

# Thermoeconomic Optimization for Three-Heat-Source Refrigerator with Linear Phenomenological Heat-Transfer Law

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**Keywords:** Finite-time thermodynamics; Three-heat-source refrigerator; thermoeconomic optimization

**Abstract.** Using finite-time thermodynamics, optimal analysis on a class of three-heat-source refrigerator for linear phenomenological heat-transfer law is done. The cooling load for the refrigerator per unit total cost is proposed as objective functions for the optimization. The optimum performance parameters which maximize the objective functions are investigated. Since the optimization technique consists of both investment and energy consumption costs, the obtained results are more general and realistic. The conclusions obtained here are of importance in the optimal design and reasonable use of the energy of real three-heat-source refrigerators.

## Introduction

The concept of an equilibrium and reversible Carnot cycle has played an important role in the development of classical thermodynamics. The coefficient of performance of a reversible inverse Carnot cycle has been used as an upper bound for refrigerator and heat pump performances. This upper bound of performance can only be achieved through the infinitely slow processes required for thermodynamic equilibrium. Therefore, it is not possible to obtain a certain amount of cooling or heating load by using heat exchangers with finite heat transfer areas. By introducing finite time processes, a new field “finite time thermodynamics” was born, and thus, more realistic limits have been developed for real heat engines, refrigerators and heat pumps. Absorption refrigerators and heat pumps utilize “low-grade” heat energy, such as waste heat in many industrial processes, heat engines, solar energy and geothermal energy. They have a large potential for saving primary energy and decreasing environmental thermal pollution [1,2]. In the last decade, many optimization and modeling studies for real absorption refrigerators, based on various performance criteria, have been performed by considering finite time and finite size constraints [1–14]. In these studies, the optimal performance characteristics have been investigated for the coefficient of performance [1–7], cooling rate [8–10], by taking into account absorption refrigerators models operating between three or four temperature levels. In this paper, the finite time thermoeconomic optimization technique [15], has been extended to irreversible absorption refrigerators.

## Irreversible three-heat-source refrigerator model

We consider an irreversible three-heat-source refrigerator operating with the temperatures of the high temperature heat source  $T_H$ , cooled space  $T_L$  and environment  $T_0$ . It is assumed that the working fluid in the refrigerator flows constantly and exchanges heat continuously with the three heat sources during whole cycle period. Owing to thermal resistance, the rates of heat transfer are finite; thus there are finite temperature differences between the working fluid and three heat sources. Assuming that the temperatures of the three isothermal processes in which the working fluid exchanges heat with the heat sources at temperatures.

$T_H$ ,  $T_L$  and  $T_0$  are, respectively,  $T_1$ ,  $T_2$  and  $T_3$  and the heat transfers obey linear phenomenological heat-transfer law, so that

$$Q_H = \alpha A_H (1/T_1 - 1/T_H) \tau \quad (1)$$

$$Q_L = \alpha A_L (1/T_2 - 1/T_L) \tau \quad (2)$$

$$Q_0 = \alpha A_0 (1/T_0 - 1/T_3) \tau \quad (3)$$

Where  $Q_H$  and  $Q_L$  are, respectively, the heats absorbed from the high-temperature source and the cooled space by the working fluid per cycle,  $Q_0$  is the heat released to the environment by the working fluid per cycle,  $\alpha$  is the heat-transfer coefficient between the working fluid and the heat sources,  $\tau$  is the cycle period,  $A_H$ ,  $A_L$  and  $A_0$  are, respectively, the heat-transfer areas between the working fluid and heat reservoirs at temperatures  $T_H$ ,  $T_L$  and  $T_0$ . The over all heat-transfer area is

$$A = A_H + A_L + A_0 \quad (4)$$

Besides the irreversibility of finite-rate heat transfer, we consider the irreversibilities inside the working fluid, i.e. the internal irreversibilities which result from the friction, eddies, mass transfer and other effects inside the working fluid. Based on the second law of thermodynamics, the total effect of the internal irreversibilities can be characterized by the entropy production  $\Delta S_i$  inside the cycle. Thus we introduce the parameter

$$I = \Delta S_0 / (\Delta S_H + \Delta S_L) \quad (5)$$

To characterize the internal irreversibility and  $I > 1$  for a real refrigerator, And  $I=1$ . In such a case, the above model represents an endoreversible three-heat-source refrigerator. This shows that the model established here is quite general and more useful.

From Eqs.(1)-(5), we can obtain the fundamental optimum relations of the cycle are obtained as

$$R = \frac{\alpha A}{(1 + 1/\sqrt{I})^2} \cdot \frac{\varepsilon}{\varepsilon + 1} \left[ \frac{1}{IT_0} - \frac{1}{T_H(1 + \varepsilon)} - \frac{\varepsilon}{T_L(1 + \varepsilon)} \right] \quad (6)$$

Using the above equations, one can obtain the expressions of the important performance parameters of the three -heat-source refrigerator.

### Thermoeconomic optimization

We consider the optimization of the cooling load per unit total costing order to account for both investment and energy consumption costs. The function to be optimized is defined as [10]

$$F = \dot{Q}_L / (C_i + C_e) \quad (7)$$

where  $C_i$  and  $C_e$  refer to annual investment and energy consumption costs, respectively. For the investment cost, we may consider the investment costs of the main system components, which are the heat exchangers and the compressor together with its prime mover. The investment cost of the heat exchangers is assumed to be proportional to the total heat-transfer area. On the other hand, the investment cost of the compressor and its driver is assumed to be proportional to its compression capacity or the required power input. Thus, the annual investment cost of the system can be given as

$$C_i = a(A_H + A_L + A_0) + b_1(\dot{Q}_H - \dot{Q}_L) \quad (8)$$

Where the proportionality coefficient for the investment cost of the heat exchangers,  $a$ , is equal to the capital recovery factor times investment cost per unit heat transfer area and its dimension is  $\text{ncu}/(\text{year m}^2)$ . The proportionality coefficient for the investment cost of the compressor and its driver,  $b_1$ , is equal to the capital recovery factor times investment cost per unit power and its dimension is  $\text{ncu}/(\text{year kW})$ . The unit  $\text{ncu}$  stands for the national currency unit. The initial investment cost is converted to equivalent yearly payments using the capital recovery factor [14,15]. The annual energy consumption cost is proportional to the power input, i.e.

$$C_e = b_2 \dot{W} = b_2(\dot{Q}_H - \dot{Q}_L) \quad (9)$$

where the coefficient,  $b_2$ , is equal to the annual operation hours times price per unit energy and its dimension is  $\text{ncu}/(\text{year kW})$ . Substituting Eqs.(1) to (4) and (10),(11) into Eq.(9), we get

$$F = \dot{Q}_L / [a(A_H + A_L + A_O) + b(\dot{Q}_H - \dot{Q}_L)] \quad (10)$$

where  $b = b_1 + b_2$  Using Eqs. (1),(2),(6) and (10) We obtain

$$F = \frac{\frac{\alpha A}{(1+1/\sqrt{I})^2} \cdot \frac{\varepsilon}{\varepsilon+1} \left[ \frac{1}{IT_0} - \frac{1}{T_H(\varepsilon+1)} - \frac{\varepsilon}{T_L(1+\varepsilon)} \right]}{aA + \alpha b \left[ A_H \left( \frac{1}{T_1} - \frac{1}{T_H} \right) - A_L \left( \frac{1}{T_2} - \frac{1}{T_L} \right) \right]} \quad (11)$$

The objective function given in Eq.(11) can be plotted with respect to the coefficient of performance of the refrigerator given in Eq.(6) for various a or b values, the economical parameter, as shown in Fig.2. As can be seen from the figure, there exists a  $\varepsilon_F$  which maximizes the objective function for a given a or b.

### Discussion

(1) From Eq.(6), when  $\varepsilon = 0$  or  $\varepsilon = \frac{T_H - IT_0}{T_H} \cdot \frac{T_L}{IT_0 - T_L} = \varepsilon_I$ , the cooling rate R is equal zero,

using the extremal conditions  $\frac{\partial R}{\partial \varepsilon} = 0$ , one can obtain, the cooling rate and the coefficient of performance, respectively, are given as

$$R_{\max} = \frac{\alpha A}{4(1+\sqrt{I})^2} \cdot \frac{T_L(T_H - IT_0)^2}{IT_0^2 T_H (T_H - T_L)} \quad (12)$$

$$\varepsilon_m = \frac{T_L(T_H - IT_0)}{T_H(IT_0 - T_L) + IT_0(T_H - T_L)} \quad (13)$$

The optimal region of the coefficient of performance should be determined by  $\varepsilon_m \leq \varepsilon < \varepsilon_I = \varepsilon_{\max}$ .

(2) The objective function given in Eq.(11) can be plotted with respect to the coefficient of performance of the refrigerator given in Eq.(6) for various a or b values, the economical parameter, as shown in Fig.2. As can be seen from the figure, there exists a  $\varepsilon_F$  which maximizes the objective function for a given a or b.

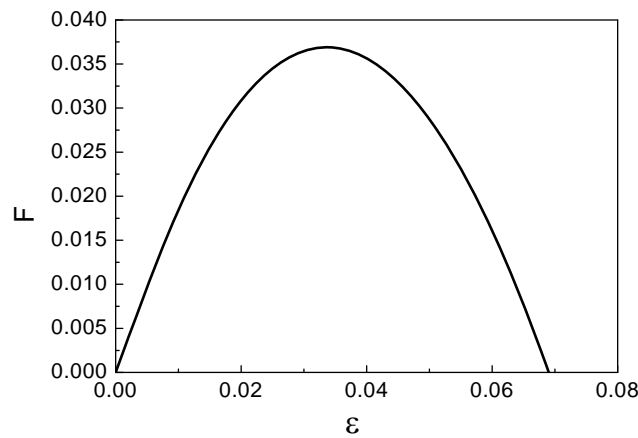


Fig.2 Variation of the objective function for the refrigerator with respect to  $\varepsilon$   
 $(I = 1.1, T_H = 400\text{K}, T_L = 250\text{K}, T_O = 350\text{K}, \alpha = 0.5\text{kW/m}^2\text{K}, a = 0.5, b = 1, A_H = A_L = A_O = 1\text{m}^2)$ .

## Summary

The irreversible three-heat-source refrigerator model devised in this paper is an important and useful cyclic model. It can be described by the two major irrversibilities which exist in a real three-heat-source refrigerator, that is, the irreversibilities of heat-transfer between the working fluid and reservoirs and the internal irreversibilities inside the working fluid. Using the thermoeconomic performance analysis to investigate the optimal performance of an irreversible three-heat-source refrigerator is effective and advantageous. A new objective function which includes both investment and energy consumption costs has been proposed. By optimizing these objective functions, the optimum operating and design conditions were determined. The economical analysis was also useful for a real three-heat-source refrigerator.

## Acknowledgements

This work was supported by the project of Science and Technology Research of Quanzhou, Fujian Provice of China (No.2009G10).

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