# Solution method of simplified classification power network model for Preserving Key Branches

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**Keywords:** security weaknesses, key branch, spectral partitioning, centroid sorting technology. **Abstract.** The key branches of power system can effectively reflect the security weaknesses of transmission line, the efficiency of system secure economic dispatch and control will be significantly improved by reducing the system to a precise model with the key branches preserved. A new network reduction method of preserving the key branches is proposed. Firstly, the spectral partitioning method is used to extract the power network feather and the preserving key branches reduction system is modeled. And then, an improved centroid sorting technology is applied for obtaining the optimal reduced model. Lastly, an algorithm based on power transfer distribution factor for computing the reduced networks flows is presented.

# Introduction

The security issues of power transmission have been the focus of electric power scientific workers. With the development scale of power system, the efficiency of dispatch and control will become lower and lower. What's worse, it will be more serious for the power system of uneven operation and weak link of transmission. In these cases, dispatch and control the weak link of the transmission system can greatly improve the efficiency of whole system. Monitoring the key branch is the most effective way to reflect the weak link of transmission system.

At the same time, equivalent simplification is particularly necessary for the complicated transmission power system. The simplified model can greatly improve the efficiency of safety analysis, which has been an important issue in theory research and engineering practice. In recent years, domestic and foreign experts and scholars have carried out a lot of research on these issues and have made a series of achievements. In order to simplify the security analysis and calculation of network, the centroid sorting technology is applied. Namely, the system will be divided into several regions. This method will control the security region, which perfectly improves efficiency; In order to simplify the security analysis, a new method for calculating the equivalent power network is proposed. The PTDF matrix of the system is used to estimate the equivalent reactance of the power network, thereby a simplified model of the system can be obtained. This method relies on the existence of the system operating points and there is a big error margin between the simplification of the trend flow and the actual network, but it provides a new method; In order to simplify the system, a fast identification method of key transmission is proposed based on the graph clustering theory. The proposed method is of great significance for improving the efficiency of system and the safety of weak links; In order to improve the precision of the simplified network model, a new method for calculating the DC flows of power system based on PTDF theory is proposed. The method does not depend on the operating points of the system, and the simplified flows obtained are of great precision. But it has some limitations on the existence of parallel branch of the simplified power network; In order to simplify the programming problems of large power system, a simplified method of bus system was developed based on the principle of the semiconductor PN junction. The simplified network which is suitable for calculating the optimal flow can not only effectively allocate the blocked branch flow, but also has high precision in the region of the power generation side and advance of marginal electricity price.

However, at present there are a little papers on the research of key branch of simplified equivalent power network. The development of the technology of Wide Area Measurement System and its

applications provide the basis for this study. On this basis, the key branch of the system is well known for the dispatcher.

In view of the above situations, this paper proposes the preserving key branch of the simplified model and the calculation method of DC power flow, assume that the key branch system is known, and ignore the condition of node voltage instability. First of all, Based on the optimal placement method of spectral clustering, the feather of the network is extracted, and the reserving key branch of simplified classification model of the power network is constructed; at last, the optimal simplified model of the power network is obtained by using the improved centroid separation method.

Part 1 of this paper is a reserving key branch of the power network of simplify classification model; Part 2 is the method to obtain the model of optimal power network; the last part is the conclusion.

#### A simplify classification model of preserving key branch of the power network

#### **Classification number**

Assume the system with T key branches, according to the connection relationship between edges and points, T edges have the most points can be connected is:

 $k_{\max} = T + 1. \tag{1}$ 

Therefore, the classification number k of power networks with T reserving key branches can be obtained is from 2 to  $k_{max}$ . While the value k is not the same, the corresponding electrical distance is not the same, either. The classification with minimum electrical distance is the optimal scheme, the corresponding scheme with  $k^*$  is the optimal classification.

## Spectral clustering algorithm for extracting the network feature

Spectral clustering algorithm is implemented by mapping the vertices of the graph with high-dimensional space to the low dimensional space to obtain classification. Its application can be described as: put the *N* nodes which belong to  $\Re^N$  space into its class subspace  $\Re^k$  ( $k \le N-1$ ) so that the weighted square of the distance between the electrical nodes can reach to the minimum, which is called the optimal placement problem mathematically, expressed as:

$$\min : F = \sum_{i=1}^{\infty} x_i^T \Phi x_i$$

$$st. \quad x_i^T x_j = \begin{cases} 1 & (i=j) \\ 0 & (i\neq j) \end{cases}$$
(2)

Among which,  $x_i$  (i = 1, 2, L, k) represents the coordinates of N-dimensional vector;  $\Phi$  is the Laplace matrix which is a real symmetric semi positive definite weight matrix for the graph. In the power system, the negative susceptance matrix can be considered as the Laplace matrix of the electricity network. Let  $\Phi$  equals -B.

Eq. 2 can be solved by using Lagrange multiplier method, the constrained optimization problem can be expressed as:

$$L = \sum_{i=1}^{k} x_i^T \Phi x_i - \sum_{i=1}^{k} I_i \left( x_i^T x_i - 1 \right).$$
(3)

Among which,  $I_i$  (i = 1, 2, L, k) is the Lagrange multiplier. Make Eq. 3 for partial differential,

$$\left(\Phi - I_{i}I\right)x_{i} = 0 \quad \left(i = 1, 2, \mathbf{L}, k\right).$$

$$\tag{4}$$

Further expressed as Eq. 5:

$$\Phi x_i = I_i x_i \quad (i = 1, 2, \mathbf{L}, k).$$
<sup>(5)</sup>

For the Eq. 5, it can be seen that,  $I_i$  (i = 1, 2, L, k) is the eigenvalue of  $\Phi$ , and  $x_i$  (i = 1, 2, L, k) is its corresponding eigenvector.

Multiply  $x_i^T$  on both sides of the Eq. 5 of the left end, make the equation in accordance with  $x_i^T x_i = 1$ , the following Eq. 6 can be acquired:

$$x_i^T \Phi x_i = I_i \quad (i = 1, 2, \mathbf{L}, k).$$
(6)

Substitute Eq. 6 into Eq. 2, the follow Eq. 7 can be obtained:

$$\min: F = \sum_{i=1}^{k} I_i .$$
<sup>(7)</sup>

Eq. 2 obtains the extreme values when taking the smallest eigenvalues of matrix  $\Phi$  in the front, the minimum eigenvalues are correspond to the feature vectors of  $x_i$  ( $i = 1, 2, \mathbf{L}, k$ ).

It is unclear of the classification of nodes after obtaining the optimal scheme by Eq. 7. Since these feature vectors  $x_i$  (i = 1, 2, L, k) are orthogonal to each other. However, they constitute the basis of space so that they can be expressed as the coordinates of the nodes in the system. Thus, we can tell the classification of the nodes through the distance between these coordinates.

However, it will cause the loss of coordinate information and the change of the order due to the projection of the points in space  $\Re^N$  to sub space  $\Re^k$ . Thus, classification error may occur. It is necessary to normalize the coordinates:

$$x'_{i} = -\lg |x_{i}| (i = 1, 2, \mathbf{L}, N).$$
(8)

$$x_i'' = x_i' / \max(x_i') .$$
<sup>(9)</sup>

Among which,  $x_i$  represents the vector which projected from space  $\Re^N$  to subspace  $\Re^k$ ,  $x_i''$  is the normalized coordinates vector.

After the simplification of Eq. 8 and Eq. 9, the node coordinates in space can play the role as the basis of classification.

#### A simplified DC power flow calculation method for preserving key branches

Preserve the network classification model of the key branch is to preserve the key branch in the simplified power network, the network nodes are classified according to their electrical distance. Critical end nodes within the branch is in a different class, there is no key branches within the same classes, but they can be aggregated with each other. Thus, the preserving key branch of simplified classification model of power network is obtained.

Assume the system with T key branches and the two ends of the nodes are respectively  $p_j$  and  $q_j$  ( $j=1,2,\mathbf{L},T$ ). The preserving T key branches can divide the system into k classes, which are as following Eq. 10:

min: 
$$D(k, N) = \sum_{i=1}^{N} \left( d_i^{C_i^m} \right)^2$$
  
s.t.  $\begin{cases} p_j \in C_{p_j}^w & (w \in [1,k]) \\ q_j \in C_{q_j}^v & (v \in [1,k]) \\ C_{p_j}^w \mathbf{I} \ C_{q_j}^v = \emptyset & (j \in [1,T]) \end{cases}$ 
(10)

Among which,

$$d_{i}^{C_{i}^{m}} = \left[\sum_{s=1}^{k} \left(A_{s}^{i} - B_{s}^{m}\right)^{2}\right]^{\frac{1}{2}}.$$

$$B_{s}^{m} = \left(\sum_{i \in m} A_{s}^{i}\right) / N_{m}.$$
(12)

D(k,N) represents the total electrical distance, divide N nodes into k classes;  $d_i^{C_i^m}$  represents the distance between the node *i* and its class  $C_i^m$ ;  $C_{p_j}^w$  represents the class containing the node  $p_j$ ,  $C_{q_i}^v$ 

represents the class containing the node  $q_i$ ;  $B_s^m$  represents the s dimensional coordinate of class m;  $N_m$  represents the total number of nodes in class m;  $A_s^i$  represents the s dimensional coordinate of node i, its value can be obtained from chapter about spectral clustering algorithm.

## Solution method of simplified classification model for power network

In order to obtain the simplified classification model of the power network. Based on the classification of the centroid sorting method, the algorithm steps are as follows:

1) According to chapter about spectral clustering algorithm, the nodes coordinate in space  $\Re^k$  can be obtained, then, they will be divided into *k* classes by the centroid sorting method;

2) Move the nodes submitted to Eq. 10 of the class  $C^m(m=1,2,\mathbf{L},k)$  to the class of  $C^v(v \neq m; v = 1,2,\mathbf{L},k)$ , compare with the reduction of each electrical distance after moving, locate and move to the class which has the largest scale of decrease of electric distance. Cycle in turn until moving any nodes which submitted to the constraint will no longer decrease. Thus, the optimal classification scheme can be obtained. Its classification of electrical distance can be written as D(k);

3)Make value k plus one, continue to calculate according to the step 2),then, optimal classification scheme can be obtained under the circumstance of k+1, the electric distance can be written as D(k+1);

4) Iterate step2) and step3) until  $k = k_{max}$ , the optimal classification scheme and the electrical distance is between D(2) and  $D(k_{max})$  after the classification is obtained. The classification scheme of the minimum electrical distance is the best. In order to make it comparable of the electrical distance of different categories,  $k = k_{max}$  is useful for feature extraction in the Eq. 7.

## **Centroid sorting method**

The coordinates of each node can be obtained based on chapter about spectral clustering algorithm, the system can be divided into k classes.

$$C_i^m = \min[k, T_i] \quad (i = 1, 2, \mathbf{L}, N; m \in [1, k])$$
 (13)

Among which,  $C_i^m$  represents the class where node *i* located;

$$T_{i} = \begin{bmatrix} 1 + \frac{k(S_{i} - S_{\min})}{S_{\max} - S_{\min}} \end{bmatrix}.$$
(14)

Symbol  $\Box$  represents rounding operation,  $S_i$  represents the algebraic sum of all dimensional coordinates of node *i*, as is in Eq. 15:

$$S_i = \sum_{s=1}^k A_s^i \,. \tag{15}$$

 $S_{\min}$  and  $S_{\max}$  represent the minimum value and maximum value of  $S_i$  (i = 1, 2, L, N), respectively.

By using this classification method, the N nodes in the system can be allocated into k classes.

## **Centroid sorting optimization**

However, the classification method both in terms of key branch and the electric distance optimal aspects cannot meet the needs of the proposed Eq. 10. For this purpose, a method for optimizing the mass of the center is proposed, and the implementation steps of this method follow above 1) 2) 3) 4) steps.

In step 2) of mobile nodes, define the electric distance variations as follows:

$$\Delta d^{-} = \left(d_{i}^{C_{i}^{m}}\right)^{2} - \left(d_{0i}^{C_{i}^{m}}\right)^{2} \left(i \in [1, N]; m \in [1, k]\right).$$
(16)

$$\Delta d^{+} = \left(d_{i}^{C_{i}^{v}}\right)^{2} - \left(d_{0i}^{C_{i}^{v}}\right)^{2} \left(i \in [1, N]; v \in [1, k]\right).$$
(17)

Among which,  $d_{0i}^{C_i^m}$  represents the electrical distance of node *i* belongs to the class  $C_i^m$  before moved out,  $d_i^{C_i^m}$  represents the distance after moved out;  $d_{0i}^{C_i^v}$  represents the electrical distance of node *i* before moved into the class  $C_i^v$ ,  $d_i^{C_i^v}$  represents the distance after moved in.

Therefore, the total variation of the electrical distance in the process of moving nodes is:  

$$\Delta d(k, N) = \Delta d^{-} + \Delta d^{+}.$$
(18)

 $\Delta d(k, N) < 0$  means a good movement. It means that the movements of the nodes make the classification of the electrical distance decreased, at this time, mark down the scale of decline. Similarly, move the nodes to other classes in turn and mark the scale of decline respectively; find and move the nodes to the class which has the largest scale of descend. According to this method, move the other nodes which submitted to Eq. 10 sequentially until electrical distance hasn't decreased. Thus, optimal classification can be obtained.

Make the number of classification plus 1, that is  $k \leftarrow k+1$ , according to the above method, further iterations will be taken until all optional class numbers from 2 to  $k_{\text{max}}$  corresponding to the optimal classification scheme from D(2) to  $D(k_{\text{max}})$  will be found. Among which, choose the minimum distance of electrical classification scheme as for the final optimal choice, that is:

$$\begin{cases}
D^*(k^*, N) & (k^* \in [2, k_{\max}]) \\
p_j \in C^m & (j \in [1, T]; m \in [1, k^*]) \\
q_j \in C^v & (j \in [1, T]; v \in [1, k^*], v \neq m)
\end{cases}$$
(19)

Among which,  $D^*(k^*, N)$  represents the minimum electrical distance.

At this time a power network classification model with the smallest electrical distance is obtained.

#### Boundary and sub region adjustment

According to the past experience, the power network classification model obtained may exist follow problems: 1) the transformer branch located in the class is a non-critical branch; 2) there are isolated nodes in class. In order to obtain good simplified model of the power network, adjustment should be applied by the following rules:

1) Let transformer branch l be connecting branch between class  $C^m$  and class  $C^v$ , the nodes at both ends are respectively p and q at the branch. If the node p is connected with r nodes which belongs to the class  $C^m$ , the node q is connected with s nodes which belongs to the class  $C^v$  and r is greater than s, the node p should be moved into the class  $C^v$  from the transformer branch.

2) For the case of isolated nodes in the class, the nodes can be moved to the minimum electrical distance by the Eq. 18, so that internal parts are connected with all the classes and boundary is clear.

According to the above adjustment, the classification model of power network can be obtained by preserving key branches and connecting all kinds of internal parts. Because of the existence of transmission security constraint in the same class, the system can be aggregated to obtain the simplified power network model for preserving key branches.

## Conclusion

In this paper, a simplified method for the power network with key branches is presented. The test shows that the method presented in this paper is clear in physical concept and the algorithm is accurate

and simple. The trend of simplified model with key branches has high precision, which has a good practical application.

# References

[1] Li Xiang, Guo Zhizhong. Power Flow at Cross-section of Transmission Line and Its Control Under N-1 Static State Secure Power Flow Restraint[J]. Power System Technology, 2005, 29(3):29-32(in Chinese).

[2] E.Fisher, R.O Neill, and M. Ferris. Optimal transmission switching [J]. IEEE Transactions on Power Systems, 2008, 23(3) :1346–1355.

[3] Zou Xiaosong,Luo Xianjue. Benders Decomposition Based Static Security and Stability-constrained Congestion Management by Coordinated Controls[J]. Proceeding of the CSEE,2009,29(25):78-85(in Chinese).

[4] K. Hedman, R. O'Neill, E. Fisher and S. Oren. Optimal transmission switching with contingency analysis[J]. IEEE Transactions on Power Systems, 2009, 24(3):1577–1586.

[5] Ding Ping,Li Yalou,Xu Dechao,Tian Fang,Yan Jianfeng,Yu Zhihong. Improved Algorithm of Fast Static State Security Analysis of Power Systems[J]. Proceeding of the CSEE,2010,30(31):77-82(in Chinese).

[6] K. Hedman, M. Ferris, R. O'Neill, E. Fisher and S. Oren. Co-optimization of generation unit commitment and transmission switching with N-1 reliability[J]. IEEE Transactions on Power Systems, 2010, 25(2):1052–1063.

[7] Wang Yang,Xia Qing,Kang Chongqing. Optimal Security Constrained Generation Scheduling Considering Closed-loop N-1 Security Correction[J]. Proceeding of the CSEE,2011,31(10):39-45(in Chinese).

[8] Liang Guishu, Ren Yu, Dong Huaying, Cui Xiang. Sensitivity Analysis of the Networks Containing Binomial From Nonuniform Transmission Lines[J]. Proceeding of the CSEE, 2004, 24(2):13-16(in Chinese).

[9] HyungSeon Oh. Aggregation of Buses for a Network Reduction[J]. IEEE Transactions on Power Systems, 2012, 27(2):705-712.

[10] Chang Kang, Han Xueshan, Wang Mengxia, Han Li. Study on the crucial element and its monotone in power grid part I: concepts and foundation [J]. Power System Protection and Control, 2009, 37(6):1-5(in Chinese).

[11] Chang Kang, Han Xueshan, Wang Mengxia, Han Li. Study on the crucial element and its monotone in power grid part II: mechanism and certification [J]. Power System Protection and Control, 2009, 37(7):1-6(in Chinese).

[12] H. Oh. A new network reduction methodology for power system planning Studies[J]. IEEE Transactions on Power Systems,2010,25(2):677–684.