The vibration control research of MRE damper for the multi-level eccentric structure

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Abstract: For the characteristics of that the center of mass and stiffness centre do not overlap and that presence of translational and torsional vibrations under earthquake to the eccentric structure, MRE damper is used to control the translation-torsion coupling effect of eccentric structure under seismic action. Comparative analysis the structural vibration response in the absence of control and without control. Numerical calculation shows that the translational and torsional vibration damping ratio of the structure under the effect of control can reach maximum of 70.41% and 26.13% respectively. Thus illustrates the MRE damper for vibration control of eccentric structure is effective.

Introduction

Due to the irregular structure layout, the center of mass and stiffness centre do not overlap, resulting the translational vibration and torsional vibration under earthquake (Dang et al.2010). All previous earthquake damage and theoretical analysis show that the ratio of structure torsional vibration damage is obvious of all forms of damage (Xiao et al.2012). So the torsional vibration response and effect control is being focus on. Traditional design method that is to increase the stiffness of the structure and adjust the structure to reduce the torsional vibration of the structure can not solve the problem of the vibration response of eccentric structure (Tang et al.2008). Nowadays the mature method is to set up control devices in structure to reduce the vibration of the structure. MRE is the solid simulation of MRF, mainly composed by matrix materials and dispersing magnetic particles. That use magneto-rheological solid material can solve the problem of magnetic particle settlement, MRE device has advantages of no need to seal, stable performance, easy to control with quick response etc (Ni et al.2010; Tu et al.2003; Long et al.2013). So the magneto-rheological dampers are widely used in engineering. In this paper, installing MRE dampers on the eccentric structure to absorb vibration energy, thereby reducing translational and torsional vibration under earthquake.

Structure Equations of Motion

For multilayer or high level biaxial eccentric structure under the seismic action, the translation-torsion coupling equations of motion of the structure is as follows:

$$\boldsymbol{M}\ddot{\boldsymbol{X}}(t)+\boldsymbol{C}\dot{\boldsymbol{X}}(t)+\boldsymbol{K}\boldsymbol{X}(t)=-\boldsymbol{M}\boldsymbol{I}\ddot{x}_g(t) \tag{1}$$

where $\boldsymbol{M}$ 、$\boldsymbol{K}$ 、$\boldsymbol{C}$ were the mass matrix、 stiffness matrix、damping matrix of structure. $\boldsymbol{x}(t)$ is the displacement column vector of structure; $\boldsymbol{I}=\{1,1\,\mathbf{L}\,1;\,l,l\,\mathbf{L}\,l;0,0\,\mathbf{L}\,0\}^T$, l is the ratio of seismic wave peak acceleration to x and y direction. $\ddot{x}_g(t)$ is the input seismic acceleration; Using Rayleigh damping assumption to set up the structural damping matrix of structural system, $\boldsymbol{C}=a_0\boldsymbol{M}+a_1\boldsymbol{K}$ in the equation (1)，where the expressions of the coefficient a_0 、a_1 are obtained by the formula(Liu et al.2005)：$x_n=a_0/2w_n+a_1w_n/2$. Given any two modal damping ratio x（Since the oscillation frequency can be obtained by mass matrix and stiffness matrix），we can get the scale factor a_0 and a_1 .

$$\boldsymbol{M}=\begin{bmatrix}\boldsymbol{m} & & \\ & \boldsymbol{m} & \\ & & \boldsymbol{J}\end{bmatrix},\ \boldsymbol{m}=\begin{bmatrix}m_1 & & & \\ & m_2 & & \\ & & \ddots & \\ & & & m_n\end{bmatrix},\ \boldsymbol{J}=\begin{bmatrix}J_1 & & & \\ & J_2 & & \\ & & \ddots & \\ & & & J_n\end{bmatrix},\ \boldsymbol{K}=\begin{bmatrix}\boldsymbol{k}_{xx} & & \boldsymbol{k}_{xj} \\ & \boldsymbol{k}_{yy} & \boldsymbol{k}_{yj} \\ \boldsymbol{k}_{jx} & \boldsymbol{k}_{jy} & \boldsymbol{k}_{jj}\end{bmatrix},$$

$$\boldsymbol{k}_{xx}=\begin{bmatrix}k_{xx}^{11} & k_{xx}^{12} & \cdots & k_{xx}^{1n} \\ k_{xx}^{21} & k_{xx}^{22} & \cdots & k_{xx}^{2n} \\ \vdots & \vdots & \ddots & \vdots \\ k_{xx}^{n1} & k_{xx}^{n2} & \cdots & k_{xx}^{nn}\end{bmatrix},\quad \boldsymbol{k}_{xj}=\boldsymbol{k}_{jx}{}^{T}=\begin{bmatrix}k_{xx}^{11}e_{y1} & k_{xx}^{12}e_{y2} & \cdots & k_{xx}^{1n}e_{yn} \\ k_{xx}^{21}e_{y1} & k_{xx}^{22}e_{y2} & \cdots & k_{xx}^{2n}e_{yn} \\ \vdots & \vdots & \ddots & \vdots \\ k_{xx}^{n1}e_{y1} & k_{xx}^{n2}e_{y2} & \cdots & k_{xx}^{nn}e_{yn}\end{bmatrix},\quad \boldsymbol{k}_{jj}=\begin{bmatrix}\boldsymbol{k}_{jj}^{11} & \boldsymbol{k}_{jj}^{12} & \cdots & \boldsymbol{k}_{jj}^{1n} \\ \boldsymbol{k}_{jj}^{21} & \boldsymbol{k}_{jj}^{22} & \cdots & \boldsymbol{k}_{jj}^{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{k}_{jj}^{n1} & \boldsymbol{k}_{jj}^{n2} & \cdots & \boldsymbol{k}_{jj}^{nn}\end{bmatrix}$$

Where $\boldsymbol{k}_{yy}$ and $\boldsymbol{k}_{yj}=\boldsymbol{k}_{jy}{}^{T}$ are similar to $\boldsymbol{k}_{xx}$ and $\boldsymbol{k}_{xj}=\boldsymbol{k}_{jx}{}^{T}$ in the expression, Just to exchange the subscript of the x and y. e_{xi}、 e_{yi} are respectively the i-th layer x and y direction of eccentricity.

Using MRE Damper to Structure Control

For the characteristics that shear modulus control is strong and the damping coefficient changes little of the magneto-rheological elastomers, the selected magneto-rheological elastomers mechanical model is a viscoelastic model which attached a magnetic shear modulus on the basis of Calvin model, as shown in figure 1.the magneto-rheological elastomer shear modulus in the model consists of two parts: the inherent shear modulus without magnetic field and magnetic shear modulus caused by magnetic field(Fang et al.2004; Fu et al.2013).

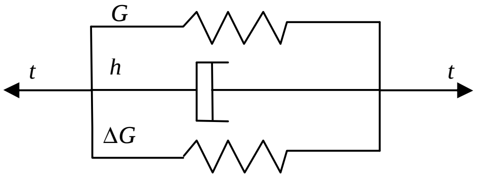

Fig. 1 MRE mechanical model

The output force calculating formula of the magneto-rheological elastomer damper:

$$F=k_0x+\Delta kx+c\dot{x}=(k_0+\Delta k)x+c\dot{x} \qquad (2)$$

Where $k_0=GA/h$, G is the inherent shear modulus without magnetic field, A is The area of the magneto-rheological elastomers, h is the thickness of the magneto-rheological elastomers. $\Delta k=\Delta GA/h$, ΔG is the magnetic shear modulus caused by magnetic field. $c=\eta A/h$, c is the damping coefficient of magneto-rheological elastomers dampers, $h=2z$ .Control system state equation:

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t)+\boldsymbol{C}\dot{\boldsymbol{x}}(t)+\boldsymbol{K}\boldsymbol{x}(t)=-\boldsymbol{M}\boldsymbol{I}\ddot{\boldsymbol{x}}_g(t)+\boldsymbol{D}\boldsymbol{F}(t) \qquad (3)$$

Where $\boldsymbol{D}$ is the position matrix of control force, $\boldsymbol{F}(t)$ is the control force vector.

Order $\boldsymbol{Y}(t)=\begin{bmatrix}\boldsymbol{x}(t)\\ \dot{\boldsymbol{x}}(t)\end{bmatrix}$, Structure equations of motion (3) can be turned into: $\dot{\boldsymbol{Y}}(t)=\boldsymbol{A}\boldsymbol{Y}(t)+\boldsymbol{B}\boldsymbol{F}(t)+\boldsymbol{L}\ddot{\boldsymbol{x}}_g(t)$,

where $\boldsymbol{A}=\begin{bmatrix}\boldsymbol{0}_{n\times n} & \boldsymbol{I}_n \\ -\boldsymbol{M}^{-1}\boldsymbol{K} & -\boldsymbol{M}^{-1}\boldsymbol{C}\end{bmatrix};\boldsymbol{B}=\begin{bmatrix}\boldsymbol{0}_{n\times n}\\ \boldsymbol{M}^{-1}\boldsymbol{D}\end{bmatrix};\boldsymbol{L}=\begin{bmatrix}\boldsymbol{0}_{n\times 1}\\ -\boldsymbol{I}\end{bmatrix}$

Control Strategy

Optimal control is that under certain conditions, to determine an optimal control strategy for a motion system, which the initial state of the motion system shifts to a predetermined target state, and performance indicators to achieve the optimal value. LQR (Liner quadratic regulator) control is an optimal control of linear quadratic optimal control, which is the optimal control of the most widely used, most highly developed control. The control object of LQR control is the linear motion systems in the form of spatial state, whose function is a quadratic function of the amount of the performance index Y (t) and control the amount of F (t). In the linear quadratic optimal control, the selected control force vector F(t) is to make performance index J to the minimum(Xu et al.2007; Zhou.2014). The performance index J is usually taken by:

$$J=\frac{1}{2}\int_0^{t_f}(\boldsymbol{Y}^T(t)\boldsymbol{Q}\boldsymbol{Y}(t)+\boldsymbol{F}^T(t)\boldsymbol{R}\boldsymbol{F}(t))dt \tag{4}$$

Where $\boldsymbol{Q}$ is Semi-definite weighting matrix, $\boldsymbol{R}$ is definite weighting matrix. Applicated of Minimax control principle, the optimal control force to make the performance index J to the minimum is as follows:

$$\boldsymbol{F}(t)=-\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P}\boldsymbol{Y}(t) \tag{5}$$

Where $\boldsymbol{P}$ is the Riccati matrix, which can be obtained by the Riccati equation $\boldsymbol{PA}+\boldsymbol{A}^T\boldsymbol{P}-\boldsymbol{PBR}^{-1}\boldsymbol{B}^T\boldsymbol{P}+\boldsymbol{Q}=0$. $\boldsymbol{Q}$、$\boldsymbol{R}$ are usually taken as a diagonal matrix:

$$\boldsymbol{Q}=a\begin{bmatrix}\boldsymbol{K} & 0\\ 0 & \boldsymbol{M}\end{bmatrix} \qquad \boldsymbol{R}=b\boldsymbol{I}$$

Where a，b can be obtained by the trial of the control system.（Select a =100，b =6.5×10^{-5}）.

Numerical Analysis

In order to discuss the control effect of the structure at MRE damper effect, we select an eight-story frame structure to analysis. The mass of each layer: $\boldsymbol{m}=[865, 865, 840, 840, 815, 805, 785, 760]\times10^3$ kg, the horizontal stiffness to X direction of each layer: $\boldsymbol{k}_x=[145, 145, 132, 132, 121, 110, 105, 95]\times10^6$ N/m,the horizontal stiffness to Y direction of each layer: $\boldsymbol{k}_y=[125, 125, 112, 112, 105, 95, 86, 75]\times10^6$ N/m, the eccentricity of each layer: $e_x=e_y=2.0m$. We installed a magneto-rheological elastomer damper on each layer in the structure, and take EI Centro seismic waves of a peak at 200cm/s^2 as the input seismic waves. Through the experiments , we select the stiffness of MRE damper ranged 2.2×10^{8}N/m-4.1×10^{8}N/m, which stiffness varies with its current relationship is shown in Table 1

Table 1 The stiffness of MRE damper

electric current /A	0	0.5	1
stiffness×10^{8}N/m	2.2	3.0	4.1

Analysis control effect of MRE damper to the structure translational and torsional vibration by Simulink toolbox in the MATLAB, and analysis the damping effect while applying the optimal control force to the each layers of the structure. MRE vibration simulation module is shown in Figure 2, the optimal control simulation module is shown in Figure 3.

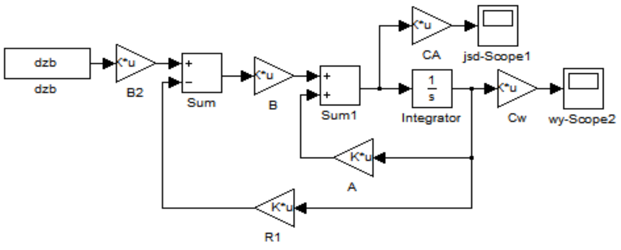

Fig. 2 MRE vibration simulation module

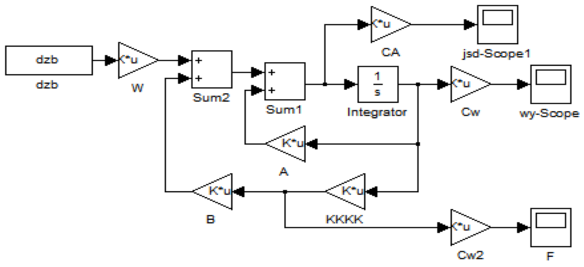

Fig.3 the optimal control simulation module

Taking the top layer as an example, the time history analysis of the translational and torsional response of the top layer in the structure in the absence of control and with MRE damper control action are as shown in Figure 4 to 6. The time history analysis of the translational and torsional response of the top layer in the structure in the absence of control and with optimal control are as shown in Figure 7 to 9. The maximum of time history analysis of the translational and torsional response of the first layer and the top layer are as shown in Table 2 and Table 3.

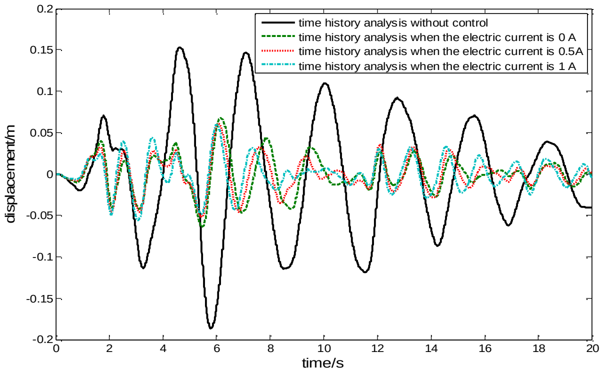

Fig.4 time history analysis of the top layer to X direction

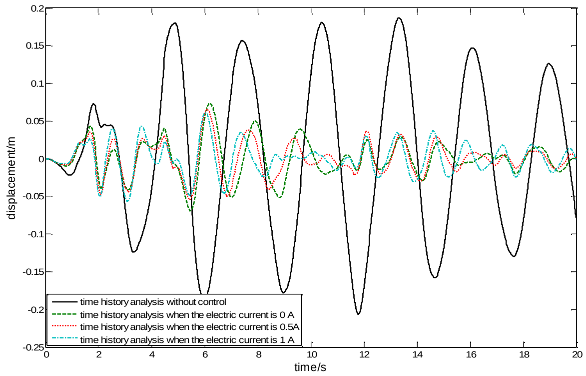

Fig.5 time history analysis of the top layer to Y direction

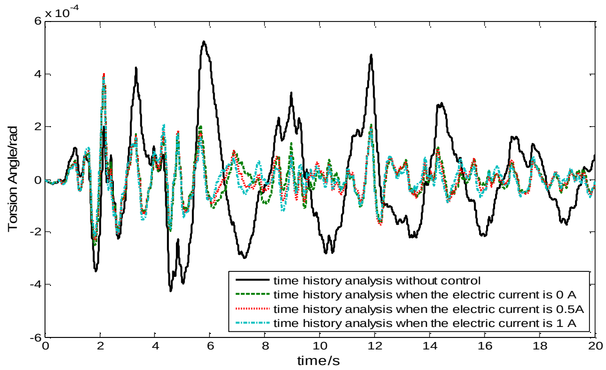

Fig. 6 time history analysis of torsional response of the top layer

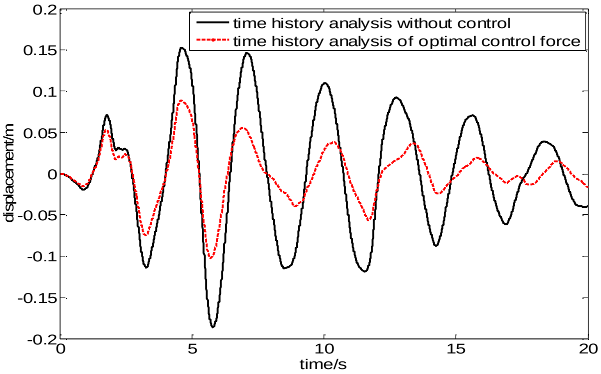

Fig. 7 time history analysis of the top layer to X direction under optimal control

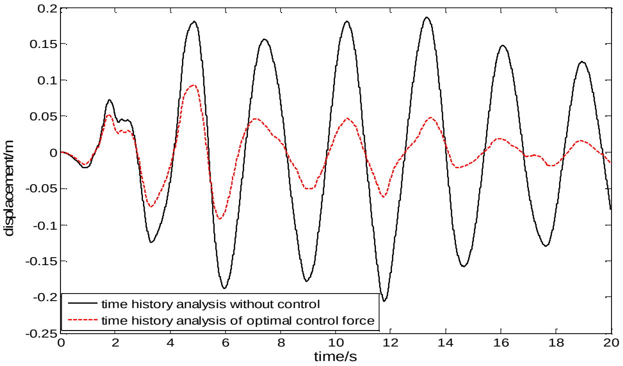

Fig. 8 time history analysis of the top layer to Y direction under optimal control

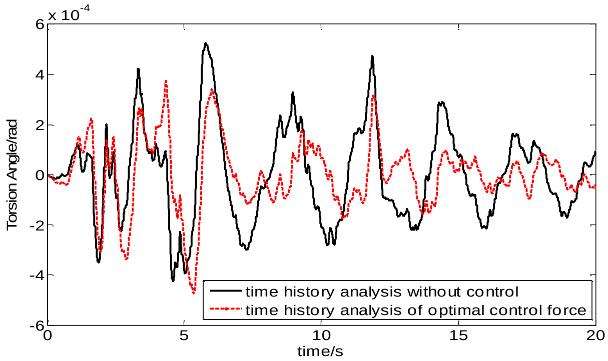

Fig.9 time history analysis of torsional response of the top layer under optimal control

Table 2 The maximum response contrast of the structure in the absence of control and with MRE damper control action

layer	Displacement to X(m)		Displacement to Y(m)		Torsional response(10^{-4}rad)	
First layer	Without control	0.0269	Without control	0.0344	Without control	1.2548
	0 A Electric current	0.0124	0 A Electric current	0.0133	0 A Electric current	1.0525
	0.5A Electric current	0.0109	0.5A Electric current	0.0116	0.5A Electric current	1.1432
	1.0A Electric current	0.0107	1.0A Electric current	0.0110	1.0A Electric current	1.2681
	The maximum damping rate	60.22%	The maximum damping rate	68.02%	The maximum damping rate	16.12%
Top layer	Without control	0.1862	Without control	0.2062	Without control	5.2484
	0 A Electric current	0.0678	0 A Electric current	0.0729	0 A Electric current	3.9641
	0.5A Electric current	0.0613	0.5A Electric current	0.0654	0.5A Electric current	4.0073
	1.0A Electric current	0.0595	1.0A Electric current	0.0610	1.0A Electric current	3.8771
	The maximum damping rate	68.05%	The maximum damping rate	70.41%	The maximum damping rate	26.13%

Table 3 The maximum response contrast of the structure in the absence of control and with control action

layer	Displacement to X（m）		Displacement to Y（m）		Torsional time history analysis （10^{-4}rad）		Applied optimal control （10^{5}N）	
	Without control	With control	Without control	With control	Without control	With control	X direction	Y direction
First layer	0.0269	0.0194	0.0344	0.0196	1.2547	1.0471	1.4274	1.6001
Top layer	0.1862	0.1021	0.2062	0.0925	5.2483	4.7423	8.0219	8.9453

Conclusion

This article focuses on vibration control effect of MRE damper to the eccentric structure, and the conclusions are as follows: (1) It has better control effect of translational and torsional vibrations of the structure while installed MRE damper in the structure. From the results of numerical examples: the damping rate of the first layer to X direction can reach a maximum of 60.22%, to Y direction can reach a maximum of 68.02%, the torsional damping ratio can reach a maximum of 16.12%; the damping rate of the top layer to X direction can reach a maximum of 68.05%, to Y direction can reach a maximum of 70.41%, the torsional damping ratio can reach a maximum of 26.13%. The vibration control effect of the damper to Y direction is better than to X direction, on the top layer is better than the first floor. (2) As the input electric current increases, the magnetic field applied in the MRE is enhanced, the MRE dampers makes the translational vibration damping effect of the structure get better. The torsional vibration damping effect is not such a law. (3) Under the optimal control action, the translational and torsional vibrations of the structure have been better controlled.

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