Pedestrian Induced Structural Vibration and the Walking Force Models Basing On Biomechanics: a Literature Review

Zhigang Song[1, a], Shuai Zhang[2,b*]

[1,2]Department of Civil Engineering, Kunming University of Science and Technology, Kunming, China

[a]zhigang_s@163.com, [b]zhang_zs5@126.com

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Abstract. The interactions of structure-human and those of human-human would change the macroscopic behavior of human walking on the structure and which, would further result the change of walking forces. The complexity of walking forces during the interaction process is the reason that lively structures exhibit some special vibration behaviors under the crowd excitation. The walking forces observed through the traditional indoor footfall experiments can't fully reflect the interaction process. However, a biomechanics based walking force models can establish the relationships between human macroscopic activities and walking forces so that it can server as an alternative method. The biomechanical models describing human body dynamics and walking forces are reviewed and the problems to be studied in the field are suggested from the views point of human-human interaction and human-structure interaction.

Introduction

The pedestrians induced structural vibrations is a worldly focused problem and which is widely solved by the forced vibration theory (FVT). However, recent studies have shown that some special structural vibration phenomena under crowds' excitation are resulted by human-human interaction (HHI) and human-structure interaction (HSI). These two interactions would change the normal gaits of human and further result the change of frequency, amplitude and phase of walking forces. However, the indoor footfall tests can't fully consider the influences of the interactions because of the experimental conditions. Combining the simulation of pedestrians' macroscopic movements, the biomechanics based walking force model (BWFM) could serve as an alternative solution.

The studies on human induced structural vibrations, human behaviors on pedestrian structures and BWFMs are reviewed in this paper. From view point of HHI and HSI, four important aspects associated with BWFMs are summarized as follows:

(1) The study of walking forces on a moving surface.
(2) The study of walking forces under different walking velocities.
(3) The study of walking forces under the change of walking directions.
(4) The study of walking forces considering the human synchronization.

Human induced structural vibrations and human behaviors

Human induced structural vibrations is a widely concerned problem [1, 2], and the forced vibration theory (FVT) which is a primary analytical method has been used in varies standards domestic and overseas [3, 4]. According to this theory, the walking forces are treated as external excitations, and the pedestrians induced vibrations are solved through forced vibration equations. Based on this theory, some similar methods have been developed, such as the time history method [5], the dynamic amplifying factor method [6], the response spectra method [7], and others [1, 8]. In these methods the walking forces are treated as external excitations and obtained from indoor footfall experiments, while the body weight is treated as the added mass [9]. During the footfall experiments, testers are required to walk in a customary way or a specific pace that the frequency prompter indicated. Therefore, the walking forces obtained from the experiments generally reflect the features of freedom walking on a fixed surface, not the case on an oscillatory surface, in other words, the FVT ignores the effects of HSI. Furthermore, the Monte Carlo simulation method is adopted to analyze crowd induced vibrations based on the FVT [10], however, the walking forces used in this method are also obtained from footfall experiments, which indicates that pedestrians' walking velocities or directions could not change even if the existence of obstacles, thus the FVT also ignores the effects of HHI.

The successive studies also shown that there are some special vibration phenomena, such as the change of structure dynamic characteristics [11, 12], human-structure phase synchronization (or "lock-in") [1, 13, 14], excessive lateral structure vibration [1, 15, 16], and the sudden termination of vibration [17], when the crowd walking on a lively structure that the FVT can't make a clear interpretation. At the same time, several researches also shown that with the varying of crowd density and structure vibrating amplitude, pedestrians' walking velocity [18, 19], walking direction [20], and the rate of synchronous people among crowd [1, 19, 21] could change, further, the change of pedestrians' macroscopic behaviors would lead the change of frequency, amplitude, and the phase of walking forces. Thus, the walking loads actually acting on the structures are different from those obtained from indoor footfall experiments. In order to interpret these phenomena, researchers are trying to study human induced vibrations with the consideration of HHI and HSI.

The study of HSI could be traced to 1960s, and researchers found the effects of the active crowd on structures could lead the increase of structure's damping ratio [22], the decrease of fundamental frequency [23], and the introduction of new vibration modes [24]. However, these phenomena couldn't be explained through regarding the human body as added mass to structures completely, and this promoted the study of human body dynamics. The single degree-of-freedom mass-spring-damper system is the simplest model describes the dynamical characteristics of human body. The researches of Matsumoto indicated that the two degree-of-freedom mass-spring-damper system showed a good agreement with the measured apparent masses and phases. Thus, it's not necessary to employ a mass-spring-damper system with more than two degree-of-freedom to simulate the dynamical features of human body [25, 26]. Based on Matsumoto's studies, many researchers adopted the mass-spring-damper model analyzed the dynamical characteristics of coupled human-structure system under different number of pedestrian with different spatial distributions [27, 28, 29]. In spite of the fact that the mass-spring-damper model only reveals the dynamical features of standing human body, and it can't obtain the walking forces directly. This model opens the way to describe human body dynamics and makes some contributions to the development of bipedal walking model. The closure of London Millennium Footbridge on its opening day due to large amplitude lateral vibrations made researchers consider that the synchronization of pedestrians' gaits to the bridge vibration frequency, or so-called "pedestrian lock-in" is the precondition mechanism leads instability of lateral vibration [9, 13]. However, the mechanism has been suffered many suspects in successive studies. On the Changi Mezzanine Bridge (CMB), Brownjohn noted that pedestrians were not observed to synchronize their motions while the bridge lateral amplitude increased significantly, however, if the "lock-in" is true, there would be at least 26% of pedestrians synchronized their paces [30]. Also, on the Clifton Suspension Bridge (CSB), Macdonald cast doubts on this assumed mechanism as follows [31]: (1) the natural frequency of the remarkable mode to be excited (0.524HZ) was half the natural walking frequency, and it seemed unlikely that pedestrians would adjust their gaits to synchronize to this low frequency, while there were two other modes closer to normal walking frequency (mode 3, 0.746HZ and mode 4, 0.965HZ); (2) If there were synchronizations to the lateral walk frequency, there would be also synchronization of vertical force at twice this frequency, while this was hardly evident in the spectra of vertical response. According to the Brownjohn's and Macdonald's work, some researchers abandoned the mechanism of synchronization during the study of human induced lateral vibrations, for example, the lateral force model based on the social force model proposed by Song [20]. Song's model considers the characteristics of pedestrian's movement on a lateral vibrating footbridge, and well predicates the critical pedestrian number that causes the instability of lateral vibration of some bridges. Further, Song combined the concept of relaxing time in social force model to explain why there was no structure with the fundamental lateral mode frequency higher than 1.2 Hz became lateral instability.

However, Song's model can't provide an interpretation about the mechanism of relaxing time and added force because of only the macroscopic movements, such as, velocity and walking direction, of pedestrians are considered and the elaborate characteristics of gait are ignored.

The study of HHI concentrates on revealing the laws of human macroscopic movements. In the last two decades, many experimental technologies or simulative models, such as the technique of video capture [32] and the compressible hydrodynamic models [33], proposed in this field have been successfully applied to simulate the evacuation processes of pedestrians in the case of emergency situations [34, 35]. Among these studies, the development in the field of transportation engineering is outstanding. The crowd behaviors are analyzed from the views point of macroscopic or microscopic [36]. The microscopic models, such as the social force model [37] and the cellular automata model [38], concentrate on the behaviors of individual. Not only can these models describe the individual movements, such as spatial position, walking velocity and walking direction precisely (Fig.1) [39], but also the crowd collective behaviors, for example, the quantitative relationship between crowd density and average walking velocity (Fig.1) [40] . Specially, these microscopic models can simulate the self-organization phenomenon, which is a typical behavior when the crowd density reaches a certain level (Fig.1) [41]. However, the study of pedestrian macroscopic behaviors can't directly obtain the walking forces that needed for the analysis of human induced structural vibrations, and how can we get the walking forces based on the pedestrians' macroscopic behaviors needs to further study.

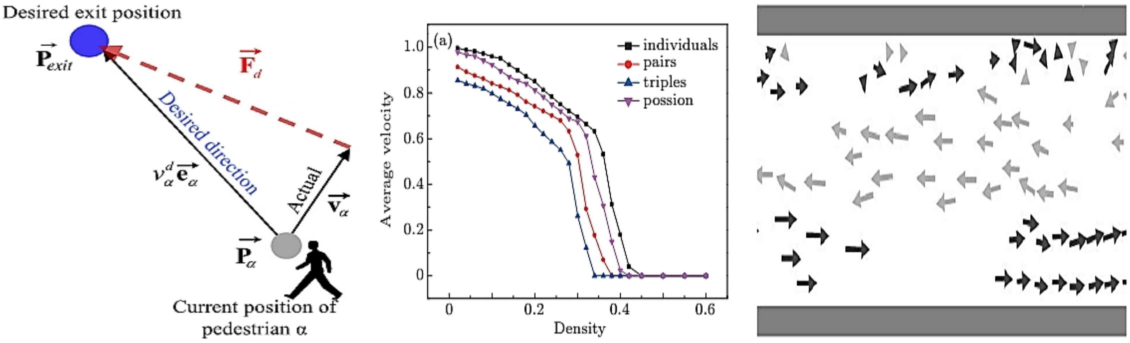

Fig.1. Individual's velocity (after [39]), the relation between crowd density and velocity (after [40]) and self-organization phenomenon (after [41])

Obviously, both HSI and HHI are important questions that the field of human induced structural vibrations should consider. The former changes the behaviors of individual through interactions between human and structures, while the latter influences the pedestrian's bipedal gait by external interferences. The change of pedestrians' gait would eventually lead the walking forces different from the indoor footfall experiments. Naturally, it's worthwhile to build the relationship between pedestrians' macroscopic behaviors and walking forces based on the precisely study of bipedal gait. To the authors' knowledge, the biomechanics based walking force model (BWFM) that discussed in the next chapters would be a finer approach to solve the problems that mentioned above.

The biomechanics based walking force model (BWFM)

The inverted pendulum model (IPM). Both the inverted pendulum model (IPM) and the bipedal walking model (BWM) discussed next parts are primary forms that simulate human body's structure in the field of biological engineering. In the early days, these two models were used to simulate the mechanisms of pedestrians' movements, the IPM for walking and the spring-mass model which is the predecessor of the BWM for running (Fig.2) [42]. As presented in Fig.2, the IPM simply comprising a lumped mass at the center of mass (COM) supported by a rigid massless leg. In the early times, the researches about walking mainly concentrated on building mechanisms that maintaining a stabilized bipedal gait [43]. It's worth noting that all of these studies were only considered the motion in the sagittal plane, and ignored the coupling with movement in the frontal plane. In fact a relative study has shown that such interconnection was weak [44]. In 2002, in the first international conference about human induced footbridge vibrations, Barker proposed that the IPM could be used to simulate the lateral walking force [45]. Based on Barker's idea, further adopted the balance control mechanism suggested by Hof [46], Macdonald used the IPM simulated the process of human body's lateral movement during walk (Fig.2), and extracted the lateral walking force [47].

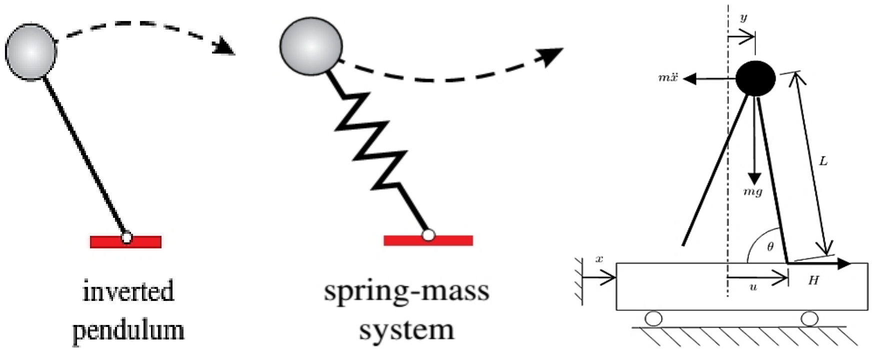

Fig.2. The inverted pendulum model (after [42]), the spring-mass model (after [42]) and the model used to simulate lateral walking movement (after [47]).

As fig.2, Macdonald established the kinematic equation about the model's COM and deduced the expression of the lateral walking force (see Eq.1 and Eq.2 respectively) when walking on a fixed surface. The position of foot placement for each step based on the lateral balance control mechanism is decided by Eq.3, which guarantees the horizontal projection of COM wouldn't exceed the zones between the placement positions of two legs during walk, as a result, the human body couldn't dump in one side [46].

$$\ddot{y} + \Omega_p^2(u-y) = 0. \tag{1}$$

$$H = -m\ddot{y} = m\Omega_p^2(u-y). \tag{2}$$

$$u = y_0 + v_0/\Omega_p^2 \pm b_{min} \text{ (+for right foot, -for left foot)}. \tag{3}$$

Where $\Omega_p = (g/L)^{0.5}$, m and L are the body mass and leg length, while y_0 and v_0 are, respectively, the initial position and velocity at the beginning of a step, and b_{min} is defined as the "margin of stability" [47]. Combining the equations described above the lateral walking force H can be solved, and its time history as represented in Fig.3. The distinguished feature of the simulation lateral walking force compared with the measured value which also described in Fig.3 represented in dotted curve is that it can't reflect the double support dynamic phase, which is the result of using of rigid legs. What's more, as showed in Fig.3, the rigid legs make the simulative vertical displacement of COM represented by dotted curve larger than measured value represented by solid line during walking. These discrepancies indicate that the IPM can't reflect the basic dynamic characteristics of human body during walking.

Further, both the first five dynamic load factors (DLFs) of the lateral walking force simulated by the IPM and the measured values offered by different authors are presented in Table 1. It can be seen that there are no components in the even multiple walking frequency of simulative values, however, Macdonald didn't make an explanation about it, and based on the analysis mentioned above this could be the result of using of rigid legs.

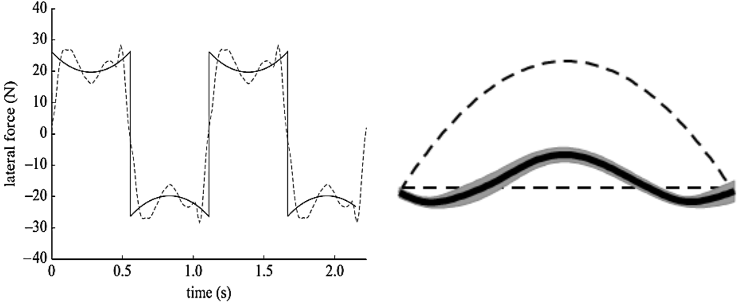

Fig.3. The lateral walking force history (after [47]) and the vertical displacement of center mass (after [42]).

Table 1. Dynamic load factors of Fourier components of lateral force (after [47])

	Dynamic load factors (% body weight)				
	f_b	2 f_b	3 f_b	4 f_b	5 f_b
Inverted pendulum model	3.9	0	1.6	0	1.0
Schneider & Chao	3.9	0	1.8	0	0.4
Bachmann & Ammann	3.9	1.0	4.3	1.2	1.5
Pizzimenti & Ricciardelli	4.0	0.8	2.3	0.4	1.1
Dallard et al.	4.0	-	-	-	-

Based on Macdonald's work, Bocian simulated the lateral walking forces under different walking frequencies, and the results similar with Macdonald's [48].

Macdonald's work offers a method to simulate lateral walking forces under different walking frequencies. However, in the development of the IPM there are several problems needed further studies:

(1) As mentioned previously, the rigid legs make it impossible to reflect the double support dynamic phase and the basic dynamical characteristics of human body during walk, which result in the discrepancies between simulative lateral walking forces and measured values, for example, no components in the even multiple walking frequencies of simulative values.

(2) Without the consideration of influences that the structure vibration on bipedal gait. Although Macdonald considered the effects that the structure vibration frequencies on lateral walking forces in successive studies, but these work are based on the condition that the vibration amplitudes are small, and there are no influences on bipedal gait. In reality, there exists the HSI, and the bipedal gait, such as stride length, walking frequency and walking direction, would change if the vibration amplitude reach a certain level, and automatically the walking forces would vary with bipedal gait.

The bipedal walking model (BWM). As mentioned above, the spring-mass model which is the predecessor of the BWM is used to simulate running. Considering that the inverted pendulum model can't reflect the basic dynamical characteristics of human body, Geyer built the famous bipedal walking model, as Fig.4, by adding a second leg to the spring-mass model [42], further, used this novel model achieved the simulation of walking under different walking velocities. Geyer's studies show that not only can the bipedal walking model reflect the double support phase and the vertical displacement of COM similar with measured values, but also have the ability to simulate the longitudinal walking force and the M-shaped vertical walking force (Fig.4).

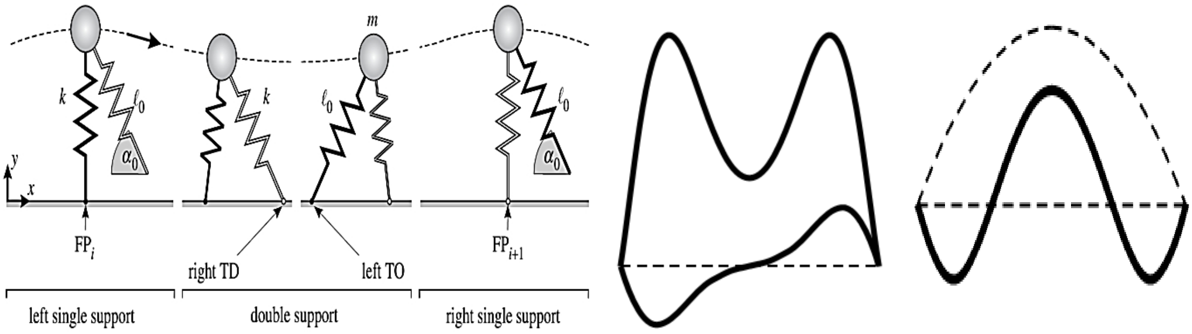

Fig.4. The bipedal walking model, walking force, and the vertical displacement of center mass (all after [42]).

Considering that the traditional walking force models ignored the HSI, Qin introduced the damping parameter to Geyer's bipedal walking model (Fig.5) [49], and built a coupled human-structure dynamic system to analyze the interaction between single pedestrian and footbridge.

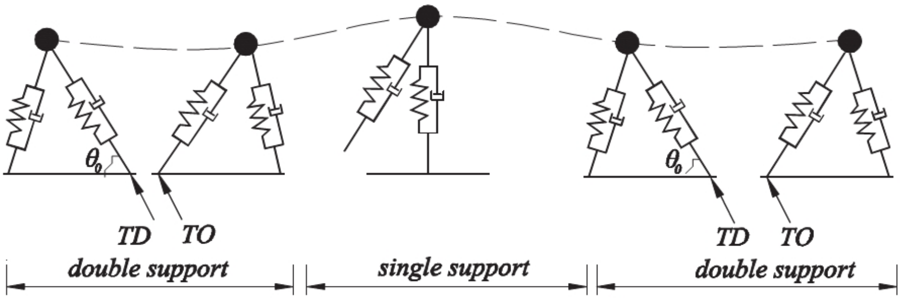

Fig.5. The bipedal walking model with damping parameter (after [49])

Due to the damping parameter, the energy would dissipate in the process of walking, and pedestrian that in the coupled dynamic system would stop movement after several walking cycles. In order to maintain a successive and stable walking gait, Qin applied a horizontal control force $F_{ctrl}(t)$ (as Eq.4) as balance control mechanism to maintain the total energy of the human body during walking. In the equation 4, E_0, $E(t)$ and $\Delta u(t)$ are the initial energy input, energy of the human body and horizontal displacement increment of COM at time 't', respectively.

$$F_{ctrl}(t) = (E_0 - E(t)) / \Delta u(t). \tag{4}$$

Using the proposed dynamic interaction model, Qin compared the footbridge dynamic response of the consideration of HSI with the case without the consideration of interaction. The results showed that the former is larger than the latter, and it's necessary to consider HSI when structures are flexible. Moreover, in the consideration of interaction the footstep force is larger than the case without consideration of interaction (as Fig.6). Also, with the pedestrian moving towards mid-span of the footbridge, the walking force gradually increase due to the increase of structure vibrations, obviously, the structure vibration affects the walking force.

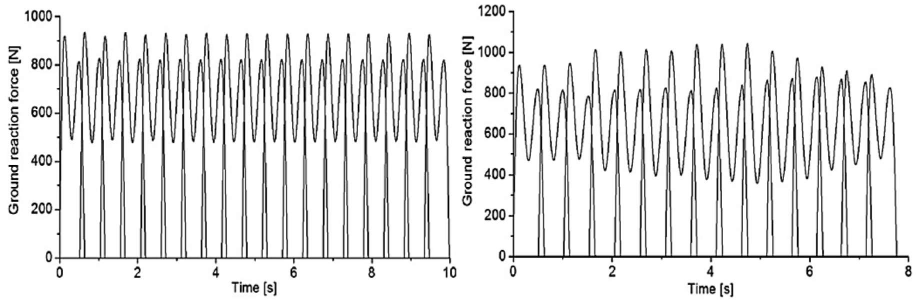

Fig.6. The vertical walking force with or without consideration of HSI (after [49])

The studies of Geyer and Qin show that the BWM is a thorough biomechanical model, and the coupled human-structure dynamic system based on this model has the ability to analyze the interaction between pedestrian and structure. However, there is a broader space for the development of the BWM:

(1) Just as other bipedal gait model, the BWM needs a balance control strategy, like the control force proposed by Qin, to maintain a stable gait during walking. However, Qin didn't analyze the effectiveness of the simulative walking force under the control of external force, and whether it has the similar dynamic characteristics with the measured values needed further study.

(2) Qin has studied the interaction between single pedestrian and structure, and the results indicated that the structure's vibration affects bipedal gait. In reality, there would be many people walking on the structure, and with the increase of crowd density the interaction among pedestrians would also influence the bipedal gaits, such as the decrease of walking velocity or change of walking direction, and the change of walking direction would lead the vary of walking force components that relative to a certain direction (Fig.7) [50]. What's more, there would be a phenomenon of synchronization among pedestrians, namely pedestrians walk in the same pace, when the crowd density reaches a certain high level, and the change of the rate of synchronization would multiple increase or decrease the walking loads exerting on structures compared to the case of single pedestrian's action [9]. How to make the bipedal walking model

reflect the influences that the structural vibrations, at the same time the environmental interferences, on pedestrians' gait needed further study.

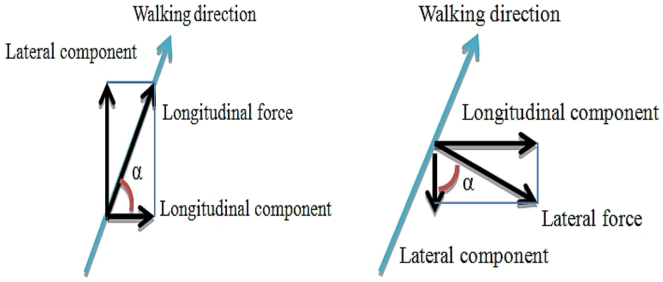

Fig.7. The walking force component relative to a certain direction (after [50])

Conclusions and expectations

The dynamic system consists of crowd and structure is a complex coupled dynamic system, which should consider the HSI and the HHI when discussing the human induced structural vibrations. In this system, the pedestrians' walking forces are the key factors that lead structural vibration. However, this process is different from the forced vibration. The structural vibrations and external interferences could lead the change of pedestrians' bipedal gaits, such as the change of walking velocity, walking frequency, walking direction and the rate of synchronization among crowd. Further, the change of bipedal gaits would lead the change of walking loads exerting on structures. Naturally, a sound walking force model should reflect the influences that the change of bipedal gaits on walking loads. As mentioned previously, the BWFMs have the potentials to solve these problems, and these types of models have been reflected several advantages in application as follows:

(1) These models can simulate the process of walking and the walking forces under different velocities or frequencies, such as Bocian used the IPM simulated the lateral walking force under different walking frequencies, while Geyer used the BWM simulated the vertical and longitudinal walking forces under different velocities.

(2) The coupled dynamic system based on the BWM can analyze the interaction between pedestrian and structure, such as reflecting the influence the structural vibration on walking force.

However, there is no doubt the fact that the space for the development of these biomechanical models is broader. As mentioned previously, the current studies concentrate on the simulation of walking forces or interactions between single pedestrian and structure in a certain movement direction, and do reflect the influences that the structure vibrations on bipedal gait. However, there are more than one pedestrian, sometimes crowd acting on structures, and the interactions among crowd could also lead the change of pedestrians' macroscopic behaviors, such as the change of walking velocity, walking direction and the collective behaviors. Though the biomechanical models have the ability to interpret the macroscopic behaviors to walking forces, for example, the walking forces in different walking velocities or frequencies, but themselves can't simulate the interactions among crowd. Thus, there needs a model, which can simulate the HHI, and offer the macroscopic parameters that the biomechanical models need to simulate bipedal gait. Fortunately, the evacuation modeling of pedestrian flow in the field of transportation engineering provides the way to simulate HHI as mentioned above, and an alternative way to analyze the crowd induced structural vibration is to combine the biomechanical bipedal walking models with the simulation of pedestrian flow models.

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References

[1] P. Dallard, A. Fitzpatrick, A. Flint, S. Le Bourva, A. Low, R. Ridsdill-Smith, M. Willford, Struct. Eng. 79 (2001) 17-33.

[2] R. Pimentel, A. Pavic, P. Waldron, Can. J. Civil. Eng. 28 (2001) 769-777.

[3] Design of timber structures part 2: bridges[S]. Eurocode 5 Belgium: European Committee for Standardization, Brussels, 1997.

[4] Assessment of Vibrational Behavior of Footbridges under Pedestrian Loading[S], Setra, 2006.

[5] B. Ellingwood, A. Tallin, J. Struct. Eng-ASCE. 110 (1984) 401-418.

[6] Zhigang Song, Yao Zhang, J. Vib. Shock. 34 (2015) 19-23. (In Chinese)

[7] Zhigang Song, Weiliang Jin, J. Build. Str. 25 (2004) 58-63. (In Chinese)

[8] M. Willford, Dynamic actions and reactions of pedestrians[C], Proceedings of the International Conference on the Design and Dynamic Behaviour of Footbridges, Paris, France, 2002.

[9] Xubin Yuan, Characteristics of pedestrian induced vibration of footbridges[D], Shanghai: Tongji University, 2006. (In Chinese)

[10] Xiaobin Qian, Acceleration response spectrum method of structure under pedestrian excitation and the analysis of crowd excitation[D], Hangzhou: Zhejiang University, 2005. (In Chinese)

[11] S. Zivanovic, I. M. Diaz, A. Pavic, Influence of walking and standing crowds on structural dynamic properties[C], Proceedings of the 27th IMC Conference, Orlando, USA, 2009.

[12] N. Jorgensen, Human structure interaction: influence of walking pedestrians on the dynamic properties of footbridge structures they occupy[D], Technical University of Denmark, 2009.

[13] J. Brownjohn, T. N. Fu, Shock. Vib. 12 (2005) 333-347.

[14] Y. Fujino, B. M. Pacheco, S. I. Nakamura, P. Warnitchai, Earthq. Eng & Struct. D. 22 (1993) 741-758.

[15] H. Bachmann, Struct. Eng. Int. 2 (1992) 118-124.

[16] L. Sun, X. Yuan, Study on pedestrian-induced vibration of footbridge[C], Proceedings of Footbridge 2008, third international conference, Porto, 2008.

[17] Zhengqing Chen, Xugang Hua, Vibration and dynamic design of footbridges, first ed., China Communications Press, Beijing, 2009. (In Chinese)

[18] S. Nakamura, T. Kawasaki, J. Constr. Steel. Res. 62 (2006) 1148-1160.

[19] F. Venuti, L. Bruno, Phys. Life. Rev. 6 (2009) 176-206.

[20] Zhigang Song, Huan Ye, J. Vib. Eng. 27 (2014) 233-237. (In Chinese)

[21] P. Dallard, T. Fitzpatrick, A. Flint, Struct. Eng. 78 (2000) 13-15.

[22] A. Polensek, Wood. Sci. 8 (1975) 140-151.

[23] K. H. Lenzen, Eng. J. AISC. 3 (1966) 133-136.

[24] B. R. Ellis, T. Ji, P. I. Civil. Eng- Str. B. 122 (1997) 1-9.

[25] Y. Matsumoto, M. J. Griffin, J. Sound. Vib. 260 (2003) 431-451.

[26] Y. Matsumoto, M. J. Griffin, J. Sound. Vib. 212 (1998) 85-107.

[27] R. Sachse, A. Pavic, P. Reynolds, Sound. Vib. 274 (2004) 461-480.

[28] Hai Wang, Ding Zhou, Shuguang Wang, Eng. Mech. 27 (2010) 14-20. (In Chinese)

[29] Jingwei Qin, Qingshan Yang, Na Yang, J. Vib. Shock. 31 (2012) 150-157. (In Chinese)

[30] J. M. W. Brownjohn, P. Fok, M. Roche, P. Omenzetter, Struct. Eng. 82 (2004) 28-34.

[31] J. H. G. Macdonald, P. I. Civil. Eng-Bridge. En. 161 (2008) 69-77.

[32] Xuan Liu, Weiguo Song, Jun Zhang, Phys. A. 388 (2009) 2717-2726.

[33] F. Venuti, L. Bruno, N. Bellomo, Math. Comp. Model. 45 (2007) 252-269.

[34] Weiguo Song, Xuan Xu, Bing-Hong Wang, Shunjiang Ni, Phys. A. 363 (2006) 492-500.

[35] Jun Zhang, Weiguo Song, Xuan Xu, Phys. A. 387 (2008) 5901-5909.

[36] Jian Sun, Keping Li, Comp. Simula. 25 (2008) 13-16. (In Chinese)

[37] D. Helbing, P. Molnar, Phys. Rev. E. 51 (1995) 4282-4286.

[38] V. J. Blue, J. L. Adler, Transport. Res. Rec. 1644 (1998) 29-36.

[39] Weiliang Zeng, Peng Chen, H. Nakamura, M. Iryo-Asano, Transpor. Res. C. 40 (2014) 143-159.

[40] Lili Lu, Gang Ren, Wei Wang, Yi Wang, Chinese. Phys. B. 23 (2014) 1-7.

[41] D. Helbing, P. Molnar, I. J. Farkas, K. Bolay, Enviro. Plan. B-Plan. Design. 28 (2001) 361-383.

[42] H. Geyer, A. Seyfarth, R. Blickhan, P. Roy. Soc. B. 273 (2006) 2861-2867.

[43] T. McGeer, Int. J. Robot. Res. 9 (1990) 62-82.

[44] C. E. Bauby, A. D. Kuo, J. Biomech. Eng. 33 (2000) 1433-1440.

[45] C. Barker, Some observations on the nature of the mechanism that drives the self-excited lateral response of footbridge[C], Proceedings of the 1st International Conference, Design and Dynamic Behavior of Footbridges, Paris, 2002.

[46] A. L. Hof, R. M. van Bockel, T. Schoppen, K. Postema, Gait & Posture. 25 (2007) 250-258.

[47] J. H. G. Macdonald, P. Roy. Soc. Lond. A. Mat. 465 (2009) 1055-1073.

[48] M. Bocian, J. H. G. Macdonald, J. F. Burn, J. Sound. Vib. 331 (2012) 3914-3929.

[49] J.W. Qin, S. S. Law, Q.S. Yang, N. Yang, J. Sound. Vib. 332 (2013) 1107-1124.

[50] Quqiang Hu, Study of crowd load model based on pedestrian flow simulation[D], Kunming: Kunming University of Science and Technology, 2014. (In Chinese)