A Reliability Index Computation Method Based on Particle Swarm Optimization

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Abstract. For the structural limit state function with the multiple most probable failure points(MPP) and the multiple most probable failure domains, the reliability computation error generated by the First Order Reliability Method (FORM) or the Second Order Reliability Method (SORM) is unacceptable in some important project designs. The fundamental reason is that they only can obtain one design point and one most probable failure domain. An algorithm to find multiple MPPs with the particle swarm optimal method is proposed in the paper, and make a choice to estimate failure probability of structural limit state function based on the multiple most probable failure domains. The performance of proposed method is compared by the some difference methods. The results show proposed method is enough precise and effective.

Introduction

For the reliability computation of single structural failure mode described by a limit state equation with only one design point and one reliability index, FORM and SORM are firstly applied in the different approximation calculation methods. However, for a structural limit state equation with multiple design points, computation accuracy of FORM and SORM rapidly declines, and may generates a huge computation error [1], because the whole failure domain is substituted by only one most probable failure domain. In such a case, FORM and SORM should cautiously used, alternate methods should be investigated. There exist two main problems need to be solved: one is to obtain the multiple design points and reliability indexes; the other is to compute the failure probability in multiple most probable failure domains. Obviously, the former one is the premise and foundation for the reliability estimation. The multiple design points solving is temporarily discussed in this paper.

To find all design points, Ditlevsen and Madsen proposed multiple FORM/SORM [2], in which change initial iteration point tactics is used. Kiureghian and Dakessian changed performance function through adding barrier functions, and then the multiple design points were obtained with the iHL-RF method[1]. Wang and Grandhi transformed the reliability index into a polynomial function to solve the multiple reliability indexes[3]. Gupta and Manohar applied series squares regression to construct the response surface of the implicit limit state function, and then the multiple design points were searched with the Bucher-Bourgund method through the slight movement of the coordinate origin[4]. Barranco-Cicilia and Castro-Prates etc. firstly applied the genetic algorithm to search design points, and then applied FORM to determine the exact design points[5]. In fact, Barranco-Cicilia method is a second optimization method. It cost more computation time, and FORM's defects still exist. Undeniably, the construction of the barrier function in Barranco-Cicilia method is more efficient than Kiureghian and Dakessian method for the general reliability computation.

Particle swarm optimization(PSO) method is simple, easy to implement and fast convergence, and has been widely used in structural reliability analysis [6-8]. Firstly, the performance function is changed with the Barranco-Cicilia method， and then apply the PSO to search all the design points in the paper. The proposed method can satisfy the design requirements without the second optimization.

Single design point with PSO

In the standard normal space, reliability index equals to the shortest distance between the coordinate origin and the limit state surface. Corresponding point on the limit state surface is called the design point or the most probable failure point[9]. Solving the reliability index and MPP has become one of the most important content in reliability analysis. The structural reliability analysis method is transformed into an optimal design and computation method. The optimal mode for solving the reliability index and MPP as follow

$$\begin{aligned} &\min \quad \beta(\mathbf{x}) = \sqrt{\mathbf{x}^{\mathrm{T}}\mathbf{x}} \\ &s.t. \; g(\mathrm{x}) = 0 \end{aligned} \tag{1}$$

where $\mathbf{x}$ is random vector, $\mathbf{x} = [x_1; x_2; \cdots; x_n]$, in which statistically mutual independent basic random variable $x_i \sim N(0,1)(i = 1,\cdots,n)$. $\beta(\cdot)$ is the objective function. $g(\cdot)$ is the given performance function.

For the constrained optimization problems, PSO firstly transform the constrained optimization mode into an unconstrained optimization mode. Penalty function method is selected to achieve the transform in the method. By means of constructing the penalty function, the Eq. 1 is changed

$$\min \quad F(\mathbf{x}) = \beta(\mathbf{x}) + rg(\mathbf{x})^2 \tag{2}$$

where r is a positive penalty factor, equals to a constant or gradually changes.

Steps to solve Eq. 2 as follow

1. random initialize the velocity and position for the each particle; position is generally located in (-10,10).
2. transform the constrained optimization mode into the unconstrained optimization mode.
3. determine the best position and velocity of each particle and the best global position in all particles.
4. calculate the new velocity for each particle with Eq. 3.

$$v_{ij}^{k+1} = k\{v_{ij}^k + c_1 rand_1(p_{ij}^k - x_{ij}^k) + c_2 rand_2(g_{ij}^k - x_{ij}^k)\} \tag{3}$$

where v_{ij}^k is the jth current velocity of the ith particle in the kth iteration. k is the shrink factor k=0.7298. p_{ij}^k is the jth best position of the ith particle in the kth iteration. g_{ij}^k is the jth best position of the all particles in the kth iteration; $rand_1$ and $rand_2$ are the random numbers in (0, 1). c_1 and c_2 are the acceleration factors, $c_1 = c_2 = 2$.

5. update the position of the each particle with Eq. 4.

$$x_{ij}^{k+1} = x_{ij}^k + v_{ij}^k \tag{4}$$

where x_{ij}^k is the jth current position of the ith particle in the kth iteration.

6. calculate the iterative number. satisfy the termination condition and turn to step 7; otherwise turn to step 2.
7. output the optimal solution, i.e. the single reliability index and design point.

Multiple design points with PSO

After the first design point is found with PSO, in the course of searching the second design point, the first design point should avoid to be obtained again. Similarly, all the former obtained designs point should avoid to be searched in searching the following design points. For this purpose, one barrier function is constructed centered at new obtained design point in time. How to select and construct the barrier function is very crucial in the proposed method. The barrier function should easily to be constructed and guarantee the design point can not been searched repeatedly. Obviously, adding barrier function to the performance function increase the difficulty and complexity in finding the reliability index and design point. Fortunately, as a kind of random intelligent optimal algorithm, PSO is not much sensitive to the complexity of optimal functions [10]. A closed hyper-sphere is selected to act as the barrier function in the proposed method.

The original constrained optimal mode Eq.1 cannot be directly used to solve multiple design points. The difference is that the built barrier functions are acted as new constrained functions to update the optimal mode in the process of the optimization design. The optimal mode for the *k*th design point in the standard normal space is expressed by

$$\begin{aligned}
&\min \quad \beta(\mathbf{x}) = \sqrt{\mathbf{x}^{\mathrm{T}}\mathbf{x}} \\
&s.t.\; g(\mathbf{x}) = 0 \\
&\quad h_1(\mathbf{x}) = R - \left\|\mathbf{x} - \mathbf{x}_{MPP1}\right\| < 0 \\
&\quad h_2(\mathbf{x}) = R - \left\|\mathbf{x} - \mathbf{x}_{MPP2}\right\| < 0 \\
&\quad \ldots \quad \ldots \quad \ldots. \quad \ldots \\
&\quad h_{k-1}(\mathbf{x}) = R - \left\|\mathbf{x} - \mathbf{x}_{MPP(k-1)}\right\| < 0
\end{aligned} \tag{5}$$

Where, $h_i(\cdot)(i=1,\cdots,k-1)$ is the *i*th barrier constrained function. $\boldsymbol{X}_{MPPi}\,(i=1,\cdots,k-1)$ is the *i*th former design point. *R* is hyper-sphere radius, a small positive number, *R* =1~3.

Application example

The two given performance functions as follow

$$g_1(X) = 5 + 0.5(x_1+2)^3 - 1.5(x_1+2)^2 - x_2 \tag{6}$$

$$g_2(X) = -0.5(x_1^{\,2} + x_2^{\,2} - x_1x_2\;) - (x_1 + x_2)/\sqrt{2} + 3 \tag{7}$$

where mutual independent x_1 and x_2 are random variables with standard normal distribution. The design points and reliability indexes obtained by PSO and other methods are listed in Table 1 and Table 2, respectively.

Table 1 Results and comparisons for g_1

Computation method	PSO		Ref. [3]		Ref. [1]	
	$\boldsymbol{X}_{MPP1}$	$\boldsymbol{X}_{MPP2}$	$\boldsymbol{X}_{MPP1}$	$\boldsymbol{X}_{MPP2}$	$\boldsymbol{X}_{MPP1}$	$\boldsymbol{X}_{MPP2}$
Design point	0;3	– 3.4305;0.4671	0;3	– 3.431;0.466	0;3	– 3.4306;0.4660
Reliability index	3	3.4621	3	3.4625	3	3.4621

Table 2 Results and comparisons for g_2

Computation method	PSO			Ref. [1]		
	$\boldsymbol{X}_{MPP1}$	$\boldsymbol{X}_{MPP2}$	$\boldsymbol{X}_{MPP3}$	$\boldsymbol{X}_{MPP1}$	$\boldsymbol{X}_{MPP2}$	$\boldsymbol{X}_{MPP3}$
Design point	1.4715 – 0.7647	– 0.7648 1.4714	2.1560 2.1561	1.4716 – 0.7645	– 0.7645 1.4716	2.1213 2.1213
Reliability index	1.6583	1.6583	3.0492	1.6583	1.6583	3

From the table1 and table2, we can find the obtained design points and reliability indexes with PSO are quite close the results of Ref.[1] and Ref.[3]. This showed the proposed method is effective and acceptable, and can be applied to estimate the structural failure probability.

Conclusions

The more design points may be obtained by expending the iteration search space of the particle, but it is unnecessary for many practical engineering designs. The accuracy of the design point is closely related with the parameters selection in PSO and the hyper-sphere radius. The proposed method strengthens the PSO application in structural reliability engineering, and theoretically make significance for improving reliability computation accuracy.

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