TRANSFER AND THE CORRESPONDING TRAJECTORY

In the Figure 3 we can see, that the first passive segment of the trajectory fully degenerates.

IV. CONCLUSION

The usage of the Fedorenko method to estimate the derivatives and the gradient method to optimize the control laws allows to determine the optimal control programs and the corresponding trajectories for the L1-L2 and L2-L1 missions. The obtained results are in good agreement with the results obtained by the usage of Pontryagin's maximum principle in the three-body task framework [6]. For ballistic optimization of mission, it is necessary to balance between fuel consumption and mission duration. The applied methods demonstrate their effectiveness for the complex optimization of the SC transfers. Findings may be used to calculate the required design-ballistic parameters of the future lunar missions.

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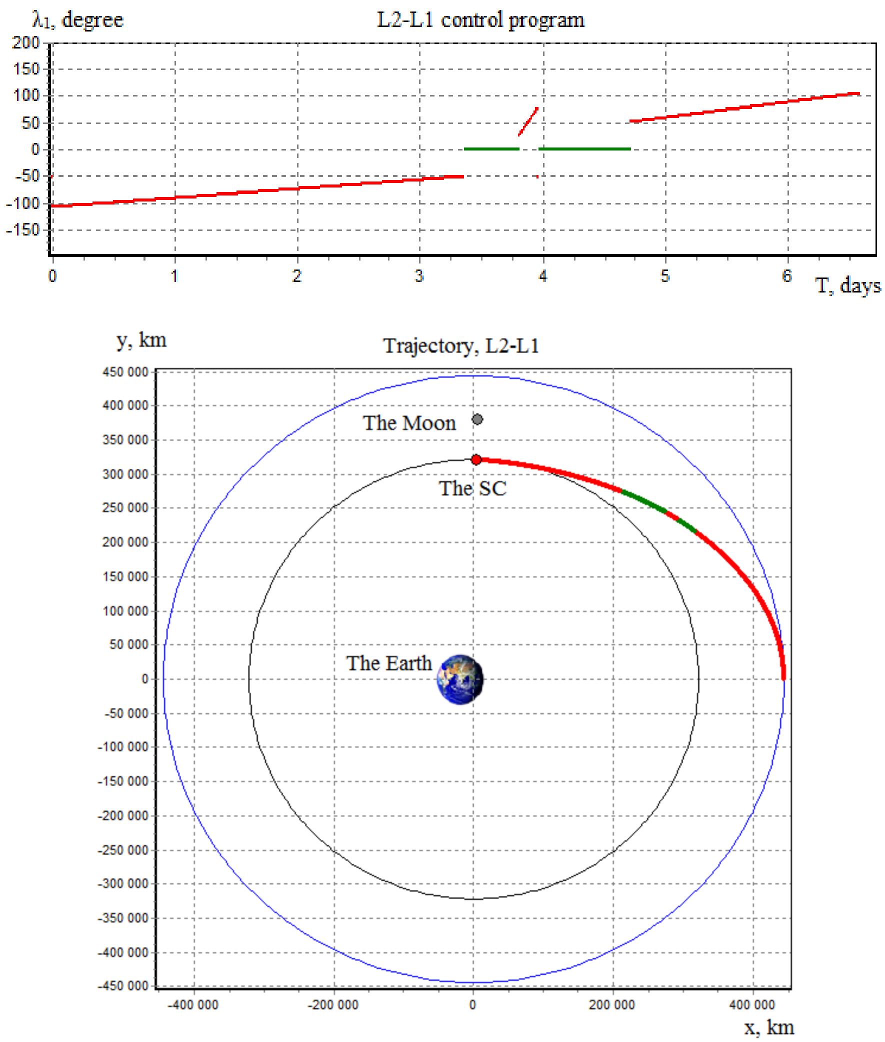

FIGURE IV. THE OPTIMAL CONTROL PROGRAM FOR THE L2-L1TRANSFER AND THE CORRESPONDING TRAJECTORY