# Fuzzy PID Control of Ship Course based on T-S Model

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**Abstract.** In order to achieve better performances of course-keeping and course-changing, the ship course maneuvering is divided into 3 typical stages, and then different Robust PID controllers and fuzzy membership functions are designed accordingly, and a comprehensive intelligent fuzzy PID course control system is formed based on T-S model. Simulation tests are done on a naval multifunctional transport ship considering wind, wave and current disturbances on course. The results indicate that the novel control system can achieve better control performances with less overshoot, swifter responding and high course-keeping precision.

## Introduction

For a given ship, there is an obvious trade-off between course changing and course keeping. The course keeping performance of ship autopilot is more important at open sea, while the course changing is more important for ship handling in restricted waters, near the waypoint or to prevent collision. In this paper, the responding motion of course to rudder can be divided into 3 typical stages, such as swift course changing, course stabilization and course keeping [1]. Course controllers based on robust PID are brought up accordingly, which are combined using T-S fuzzy model [2,3]. Then an intelligent fuzzy PID system is formed, the control process of which is shown in Fig.1, which has better maneuverability in course changing and less consumption with good tracking precision in course keeping.

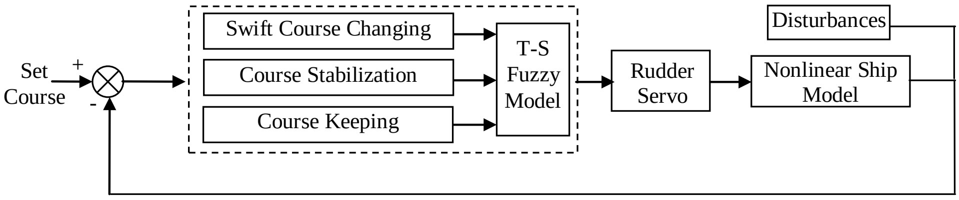


Fig.1 The ship course control based on fuzzy PID control

## Mathematical Model of Ship Motion

Nonlinear ship mathematical model describes the ship motion more precisely, while the controller design is generally based on linear model. For course controller, Nomoto model is popular, just as shown in (1).

$$G(s) = \frac{K_0}{s(T_0 s + 1)} \tag{1}$$

Where, $K_0, T_0$ are maneuverability indices, if a constant disturbance $\varepsilon$ is taken into account [4], the Nomoto model can be extended into (2).

$$G'(s) = \frac{K_0}{T_0 s^2 + s + \varepsilon} \tag{2}$$

Perez and Blanke presented a nonlinear ship model with four degrees of freedom using a roll planar motion mechanism (RPMM) [5]. The basic equations are formulated as follows

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$$
\begin{aligned}
&m(\dot{u} - vr - x_G r^2 + z_G pr) = X \\
&m(\dot{u} + ur + x_G \dot{r} - z_G \dot{p}) = Y \\
&I_z \dot{r} + m x_G (\dot{v} + ur) = N \\
&I_x \dot{p} - m z_G (\dot{v} + ur) + \rho g \nabla \overline{GM} \phi = K
\end{aligned}
\tag{3}
$$

Where, $m$ is ship mass, $\nabla$ is ship displacement, $g$ is gravity constant, $\rho$ is water density, $I_x, I_z$ are the inertias with respect to $x$-axis and $z$-axis, ($x_G, z_G$) is the center of gravity, $u, v$ are surge and sway speeds, $\phi, p$ are angle and angular velocity of roll, $\psi, r$ are angle and angular velocity of yaw motion. $X, Y, N, K$ are hydrodynamic forces and moments, defined as follows

$$
\begin{aligned}
X &= f(u, \dot{u}, v, \dot{v}, r, \dot{r}, p, \dot{p}, \phi, \delta, ...) \\
Y &= f(u, \dot{u}, v, \dot{v}, r, \dot{r}, p, \dot{p}, \phi, \delta, ...) \\
N &= f(u, \dot{u}, v, \dot{v}, r, \dot{r}, p, \dot{p}, \phi, \delta, ...) \\
K &= f(u, \dot{u}, v, \dot{v}, r, \dot{r}, p, \dot{p}, \phi, \delta, ...)
\end{aligned}
\tag{4}
$$

### PID Course Controller Design

With the development of automatics, the control algorithm of steering autopilot changes with each passing day. However, the above 90% products in industrial control are designed based on PID, given as follows

$$
K = K_p e + K_i \int e dt + K_d \dot{e} \tag{5}
$$

Where, course error $e = \psi_d - \psi$, $\psi_d$ is set course, $K_p, K_i, K_d$ are PID parameters, which can be derived by trail and error method. An empirical formula for PID is derived from extended Nomoto model based on closed-loop gain shaping algorithm (CGSA), and the dynamic control performance can be improved when proportionality coefficient is added a small positive $\rho$.

$$
K_p = \frac{1}{K_0 T_1} + \rho \tag{6}
$$

$$
K_i = \frac{\varepsilon}{K_0 T_1} \tag{7}
$$

$$
K_d = \frac{T_0}{K_0 T_1} \tag{8}
$$

The control plant is a multi-functional naval ship, the main data of which is indicated in Tab.I. 3 course controllers, such as course changing $C_c$, course stabilization $C_s$, and course keeping $C_k$, are designed according to different stages based on GA optimization of section V,and the results are shown in Tab.II, Fig.2 and Fig.3.

TABLE I. PARTICULARS FOR A NAVAL MULTIFUNCTIONAL TRANSPORT SHIP

| | | | | | |
|---|---|---|---|---|---|
| Length | 51.5m | Displacement | 355.88 $m^3$ | Rudder Speed Limit | ±5°/s |
| Beam | 8.6m | Rudder Area | 2.6 $m^2$ | $\varepsilon$ | 0.001 |
| Draft | 2.29m | Block Coefficient | 0.39 | $K_0$ | -0.22 |
| Speed | 15.0kn | Hard Over Stop | ±35° | $T_0$ | 4.59 |

TABLE II. THE CONTROLLER DESIGNING OF DIFFERENT COURSE-CHANGING STAGE

| Controller | $K_p$ | $K_i$ | $K_d$ | Rise Time | Overshoot | Rudder angle |
|---|---|---|---|---|---|---|
| $C_c$ | -0.95 | $-4.50\times10^{-4}$ | -2.10 | 25s | 3.3° | 22.9° |
| $C_s$ | -0.10 | $-9.09\times10^{-5}$ | -0.42 | 84s | 1.6° | 3.1° |
| $C_k$ | -0.05 | $-2.27\times10^{-5}$ | -0.10 | 240s | 0° | 0.78° |

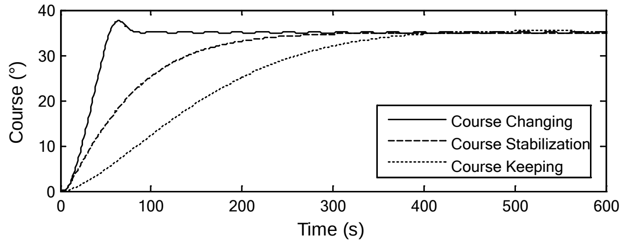

Fig.2 The output course of different controllers

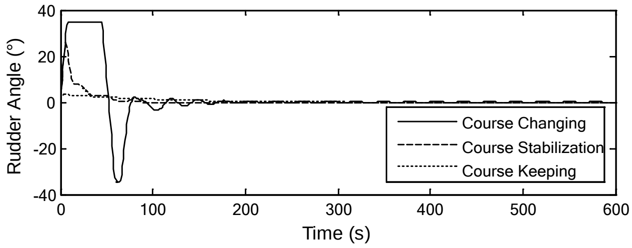

Fig.3 The input rudder of different controllers

## Controller Design based on T-S Model

The input and output data of a MIMO T-S fuzzy inference systems is defined as [6, 7]

$$\{(x_l, y_l), l = 1, 2, 3..., n\} \tag{9}$$

$x_l = (x_{l1}, x_{l2}, ..., x_{ln})$ is input, $y_l$ is output. Assume $i^{th}$ input variable is divided into $K_i$ fuzzy sets.

$$A_{i1}, A_{i2}, ...A_{ij}, ..., A_{iK_i}; i = 1, 2, ..., n; j = 1, 2, ..., K_i \tag{10}$$

Fuzzy rules of T-S model is defined as

$$\begin{aligned} &\mathrm{R}^1: \ if \ x_1 \ is \ A_{11} \ ... \ and \ x_r \ is \ A_{1r} \quad \text{then } u = f_1\left(x_1, \cdots, x_r\right) \\ &\mathrm{R}^2: \ if \ x_1 \ is \ A_{21} \ ... \ and \ x_r \ is \ A_{2r} \quad \text{then } u = f_2\left(x_1, \cdots, x_r\right) \\ &\cdots \\ &\mathrm{R}^m: \ if \ x_1 \ is \ A_{m1} ... \ and \ x_r \ is \ A_{mr} \quad \text{then } u = f_m\left(x_1, \cdots, x_r\right) \end{aligned} \tag{11}$$

Where, $x_1, x_2, \cdots, x_r$ are fuzzy antecedents, corresponding domains are $Z_1, Z_2, \cdots, Z_r$, $A_{ij} \in F(Z_i)$ is the fuzzy set of $x_i$, and $i = 1, 2, \cdots, r$, $j = 1, 2, \cdots, m$. $u$ is output control variable. Centroid method is used in defuzzification, just as shown as

$$\sum x = \frac{\sum_{j=1}^{m} \omega_j f_j\left(x_1, x_2, \cdots, x_r\right)}{\sum_{j=1}^{m} \omega_j} \tag{12}$$

Where, $\omega_j$ is membership function for $j^{th}$ rule, $\sum x$ is the comprehensive output. The fuzzy consequents $f_j\left(x_i\right)$ take the form of polynomial, $\omega_j$ is deduced in Sum-Product.

$$\omega_j = A_{j1}(x_1) \cdot A_{j2}(x_2) \cdots A_{jr}(x_r) \tag{13}$$

3 course control outputs are connected in parallel, using T-S fuzzy rule and corresponding membership functions. $\gamma = \psi / \psi_d$ is fuzzy antecedents, the domain is [0 1]. For swift course changing stage, the membership function is set to Z type, while the bell shape and Sigmoid type membership functions are separately applied on course stabilization and course keeping. Exact functions are shown as (14) to (16), and the parameter configuration is indicated in Fig.4.

$$
f(x,a,b)=\begin{cases} 1, & x \le a \\ 1\text{-}2\left(\dfrac{x-a}{b-a}\right)^2, & a \le x \le \dfrac{a+b}{2} \\ 2\left(b-\dfrac{x}{b-a}\right)^2, & \dfrac{a+b}{2} \le x \le b \\ 0, & x \ge b \end{cases} \tag{14}
$$

$$
f(x,a,b,c)=\frac{1}{1+\left|\dfrac{x-c}{a}\right|^{2b}} \tag{15}
$$

$$
f(x,a,c)=\frac{1}{1+e^{-a(x-c)}} \tag{16}
$$

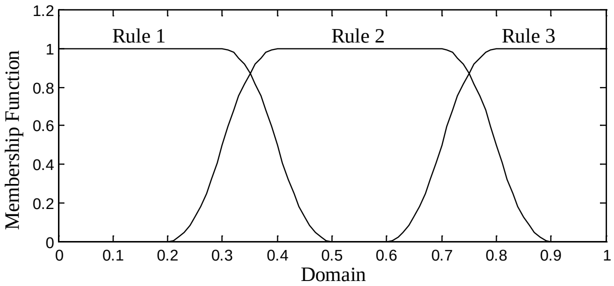


Fig.4 The diagram of membership functions

## The Steps of GA Optimization

Generic algorithm can simulate the natural evaluation by the basic operation, such as reproduction, crossover and mutation. The search procedure of GA is highly parallel, random and global adaptive. The detailed search procedure is as follows [8].

(1) improved genetic algorithm used two-dimensional code strategy is applied to initialize the population, which can improve the convergence speed and global searching ability;

(2) the individuals are selected and evaluated according to the fitness function, which is defined to minimize the global variance between the output and the reference data, that is:

$$
\min f(N)=\sum_{i=1}^{m}(U_i - U_{0i})^2 \tag{17}
$$

Where, $N$ is the population number, $m$ is the number of samples, $U_i$ is the $i^{\text{th}}$ output data, and $U_{0i}$ is the $i^{\text{th}}$ reference data.

(3) new populations are produced following the rules of reproduction, crossover and mutation according to the genetic probability. The crossover operator is combined with arithmetic cross and adjacent floating point crossover. The crossover rate $P_c$ and the probability of mutation are adaptively adjusted according to the following rules:

$$
\begin{cases} P_c^n = P_c^{n-1} - (P_c^0 - 0.6)/Q \\ P_m^n = P_m^{n-1} - (0.1 - P_m^0)/Q \end{cases} \tag{18}
$$

Where, $n$ is iterations, $Q$ is the maximum number of generations, $P_c^0, P_c^n$ are the initial value and the $n^{th}$ iterated value of crossover rate, $P_m^0, P_m^n$ are the initial value and the $n^{th}$ iterated value of probability of mutation.

(4) repeat the steps (2) and (3) until the termination conditions are fulfilled, the best individual is recognized as the result of the optimization based on GA.

When the ship is navigating in open waters, the course-keeping performance is more important for ship autopilot system. The variance of the sea state has direct influence to the course-keeping precision. In order to improve the performance, it is reasonable to minimize the course deviation and the steering gear wear. Then the fitness function can be defined with a minimum sum of the course error variance $\Delta\psi_i^2$ and the rudder angle variance $\delta_i^2$.

$$\min f_1(N)^{-1} = \left[\sum_{i=1}^{N} (\Delta\psi_i^2 + \delta_i^2)\right]^{-1} \tag{19}$$

If the ship is encountering severe sea state, the roll amplitude is increasing heavily to affect the safety of the ship and the cargo on it. In such case, the ship maneuvering strategy is to heading the wind and waves to avoid further rolling. Consequently, a course-keeping autopilot with RRD function does great good to the navigation safety in heavy seas. The fitness function for RRD can be defined to minimize the variance of roll angle $\Delta\phi_i^2$.

$$\min f_2(N)^{-1} = \left[\sum_{i=1}^{N} (\Delta\psi_i^2 + \delta_i^2) + 0.01\right]^{-1} \tag{20}$$

## Simulation and Results

In the simulation tests, the wind is divided into mean wind and fluctuating wind. The fluctuating wind is considered as white noise. The mean wind is taken as wind induced rudder angle $\delta_{\text{wind}}$, which has a empirical expression as (21).

$$\delta_{\text{wind}} = K^0 (\frac{V_{\text{R}}}{U_0})^2 \sin\gamma_w \tag{21}$$

Where, $K^0$ is leeway coefficient, $V_{\text{R}}$ is wind speed, $U_0$ is ship speed, $\gamma_w$ is the wind angle. The wind force is set to 6, $V_{\text{R}}$ is 12m/s, $K^0$ is 0.05, and the wind is from north.

The nonlinearity of steering gear includes saturation and backlash. The setting is as follows

$$\begin{gathered} \delta_{\max} = 35^\circ \\ 2\frac{1}{3}(\text{deg}/s) \le \dot{\delta}_{\max} < 7(\text{deg}/s) \end{gathered} \tag{22}$$

The testing program is shown in Tab.III. Contrast test 4 is based on CGSA, and T1 is set to 20s.

TABLE III. THE MODEL PERTURBATION AND DISTURBANCE SETTING

| Test No. | No 1 | No 2 | No 3 | No 4 |
|---|---|---|---|---|
| Set Course | 20° | 50° | 90° | 50° |
| Controller | Fuzzy PID | Fuzzy PID | Fuzzy PID | CGSA |

The simulation results are indicated in Fig.5-Fig.7. The time of course adjusting are 25s, 40s and 54s for test 1 to 3 with fast responding speed. The course overshoots are less than 3°, and the course keeping has no steady-state error and precision is within ±0.5° under the wind disturbance. The rudder angle input is shown in Fig.6, and rudder movements are reasonable with swift starting and checking. The largest rudder angle is proportional to set course magnitude. When the course error is small, the rudder control is set to course keeping model, which consumes less energy.

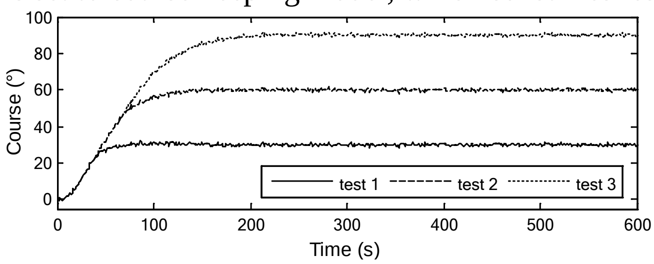


Fig. 5 Course simulation results of 3 different tests

The results of contrast test are indicated in Fig.7. Compared to standard CGSA, the fuzzy PID control system can achieve better performance of course changing and course keeping.

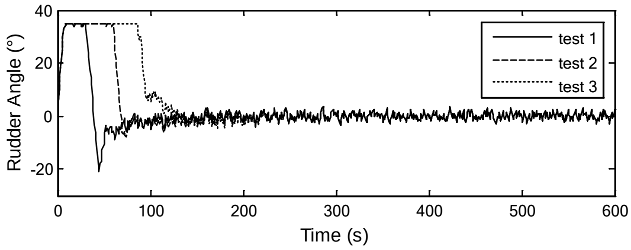

Fig.6 Rudder angle simulation results of 3 different tests

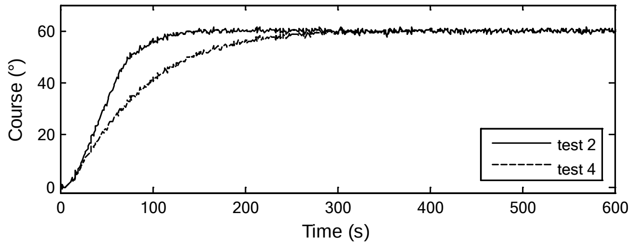

Fig. 7 Simulation results comparison with a 50° turn

## Conclusions

In order to achieve better performances of course-keeping and course-changing, the ship course maneuvering is divided into 3 typical stages, and then different Robust PID controllers and fuzzy membership functions are designed accordingly, and a comprehensive intelligent fuzzy PID course control system is formed based on T-S model. Simulation tests are done on a naval multifunctional transport ship considering wind, wave and current disturbances on course. The results indicate that the novel control system can achieve better control performances with less overshoot, swifter responding and course-keeping.

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