Three Models to Analyze the Uncertainty of Boiler Emission Flue Gas Heat Loss

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***Abstract*—Uncertainty assessment is a quantitative method about accuracy grade. This essay mainly focused on the principle of uncertainty analysis, including establish the uncertainty analysis model, assess the uncertainty components of type A and type B, and calculate the sensitivity coefficient and combined standard uncertainty. Because the result is easy to go wrong when using partial differential to solve the sensitivity coefficient, two kinds of mathematical models without using partial differential are put forward. Based on the uncertainty assessment of boiler emission flue gas heat loss, three models are used to calculate the combined standard uncertainty of boiler flue gas heat loss respectively, the results are contrasted and analyzed which certified the correctness of the two new models. In this way, a new calculation method is bought up, as well as simplifying the work of uncertainty assessment.**

Keywords-uncertainty; mathematical model; sensitivity coefficient; emission flue gas heat loss

I. INTRODUCTION

Generally, after the measurements and experimental work is achieved in the laboratory or on the scene, only the relevant test data are given in the test reports, but no description of the credibility of the test results is made. Strictly speaking, the testing effort is only half done, and the evaluation and analysis of test results is not conducted. Due to the existence of various uncertain factors, there is an error between the true value and the measured test data. Error analysis methods of the results are mostly used for error analysis. In recent years, uncertainty analysis error analysis has gradually replaced error analysis methods [1-2], for evaluation of the test results. Uncertainty is the reliability of the measurement results to characterize reasonably be attributed to the dispersion of the measured values, it is a parameter containing the measurement results, it is possible to characterize the quality of the test results quantitatively. In general, the uncertainty, the higher the quality of the test results, the more advanced test level, the higher the value.

In recent years, error analysis method has been gradually replaced by uncertainty analysis for the evaluation of the test results. In 1993, a standard, Guide to the Expression of Uncertainty in Measurement (GUM), was issued by seven international organizations: the International Organization for Standardization (ISO), the International Electrotechnical Commission (IEC), BIPM (BIPM), the International Organization of Legal Metrology (OIML), the International Federation of physics and chemistry and Applied Chemistry (IUPAC), the International Federation of Clinical Chemistry (IFCC), the International Federation of Physical and Chemical Physics and Applied Physics (IUPAPA). In 1995, GUM has been amended [3-4]. A unified standard for assessing, expressing and comparing the test results is provided by GUM with international common viewpoints and methods. In 1999, measurement specifications JJF1059-1999 "Measurement Uncertainty and Representation " is released by on the basis of GUM, used in place of JJF1027-1991 "measurement error and data processing" in the measurement error part.

II. THE PRINCIPLE OF UNCERTAINTY ANALYSIS

A. Mathematical Model

Before the uncertainty analysis, a mathematical model which meets the requirement of the measurement uncertainty should be built. Here the measured parameters Y is determined by a function of n inputs, X_1, $X_2 \cdots$:

$$Y = f(X_{1,} X_{2,} \cdots X_{n,}) \quad (1)$$

The mathematical model should meet the following requirements: (1) Include all input parameters that effect the measurement result. (2) No omission of any uncertainty components that effect the measurement result. (3) No repeat of any uncertainty components that effect the measurement result. (4) The model can be written into different forms when select different inputs.

B. Assessment in Type A or Type B of Uncertainty

The measurement uncertainty can be divided into type A and type B according to the assessment and expression.

- Measurement uncertainty assessment of type A

The type A uncertainty component (u_A) is measured by statistical methods after n times individual measurements. With n times individual measurement of the measured parameter x under the repeatability condition, the sample standard deviation ($s(x_i)$) which characterizing the results of each measurement can be get using the Bessel formula:

$$s(x_i) = \sqrt{\frac{\sum_{i=1}^{n} v_i^2}{n-1}} = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}} \quad (2)$$

And the standard uncertainty of the measurement is:

$$s(x)=\frac{s(x_i)}{\sqrt{n}}=\sqrt{\frac{\sum_{i=1}^{n}(x_i-\overline{x})^2}{n\times(n-1)}}=u(x) \tag{3}$$

- Measurement uncertainty assessment of type B

The assessment of type B uncertainty is mainly according to the following basis: (1) Previously observed data. (2) The experience and knowledge about the relevant technical materials and characteristic of the measurement instruments. (3) Documents from the production department. (4) Knowing of the acting rules of some influence factor on the system. (5) Data, accuracy class and levels from calibration certificate, test certificate or other documents, including the limit error currently in use. (6) Reference data and uncertainty from manual or some material. (7) The repeatability limit r and reproducibility limit R from national standards or technical documents that prescribes the test methods.

By the method above we get the $u(x_i)$, named measurement uncertainty of type B.

- Combined measurement uncertainty assessment

When the measurement result f is measured indirectly with several inputs x_i which are independent or uncorrelated, the combined standard uncertainty $u_c(y)$ can be obtained from this formula:

$$u_c(y)=\left[\sum_{i=1}^{n}\left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i)\right]^{1/2}=\left[\sum_{i=1}^{n}c_i^2u^2(x_i)\right]^{1/2} \tag{4}$$

Here the $c_i=\dfrac{\partial f}{\partial x_i}$, named as sensitivity coefficient is a partial derivative of the function f with respect to x_i.

- Extend measurement uncertainty assessment

The extend measurement uncertainty U is expressed by the multiples of the combined standard uncertainty. Here U is a numerical scale in a scale of $[y\text{-}U, y+U]$ the probability is high enough to cover most of the measurement results. For comparison we extend to a confidence level of about 95%, normally include the factor $k=2$ [5-6]:

$$U=ku_c \tag{5}$$

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III. The Simplified Assessment Of The Combined Standard Uncertainty

In the assessment process throughout the measurement uncertainty, the key part is to calculate the combined standard uncertainty, combined standard uncertainty required sensitivity coefficient is calculated, and the sensitivity coefficient required to solve partial differential, if more complex mathematical model, with partial differential calculation is too much trouble, prone to error, this paper proposes two methods to avoid solving partial differential, a relative standard uncertainty method, a method for the numerical interference. Analyze specific issues, you can choose different calculation model based on the actual problem. This paper utilizes three methods of heat losses uncertainty were assessed and compared.

A. Method of Obtaining the Relative Standard Uncertainty

Using the relative standard uncertainty, the combined standard uncertainty model can be simplified into three simpler model: the linear model only involves the sum or difference, the model involves only the product or quotient and the complex model.

- Definition of the relative standard uncertainty

The relative standard uncertainty is the key to solve the combined standard uncertainty $u_{rel}(x_i)$. So here introducing its definition as a ratio of a variable's uncertainty and the variable itself. The relationship is:

$$u_{rel}(x_i)=\frac{u(x_i)}{x_i} \tag{6}$$

- For the linear model only involves the sum or difference

For the linear model with a functional form only involves the sum or difference such as $y=c_1x_1+c_2x_2+\cdots c_nx_n$, the combined standard uncertainty is:

$$u_c(y)=\sqrt{c_1^2u^2(x_1)+c_2^2u^2(x_2)+\cdots c_n^2u^2(x_n)} \tag{7}$$

For this kind of model, a method of sum and root can be applied to get the result.

- For the model involves only the product or quotient

For the linear model with a functional form only involves the product or quotient such as $y=mx_1^{r_1}x_2^{r_2}\cdots x_n^{r_n}$, the combined standard uncertainty is:

$$u_{rel}(y)=\frac{u_c(y)}{y}=\sqrt{\sum_{i=1}^{n}\left[r_i\frac{u(x_i)}{x_i}\right]^2}=\sqrt{\sum_{i=1}^{n}[r_iu_{rel}(x_i)]^2} \tag{8}$$

In this formula, m is a constant and the exponent r_i can be positive or negative (ignoring the uncertainty of

r_i). $u_{rel}(x_i)=\frac{u(x_i)}{x_i}$ is the relative standard uncertainty whose sensitivity coefficient $|c_i|=|r_i|$.

$$u_c(y)=y\sqrt{\sum_{i=1}^{n}[r_i u_{rel}(x_i)]^2} \tag{9}$$

- For the complex model

The function of the complex model is over the limit of formula (7) and (8). It contains both the sum and difference part and the product and quotient part. Here is an example: $y=\frac{x_1+x_2}{x_3}$. When dealing with the combining of the uncertainty component in this model, we can decompose the original mathematical model for convenience. For example decompose $y=\frac{x_1+x_2}{x_3}$ into x_1+x_2 and x_3 so that the principle one or two can now be applied. Then by principle one we can obtain the temporary uncertainty of each part and by principle two we combined these temporary uncertainties together into the combined standard uncertainty.

$$\begin{aligned} u_{rel}(y)=\frac{u_c(y)}{y}&=\sqrt{[\frac{u(x_1+x_2)}{x_1+x_2}]^2+[\frac{u(x_3)}{x_3}]^2} \\ &=\sqrt{[\frac{\sqrt{u(x_1)^2+u(x_2)^2}}{x_1+x_2}]^2+[\frac{u(x_3)}{x_3}]^2} \end{aligned} \tag{10}$$

B. Numerical Method of Perturbation

As the calculation of the sensitivity coefficient is very complicated, someone suggested that [7] using the numerical method of perturbation to estimate the value in the calculation of combined standard uncertainty. Assume a variables x_i corresponding to a calculation value of f_1, when x_i changes a bit Δx (normal set as 1% of x_i), f_1 changes to f_2. So the sensitivity coefficient can be calculated as:

$$c_1=\left|\frac{f_1-f_2}{\Delta x}\right| \tag{11}$$

IV. Standard Uncertainty Of Boiler Flue Gas Heat Loss Calculation

Boiler emission heat loss is an indirect measurement values, involving the direct measurement of multiple parameters, for example, boiling coal composition, emission flue gas temperature, emission flue gas oxygen content, atmospheric pressure, atmospheric temperature and humidity, flue gas heat and water vapor heat. Therefore the assessment of uncertainty to boiler emission flue gas heat loss is a complicated process. Three uncertainty assessment method is used to assess the uncertainty. Assumption: the known conditions shown in Table I has been a type A and type B assessment.

TABLE I. Known Conditions Of Uncertainty Assessment

Measured Parameters	value	uncertainty
As received basis carbon（%）	44.15	0.43
As received basis hydrogen（%）	2.37	0.15
As received basis nitrogen（%）	0.76	0.09
As received basis oxygen（%）	3.85	1.11
As received basis sulphur（%）	0.56	0.06
As received basis total moisture（%）	11.2	0.27
As received basis ash（%）	37.11	0.97
Lower heating value（kJ/kg）	15860	135
Fly ash proportion（%）	90	5
Fly ash combustible content（%）	1.56	0.3
Slag proportion（%）	10	5
Slag combustible content（%）	3.84	0.39
Emission flue gas temperature（℃）	134.8	2.4
oxygen content oxygen content（%）	4.5	0.4
Air temperature（℃）	28.2	0.7
absolute humidity of air（kg/kg）	0.01	0.001
Dry flue gas Specific heat（kJ/m3.k）	1.38	0.04
Specific heat（kJ/m^3.k）	1.58	0.05

On the base of GB10184-88 and the principle of boiler [8], Emission flue gas heat loss can be calculated as follow:

TABLE II. Main Calculation Formula

Calculated Parameters	Calculation formula

Average ash carbon content（%）	$\overline{C}=\frac{\alpha_{lz}C_{lz}}{100-C_{lz}}+\frac{\alpha_{fh}C_{fh}}{100-C_{fh}}$
Actual burned carbon content（%）	$C_r=C_{ar}-\frac{A_{ar}\overline{C}}{100}$
Excess air coefficient	$\alpha_{py}=\frac{21}{21-O_2}$
Theoretical dry air flow（m3/kg）	$V_{gk}^0=0.089(C_r+0.375S_{ar})$ $+0.265H_{ar}-0.0333O_{ar}$
Theoretical dry flue gas flow（m3/kg）	$V_{gy}^0=1.866\frac{C_r+0.375S_{ar}}{100}$ $+0.79V_{gk}^0+0.8\frac{N_{ar}}{100}$
Actual dry flue gas flow（m3/kg）	$V_{gy}=V_{gy}^0+(\alpha_{py}-1)V_{gk}^0$
Steam volume in the flue gas（m^3/kg）	$V_{H_2O}=1.24(\frac{9H_{ar}+M_{ar}}{100}$ $+1.293\alpha_{py}V_{gk}^0d_k)$
Emission flue gas heat loss (kJ/kg)	$q_2=\frac{V_{gy}C_{p,gy}(t_{py}-t_0)}{Q_{net,ar}}$ $+\frac{V_{H_2O}C_{p,H_2O}(t_{py}-t_0)}{Q_{net,ar}}$

According to the above formula, first, in order to solve the uncertainty of boiler emission flue gas heat loss, the original method of definition is used. After that both relative standard uncertainty and numerical perturbation method are used to calculate the uncertainty and make a comparison.

The combined standard uncertainty of average ash carbon content $u_c(\overline{C})$ is calculated as an example.

- By using partial differential procedure, the uncertainty $u_c(\overline{C})$ can be expressed as follows:

$$u_c^2(\overline{C})=c_{\alpha_{fh}}^2u_c^2(\alpha_{fh})+c_{\alpha_{lz}}^2u_c^2(\alpha_{lz})$$
$$+c_{C_{fh}}^2u_c^2(C_{fh})+c_{C_{lz}}^2u_c^2(C_{lz})$$
$$=(\frac{C_{fh}}{100-C_{fh}})^2u_c^2(\alpha_{fh})+(\frac{C_{lz}}{100-C_{lz}})^2u_c^2(\alpha_{lz})$$
$$+[\frac{100\alpha_{fh}}{(100-C_{fh})^2}]^2u_c^2(C_{fh})+[\frac{100\alpha_{lz}}{(100-C_{lz})^2}]^2u_c^2(C_{lz})$$

Where $c_{\alpha_{fh}}$, $c_{\alpha_{lz}}$, $c_{C_{fh}}$ and $c_{C_{lz}}$ stand for fly ash quantity, slag quantity, carbon content in fly ash, the sensitivity coefficients of carbon content in fly ash, respectively. The result of calculation is:

$$\overline{C}=1.83\%$$
$$u_c(\overline{C})=0.354\%$$

- Calculate the sensitivity coefficient using the perturbation method. Here set the perturbation as 1% of the variable. The results are:

$$c_{\alpha_{fh}}=0.01585\ c_{\alpha_{lz}}=0.03993,$$
$$c_{C_{fh}}=0.9289,\ c_{C_{lz}}=0.1082.$$

$$u_c^2(\overline{C})=c_{\alpha_{fh}}^2u_c^2(\alpha_{fh})+c_{\alpha_{lz}}^2u_c^2(\alpha_{lz})$$
$$+c_{C_{fh}}^2u_c^2(C_{fh})+c_{C_{lz}}^2u_c^2(C_{lz})$$

So $u_c(\overline{C})=0.354\%$.

Calculating using relative standard uncertainty method. Because the function here contains product and quotient, we use the complex mode in the relative standard uncertainty calculation. Set that: $y_1=\frac{\alpha_{lz}C_{lz}}{100-C_{lz}}$, $y_2=\frac{\alpha_{fh}C_{fh}}{100-C_{fh}}$.

So

$$u_c(y_1)=\alpha_{lz}C_{lz}u_{rel}(\alpha_{lz}C_{lz})=$$
$$\alpha_{lz}C_{lz}\sqrt{[\frac{u(\alpha_{lz})}{\alpha_{lz}}]^2+[\frac{u(C_{lz})}{C_{lz}}]^2}$$

$$u_c(y_1)=y_1u_{rel}(y_1)=$$
$$\frac{\alpha_{lz}C_{lz}}{100-C_{lz}}\sqrt{[\frac{u_c(\alpha_{lz}C_{lz})}{\alpha_{lz}C_{lz}}]^2+[\frac{u(C_{lz})}{(100-C_{lz})}]^2}$$

By the same way: $u_c(\alpha_{fh}C_{fh})$ 与 $u_c(y_2)$.

$$u_c(\overline{C})=\sqrt{u_c^2(y_1)+u_c^2(y_2)}$$

The result is: $u_c(\overline{C})=0.351\%$.

The calculation above tells that the two new method of combined standard uncertainty have reasonable error with the original method using the definition. Using these three methods to assess the standard uncertainty in the heat loss in the smoke extracting gives the result table blow.

TABLE III. RESULT OF UNCERTAINTY OF BOILER EMISSION FLUE GAS HEAT LOSS

Calculated Parameters	Value	Uncertainty		
		definition	numerical perturbation method	relative standard uncertainty
Average ash carbon content （%）	1.83	0.354	0.354	0.351
Actual burned carbon content （%）	43.47	0.450	0.450	0.448
Excess air coefficient	1.27	0.031	0.031	0.031
Theoretical dry air flow （m^3/kg）	4.387	0.073	0.073	0.073
Theoretical dry flue gas flow （m3/kg）	4.287	0.111	0.111	0.111
Actual dry flue gas flow （m^3/kg）	5.471	0.177	0.177	0.177
Steam volume in the flue gas （m^3/kg）	0.473	0.019	0.019	0.020
Emission flue gas heat loss (kJ/kg)	5.58	0.257	0.257	0.261

The value of all uncertainties is calculated by the definition method.

According to the calculations, not only the principle of numerical perturbation method is consistent with partial differential method, but also the calculation result is equal, moreover, numerical perturbation method was able to avoid for solving partial differential and simplifying calculation. The calculation error of relative standard uncertainty method and the partial differential method is so small that can be ignored, however, there is a little deviation when assessing the uncertainty about multivariate multiplied. The two methods are completely equal the sum or difference of assessment as well. By comparing three uncertainty calculation method, all of the method can be used in evaluating combined standard uncertainty, and the error is within reasonable limits. But every method has advantages and disadvantages, only if different calculation models are used for different issues suitably, will uncertainty evaluation work processes can be greatly simplified.

V. CONCLUSIONS

Based on the relevant literature, the procedure of uncertainty assessment is described, including the establishment of uncertainty model, type A or type B of uncertainty assessment, the calculation of combined standard uncertainty and expanded uncertainty, and so on. By solving sensitivity coefficients partial differential formula, two new calculation models of the combined standard uncertainty are proposed. In this way, there is no need solving sensitivity factor directly, and the average error of the two models is within reasonable limits. Therefore, the two models can be used to assess the combined standard uncertainty. If the appropriate model is used for assessing uncertainty of specific issues, it may greatly simplify the uncertainty evaluation workload.

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