Grey Correlation Coefficient Analysis and Estimation of Urban Living Garbage in Harbin

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Abstract. The amount of garbage conforms to the gray data characteristics because of the uncertainty and dispersion of urban living garbage produced, the corresponding dynamic prediction model based on Grey model can be established due to the uncertainty and dispersion of the collected amount in line with Grey feature. First the method of Grey correlation coefficient analysis was used to quantitatively analyze the impacts of waste generation factors, the result shows that the first order of the precision is per capita consumption expenditure; Second the gray GM(1.1) prediction model was established to predict the amount of garbage produced in Harbin for the next few years. The result is helpful for the management and the planning of Harbin.

Introduction

In our country, with the rapid development of urban economy, the social problems and the environment problem caused by urban living garbage are increasingly highlighted, discharge of municipal solid waste is the important basis of planning and construction of urban environment health, the prediction of Municipal solid waste output is one of the important reference index accepted as city life garbage disposal and transportation development planning and urban development and planning, it's an important fundamental research work. The uncertainty and dispersion by urban living garbage producing makes its QingYunLiang to conform to the characteristics of grey data[1], the Urban living garbage output can use gray theory to analyze and set up a corresponding grey model to forecast.

Grey Relational Analysis

Grey relational coefficient and correlation degree. Correlation degree was used to describe degree of correlation of two systems in the process of development and changes. Using grey correlation analysis method can analyze more accurately the correlation between the two systems through a small amount of data. T correlation coefficient was used to Compare the close degree of two sequences[2] . let $Y_0(t), t=1,2,\cdots n$ to be the mother series for Garbage output, $Y_i(t)$ $(i=1,2,\cdots m)$ is the subsequence, the correlation coefficient of the subsequence comparing $Y_i(t)$ to the mother series $Y_0(t)$ at t time is:

$$\zeta_i(t)=\frac{\min\min\left|y_0(t)-y_i(t)\right|+\rho\max\max\left|y_0(t)-y_i(t)\right|}{\left|y_0(t)-y_i(t)\right|+\rho\max\max\left|y_0(t)-y_i(t)\right|} \quad (1)$$

Among $\min\min\left|y_0(t)-y_i(t)\right|$ t $\max\max\left|y_0(t)-y_i(t)\right|$ he minimum differential and the biggest difference between two levels , ρ is resolution, $0<\rho<1$ and usually take $\rho=0.5$. According to the correlation coefficient formula of the above equation (1) , the correlation degree of two sequences is

$$R_i=\frac{1}{n}\sum_{t=1}^{n}\zeta_i(t). \quad (2)$$

Correlation degree analysis about Harbin city life garbage output. Using correlation analysis method to analyze urban living garbage, several indicators[3]were selected generally from the

population, urban construction and economic factors, then collecting the relevant historical data in Harbin, specific data was listed in table 1: According to equation (1) and (2) grey correlation degree was calculated, the results were shown in table 2:

Table 1 Harbin city life garbage output and each influence factor from 2004 to 2013

year	garbage output	GDP	population	City gas rate	Total retail sales of consumer goods	Annual per capita consumption expenditure
2004	98	646.13	688.01	81.51	360.42	5445.7
2005	131.6	733.85	694.84	85.43	365.93	5815.66
2006	126	823.5	702.59	87.52	409.39	6419.21
2007	147	941.6	716.58	91.23	440.05	6805.3
2008	133	1102.39	725.01	91.2	578.6	7427.82
2009	145	1313.93	741.73	91.3	670.56	7899.81
2010	119	1538.94	753.11	92.62	784.95	8986.87
2011	147	1856.63	764.25	98.6	936.21	10097.95
2012	152	2318.14	772.3	97.66	1176.58	12015.81
2013	179	2724.08	781.67	98.15	1381.12	14250.78

Table 2 correlation degree and rank of Five indicators and garbage output

indicator	correlation degree	rank
Annual per capita consumption expenditure of urban households	0.697031	1
Total retail sales of consumer goods	0.632324	2
GDP	0.629015	3
City gas rate	0.627936	4
population	0.607275	5

By the above result, the biggest influence on garbage output factor is the per capita consumption expenditure, followed by total retail sales of social consumer goods, GDP and so on, Thus it can be seen economic factors on behalf of the residents living standard are the main factors influencing the life garbage output.

GM (1, 1) Model[3] about Garbage Output. Let waste output before k years to be the original series $x^{(0)} = (x^{(0)}(1), x^{(0)}(2), \cdots x^{(0)}(k))$, grey model are defined as following differential equation:

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = u \ . \tag{3}$$

Among $x^{(1)}(k)$ is the accumulation generation sequence of $x^{(0)}(k)$, $x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i)$, a is called the development coefficient, u is called grey action quantity. calculation of the parameters , a, u of the model was used the least square method, the above model by discretization was expressed by matrix for:

$$y_N = BP \ . \tag{4}$$

Among equation (4) $P = \begin{pmatrix} a \\ u \end{pmatrix} = (B^T B)^{-1} B^T y_N$, $B = \begin{pmatrix} -z^{(1)}(2) & -z^{(1)}(3) \cdots z^{(1)}(n) \\ 1 & 1 \quad \cdots \ 1 \end{pmatrix}^T$

$y_N = \left(x^{(0)}(2), x^{(0)}(3) \cdots x^{(0)}(n)\right)^T$, $z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1), k = 2, 3, \cdots n$

The model of the continuous variable is

$$x^{(1)} = \left(x^{(0)}(1) - \frac{u}{a}\right)e^{-at} + \frac{u}{a} \ , \tag{5}$$

model by Discretization is

$$x^{(1)}(k+1)=\left(x^{(0)}(1)-\frac{u}{a}\right)e^{-ak}+\frac{u}{a}\ ,\ k=0,1,2,\cdots,n\ , \tag{6}$$

After reduction the model is

$$x^{(0)}(k+1)=e^{-a(k-1)}\left(x^{(0)}(1)-\frac{u}{a}\right)(e^{a}-1)\,. \tag{7}$$

GM (1, 1) Model Prediction of Garbage Output on Harbin . Let garbage output data In 1997-2013 in Harbin to be the original sequence, as shown in table 1.Using the equation (4)-(7) to forecast Harbin city life garbage output[4] and obtain the following models:

$$x^{(1)}(k+1)=461.2346e^{0.0187k}-452.9946\,,\quad k=0,1,2,\cdots,n \tag{8}$$

The results was shown in table 3.

table 3 the actual and estimated values and the relative error of garbage output from 1997 to 2013 in Harbin

year	1997	1998	1999	2000	2001	2002	2003	2004	2005
estimated value / 10^4t	8.24	8.71	8.87	9.03	9.21	9.39	9.56	9.74	9.92
Actual value/ 10^4t	8.24	8.55	8.95	9.16	9.27	9.39	9.49	9.68	9.87
relative error / %	0.00	1.87	-0.89	-1.41	-0.65	0.00	0.74	0.62	0.51
year	2006	2007	2008	2009	2010	2011	2012	2013	
estimated value / 10^4t	10.11	10.3	10.49	10.7	10.9	11.1	11.31	11.53	
Actual value / 10^4t	10.07	10.29	10.57	10.86	10.97	11.01	11.21	11.41	
relative error / %	0.4	0.097	-0.76	-1.47	-0.64	0.81	0.89	1.05	

Precision Test . Grey model inspection methods commonly used posterior precision posterior difference test (the variance ratio). Namely, respectively, the variance of the original data sequence and residual error sequence S_1, S_2, let $C=\frac{S_2}{S_1}$ to be a posteriori difference test data. according to the formula $p=\left\{\left|e(k)-\overline{e}\right|<0.6745S_1\right\}$ to calculate the Small error probability. the precision of the model was usually depicted by c and p , and according to its value to classify the accuracy .The model precision grade standard was shown in table 4.

table 4 The model precision grade judging table

model precision grade	P	C
1	$p\geq 0.95$	$C\leq 0.35$
2	$0.80\leq p<0.95$	$0.35<C\leq 0.50$
3	$0.70\leq p<0.80$	$0.50<C\leq 0.65$
unqualified	$p<0.70$	$C>0.65$

Using residual analysis and a posteriori difference test the accuracy of the model, let the original data to be $x^{(0)}=(x_1^{(0)},x_2^{(0)},\cdots x_n^{(0)})$, then the residual error is $e(k)=x^{(0)}(k)-\hat{x}^{(0)}(k)$, the mean of The original sequence is $\overline{x}^{(0)}=\frac{1}{17}\sum_{k=1}^{17}x_k^{(0)}=9.9406$, the average error is $\overline{e}=\frac{1}{17}\sum_{k=1}^{17}e(k)=-0.00706$, he variance of the original data sequence is $S_1^2=\frac{1}{17}\sum_{k=1}^{17}\left(x_k^{(0)}-\overline{x}^{(0)}\right)^2=0.8705$, variance of residual error sequence is

$S_2^2 = \frac{1}{17}\sum_{k=1}^{17}\left(e(k)-\overline{e}\right)^2 = 0.0081$,so $C = \frac{S_2}{S_1} = 0.09626$, $p = \left\{\left|e(k)-\overline{e}\right| < 0.6745S_1\right\}$

According to table 4 the model precision[5] is grade 1 .According to this model to predict the waste output with 2015-2020, the results were shown in table 5, in the next few years the output will be a trend of increasing year by year.

table 5 Harbin garbage output prediction value In 2015-2020

year	2015	2016	2017	2018	2019	2020
Prediction value/10^4 t	11.96	12.19	12.42	12.65	12.89	13.13

Conclusion.

Per capita consumption expenditure is the greatest impact on Harbin city life rubbish emission through the calculation of grey correlation degree , followed by total retail sales of social consumer goods, so the economic factor on behalf of the residents living standard is the main factor that influence the production of household waste output. using GM (1, 1) model to forecast the Harbin city garbage output, model prediction precision and prediction accuracy is higher, the Harbin city life garbage output in the coming years will be a trend of increasing year by year.

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