Demo and Curve Drawing Model of Relationship between Tangent and Cotangent Function with Varying Period Amplitude

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Abstract. In accordance with the principle of arc expansion and the displacement of projection, the demo and curve model of relationship between tangent and cotangent function shall be designed and drawn. The paper introduces the composition structure of the model, analyzes the main parameters of the system and describes the working principle of the system so as to illustrate the specific application of the model on the basis of the different periods and amplitudes.

Introduction.

Function image is regarded as the geometry representation of function in the rectangular coordinate system. For unary monotonic function, its image is usually a plane curve, and a clear and accurate curve will bring great convenience to study the function. So, using dynamic form to demonstrate function relations can impress the learners deeply with curve drawing, thus helping us learn and research function. In functions, it is a bit difficult to demonstrate tangent and cotangent function and their generic forms, and draw their curves because the demonstration and curve not only reflects the right angle corresponding relationship between angle and function value but also indicates the different effects caused by other factors, such as initial phase, period and amplitude. An entity model is designed and made as demonstrating and drawing. That is, based on trigonometric definition, the increase of arc length is rotated into linear expansion by mechanical means and the parallel and independent movement of vertical rods is used to conduct displacement mapping of function value so as to project the circular motion of dot on the plane as orthogonal synthetic movement of the dot in rectangular coordinate system. And its trajectory entails function curve. At this stage, such methods as drawing dots and computer software assignment are taken to draw[1,2]. Selecting dots is complicated and the calculating results are mainly approximations with broken lines as the connecting lines among fixed dots with the result that the curve full picture cannot be seen. So, drawing is inefficient with poor applicability. The paper aims to design the tangent (cotangent) function curve tracer on the basis of arc development and isometric transformation principle[2,3]. By setting the initial state of the model, as long as the handle of the driving wheel is shaken, a coherent and complete standard function curve shall be drawn on paper or drawing board. The image of simple function $y = \tan x, y = \cot x$ can be drawn and that of complicated function $y = \tan(x + \varphi)$, $y = a\tan x$ drawn. Model curve drawing process directly reflects the formation process of the function. Try the models with different specifications so as to draw large-scale curves on the classroom blackboard, draw miniature curves on students' exercise books, and even draw practical curves in line with the requirement of engineering and technology. The device can be used as teaching aids or drawing tools for teaching and engineering.

Circular movement and displacement mapping.

In circular movement, $O_1x_1y_1$ in rectangular coordinate system (Figure 1.), supposed that the moving point rotates around fixed one O_1 to do circular movement, the trajectory equation is $x_1^2 + y_1^2 = 1$. Among them, unit circle is denoted as $\odot O_1$ .S_1 and C_1 are two points on the circumference.

Given $O_1S_1 \perp O_1C_1$,the rays through radius are $O_1Q_1$and O_1Q_2 respectively. The intersections between $\odot O_1$ and axis O_1x_1 are A_1 and A_2. The lines, through A_1 and A_2, which are perpendicular to the axis O_1x_1, are A_1B_1 and A_2B_2. And the intersections between O_1Q_1 and A_1B_1, O_1Q_2 and tangent A_2B_2 of $\odot O_1$ are respectively A_1 and A_2. If $\angle A_1O_1S_1 = x$, the length of arc is x. So, there are $A_1T_1 = \tan x$ and $A_2M_1 = \cot x$ according to the definition of trigonometric function.

Projection trajectory. once rectangular coordinate system Oxy (Figure 1) is set, whose unit is same as $O_1x_1y_1$, the axes of Oy^+ and $O_1y_1^+$ are in the same direction with Ox^+ and $O_1x_1^+$ overlap and $|A_1O| \geq \pi$. A_1 of the circumference $\odot O_1$ is correspondingly fixed on the origin point O and the arc A_1S_1 is straightened along axis Ox .The point corresponding to S_1 is X , that is $OX = x$ .When S_1 moves from point A_1 counterclockwise, A_1S_1 straightens along the axis of Ox^+, that is $OX = x \geq 0$. When S_1 moves clockwise, A_1S_1 straightens along the axis of Ox^-, that is $OX = x \leq 0$ .The line $OX = x \leq 0$ through point X is perpendicular to the axis Ox and the projection of point T_1 and M_1 on the XQ is T and M respectively. So, $XT = A_1T_1$ and $XM = A_2M_1$. When S_1 does circular motion on $\odot O_1$, the trajectory equations of projection point T and M are as follows, $y = XT = tan\,x$, $y = XM = cot\,x$.[4,5]

The projection XT and XM of displacement vector $\overrightarrow{A_1T_1}$ and $\overrightarrow{A_2M_1}$ from moving points T_1 and M_1 to axis O_1x_1 on moving vector $\overrightarrow{XQ}$ is named as a displacement map.

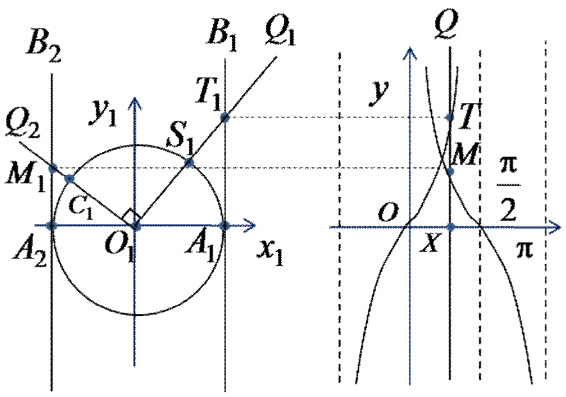

Fig.1. Circular arc projection point trajectory.

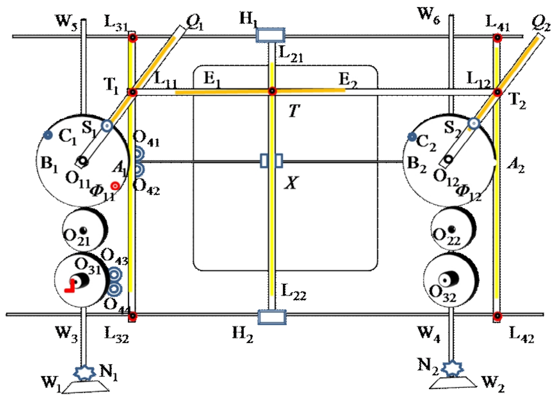

Fig.2. Tangent curve plotting model structure

Circular movement and displacement mapping.

According to the circular linear expansion and displacement mapping principle, a system model can be constructed to express the movement of each point. The model consists of a bracket, spinning wheel disc, driving wheel disc, driving gear, chute dowel lever, brush and link, etc., as shown in Figure 2.

Support. Bracket is composed of anchors W_1 and W_2, vertical rods W_3W_5 and W_4W_6 and lateral poles W_3W_4 and W_5W_6, etc. And there are spinning wheels N_1 and N_2 between the anchor and vertical rod. And there are vertical shafts $O_{11}z$ and $O_{12}z$ on the points of O_{11} and O_{12} of vertical rods W_3W_5andW_4W_6. With the middle of vertical rods W_3W_4 and W_5W_6, on it, there are sliders H_1 and H_2 and a flat panel E_1E_2 between W_3W_4 and W_5W_6 is equipped.

Spinning wheel disc. the disc of the same radius R is superimposed on circular gear to form it. They have a unified central shaft hole and there is a circle of bolt holes on the front side of the outer edge of the disk plane. The wheel discs, which are equipped on the shafts $O_{11}z$ and $O_{12}z$, are denoted as $\odot O_{11}$ and $\odot O_{12}$,which are identical in size and shape. The bolt holes on the disc plane $\odot O_{11}$ are denoted as S_1, C_1 and those on the Φ_{11} as S_2,C_2 andΦ_{12}. Among them, $O_{11}S_1 \perp O_{11}C_1$， $O_{12}S_2 \perp O_{12}C_2$, the radius is R from bolt hole center to the center of the wheel disk.

Driving wheel disc. a large gear is superimposed on the disc and pinion. And pinion is rectangular alveolus with the disc in the middle of two gears, whose radius is same as that of the large gear. And the large gear can mesh with the circular gear of spinning wheel. Circular disc and large gear and pinion are fixed together to share a shaft, one end of which can be installed anywhere in the bracket and the other end of which can insert into the spinning handle. According to the radius difference of large gears, there are many different variety of models, among which the two driving wheel discs with radius as 0.5R are denoted as$\odot O_{21}$and $\odot O_{22}$, and those with radius as 2R as$\odot O_{23}$and$\odot O_{24}$. The pinions are same so they can be connected through rectangular toothed belts.

Driving gear is a circular one with a smaller radius, which functions as coordinating that the turning of the driving wheel disc and spinning one is consistent, and its gears can mesh with those on spinning wheel disc and the teeth on large gears. Those, which are fixed on brace W_3W_5 and W_4W_6, are denoted as driving wheel $\odot O_{41}$ and $\odot O_{42}$.

Chute dowel lever is a long pole with rectangular cross section and the middle is vacant with the chute in the inner edge. There are three levers which is vertical to axis $O_{11}O_{12}$, two ends of which with clamps can be fixed on the lateral poles W_3W_4 and W_5W_6, which are denoted as $L_{31}L_{32}$ and $L_{41}L_{42}$ respectively. The two ends of the levers which connect with sliders H_1 and H_2 are vertical chute dowel lever, which are denoted as $L_{21}L_{22}$ or H_1H_2. Its intersection with $O_{11}O_{12}$ is denoted as X. There is a blank respectively on the frame corresponding to X, among which, the one close to O_{11} is denoted as X_{11} and the other as X_{12}. That level, parallel to the axis $O_{11}O_{12}$, is lateral chute dowel one as $L_{11}L_{12}$, two ends of which are connected with the sliders in chute dowel levers $L_{31}L_{32}$,$L_{41}L_{42}$. The levers which are fixed on the radius of spinning gears $\odot O_{11}$ and $\odot O_{12}$ are radial chute dowel levers as $O_{11}Q_1$ and $O_{12}Q_2$. They are same in size and shape with the length larger than the radius of driving wheel disc.

Slider is a rectangular object, which is embedded in the chute of chute dowel lever. There is a shaft hole in the middle of each chute. The sliders of chute dowel levers $L_{11}L_{12}$ and $L_{21}L_{22}$ are denoted as K_{11} and K_{21}, which are connected through shaft T. The sliders of chute dowel levers $L_{31}L_{32}$ and $O_{11}Q_1$are denoted as K_{12} and K_{22}, which are connected through pin shaft T_1. And the sliders of chute dowel levers $L_{41}L_{42}$ and $O_{12}Q_2$ are denoted as K_{13} and K_{23}, which are connected through pin shaft T_2.

Brush is a conical object whose nib is set at the top of the cone so as to draw traces of color in contact with such planes as paper or drawing board. The rack is set at the bottom of the cone, which can be directly inserted into the shaft hole. The bush installed on the shaft T is denoted as brush Y.

Pulley. a pair of externally tangent fixed pulley is fixed on the ┤ frame vertical pole with externally tangent point at the intersection of lateral and vertical pole. On the lateral pole, movable clamp is mounted so as to fix externally tangent pulley on any position of the bracket. The pulley mounted at R of center line $O_{11}O_{12}$ is denoted as$\odot O_{41}$,$\odot O_{42}$ and the pulley mounted on the center line $O_{21}O_{22}$ as $\odot O_{43}$ and $\odot O_{44}$.

Link is a soft inelastic rope for pulling lever H_1H_2 to slide crosswise, whose top is fixed on the cylindrical surface of the disc and whose end is fixed at the X of pole H_1H_2 through the pulley $\odot O_{32}$ and $\odot O_{31}$.

Working principle of the model.

Initial state setting: to adjust the spinning wheel N_1N_2 of the bracket anchor so as to make the shaft $O_{11}O_{12}$ of spinning disc at the horizontal state. On the drawing board E_1E_2, rectangular coordinate system Oxy is drawn, among which, the axis Ox^+ overlaps $O_{11}O_{12}$ and the axis Oy is parallel to the pole H_1H_2. The distance from $L_{31}L_{32}$ and $L_{41}L_{42}$ to the center of disc$\odot O_{11}$ and $\odot O_{12}$ is same, at the same side of vertical pole W_3W_5 and W_4W_6. the lateral displacement of the point of slider bar $L_{21}L_{22}$ on the axis Oy is denoted as x and the vertical displacement of the point of slider bar $L_{11}L_{12}$ on the axis Ox as y . The initial positions before the discs$\odot O_{11}$ and $\odot O_{12}$ rotate are set as $O_{11}A_1$ and $O_{12}A_2$ with the overlap of the point A_1 on the disc$\odot O_{11}$ and the tangent point of pulley of $\odot O_{31}$ and $\odot O_{32}$. The circuitous direction of the link is set so when the top of the link rotates clockwise on the disc with the value of the angle $\angle S_1O_{11}A_1 = x$ negative, the end of the link is fixed at X_{11} with the expansion point X of arc moving to the left horizontally along Ox^-. When the top of the link rotates counterclockwise with the value of the angle $\angle S_1O_{11}A_1 = x$ positive, the end of the link bypasses revolving shaft and is fixed at X_{12} with the expansion point X of arc moving to the right horizontally along Ox^+.

Model job requirements: when the system is working, the spinning wheel discs $\odot O_{11}$ and $\odot O_{12}$ keep synchronous motion, namely their angular velocity is same with the slider of each pole can move freely so as to make pole $L_{11}L_{12}$ parallel to axis Ox and pole $L_{21}L_{22}$ to axis Oy.

The variables operation in the model perform as follows: rotating disc is denoted as a unit circle and unit arc length and the number of central Angle radian are same. Then the link winding up the disc directly reflect the changes in the angle, namely the angle is converted into horizontal displacement through the arc length. And the link pulls in equal length to make pole H_1H_2 move in parallel so the change of the arc length is converted into that of the straight line segment. Intuitively it is to make a horizontal displacement between point X and the origin O. When wheel disc $\odot O_{11}$ rotates, angle change is, $\angle S_1O_{11}A_1 = \overset{\frown}{A_1S_1} = OX = x$. Because pole $L_{21}L_{22}$ makes side-to-side movement and pole $L_{11}L_{12}$ moves up and down, the intersection of two poles is the synthesis of two orthogonal motion, which can be reflected through the brush Y sliding on the Oxy. In this way, the side of the triangle is converted into longitudinal displacement by parallel bar, expressed with the function $y = f(x)$.

Limit state in the system: due to the limit by the length of longitudinal and radial poles, the system model can be drawn only with limited curve, for example, the curve at the side of Angle of $x = \pm\pi/2$ in the limit state, the reason is that when radial pole rotates nearly parallel to longitudinal pole, their intersection point tends to be infinite.

The working process of the system.

Function $y = tan x, (-\pi/2 < x < \pi/2)$ image. As shown in Figure 3, by adjusting the spinning wheel N_1N_2 of the anchor, disc axis $O_{11}O_{12}$ is in a level state. In the middle of the drawing board E_1E_2 rectangular coordinate system Oxy is drawn with axis Ox^+ and $O_{11}O_{12}$ overlap. Driving wheel discs $\odot O_{21}$ and $\odot O_{22}$ are mounted on poles W_3W_5 and W_4W_6 respectively to make them mesh with the external part of the gear spinning wheel disc $\odot O_{41}$ and $\odot O_{42}$ of gear meshing. And the gear belt is installed between $\odot O_{21}$ and $\odot O_{22}$ to make the wheel discs $\odot O_{11}$ and $\odot O_{12}$ become synchronous rotation system. Pulleys $\odot O_{31}$ and $\odot O_{32}$ are mounted on shaft $O_{11}O_{12}$ to make the tangent point at the circumference A_1 of wheel disc $\odot O_{11}$. The longitudinal poles $L_{31}L_{32}$ and $L_{41}L_{42}$ are fixed at the point A_1 and A_2 of the right side of the disc $\odot O_{11}$ and $\odot O_{12}$ to make it tangent with the disc. The radial poles $O_{11}Q_1$ and $O_{12}Q_2$ are fixed at the $O_{11}S_1$ and $O_{12}S_2$ of the radius of the disc $\odot O_{11}$ and $\odot O_{12}$ respectively to make them link with sliders with shafts T_1 and T_2. The two ends of pole $L_{11}L_{12}$ are connected with shafts T_1 and T_2 respectively to make the sliders K_{11} and K_{21} link together with shaft T. The top of the link $S_1A_1O_{12}X_{12}$ is fixed at cylinder S_1 of $\odot O_{11}$ and its end is fixed at X_{12} of pole H_1H_2 through pulley $\odot O_{32}$ (initial point A_1) bypassing a shaft O_2z with the pole H_1H_2 and the shaft Oy on E_1E_2 overlap. Adjusting the wheel discs $\odot O_1$ and $\odot O_2$ is to make the points of S_1 and S_2 respectively coincide with the initial ones A_1 and A_2 with pole $L_{11}L_{12}$ and shaft Ox overlap and the tope of brush Y is fixed at the origin O. Shaking the handles of the driving wheel disc $\odot O_{21}$ counterclockwise is to drive wheel disc $\odot O_{11}$ and $\odot O_{12}$ move synchronically counterclockwise. The angle $\angle S_1O_{11}A_1$ expands positively and one end of link $S_1A_1O_{12}X_{12}$ at the same part of S_1 rotates with wheel disc $\odot O_{11}$ with subsequent link winding on the arc $\overset{\frown}{A_1S_1}$ of the disc. Its other end pulls the pole H_1H_2 to move to the right and the radial poles $L_{31}L_{32}$ and $L_{41}L_{42}$ push the sliders K_{22} and K_{23} inside to move so sliders K_{22} and K_{23} drive the slider K_{12} and K_{13} through shafts T_1 and T_2 to move up in longitudinal poles $L_{31}L_{32}$ and $L_{41}L_{42}$. Poles H_1H_2 and $L_{11}L_{12}$ drive brush Y to leave a mark on the drawing board E_1E_2, which is the image of function

$f(x)=tanx,(0\le x<\pi/2)$. Afterwards, the system is reset to the initial state (brush Y stays in a state of origin at O and others keep on). Link $S_1A_1X_{11}$ is changed with its top still and its end connected directly with the X_{11}through pulley ⊙O_{31}. Shaking the handle of driving wheel disc ⊙O_{21} clockwise is to drive the wheel discs ⊙O_{11} and ⊙O_{12} to rotate synchronously clockwise. The angle $\angle S_1O_{11}A_1$ expands reversely. One end of link $S_1A_1X_{11}$ at the same part of S_1 rotates with wheel disc ⊙O_{11} with subsequent link winding on the arc $\overset{\frown}{A_1S_1}$ of the disc and its other end pulls the pole H_1H_2 to move to the left so brush Y can draw the curve on the drawing board E_1E_2 as the image of the function $f(x)=tanx,(-\pi/2<x<0)$. The above-mentioned two processes are combined to make the image of the function $y=tanx,(-\pi/2<x<\pi/2)$.

Image of function. $y=\cot x,(0\le x\le\pi)$. As shown in Figure 4, the coordinate system Oxy is drawn at the left side of drawing board E_1E_2 with axis Ox^+ and $O_{11}O_{12}$ overlap. The relationship between the wheel discs and gears is same as that of pulley position and drawing curve $f(x)=tanx,(0\le x<\pi/2)$. The longitudinal poles $L_{31}L_{32}$ and $L_{41}L_{42}$ are fixed at the left points B_1 and B_2 of the wheel disc ⊙O_{11} and ⊙O_{12} respectively to make it tangent with the disc. The radial poles $O_{11}Q_1$ and $O_{12}Q_2$ are respectively fixed at the radius $O_{11}C_1$ and $O_{12}C_2$ of wheel discs ⊙O_{11} and ⊙O_{12} with the sliders K_{12} and K_{22} connecting with the left end of pole $L_{11}L_{12}$ through shaft T_1, the slider K_{13} and K_{23} connecting with the right end of pole $L_{11}L_{12}$ through shaft T_2, and the slider K_{11} connecting with K_{21} through the shaft T. The top of the link $S_1A_1O_{12}X_{12}$ is fixed at cylinder S_1 of ⊙O_{11} and its end is fixed at X_{12} of pole H_1H_2 through pulley ⊙O_{32} (initial point A_1) bypassing a shaft O_2z. Adjusting the wheel discs ⊙O_1 and ⊙O_2 is to make the points of S_1 and S_2 respectively coincide with the initial ones A_1 and A_2 with longitudinal pole $L_{31}L_{32}$ and $L_{41}L_{42}$ almost vertical to O_{11} and O_{12} with pole $L_{11}L_{12}$ at the top of the model. The pole H_1H_2 almost overlaps the shaft Oy of E_1E_2 so the top of brush Y is near axis Oy at the highest level of the system model. By shaking the handle of driving wheel disc ⊙O_{21} counterclockwise, the system operation process is same as drawing curve $f(x)=tanx,(0\le x<\pi/2)$. Eventually, the trace with brush Y to draw on the drawing board E_1E_2 is the image function $y=cot\,x,(0\le x\le\pi)$.

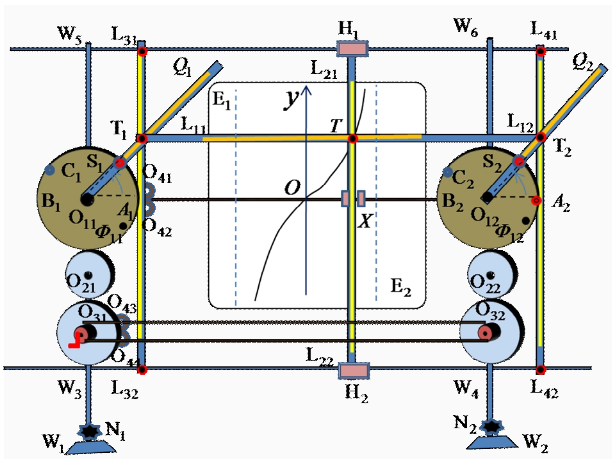

Fig.3. y= tanx images depicting the process.

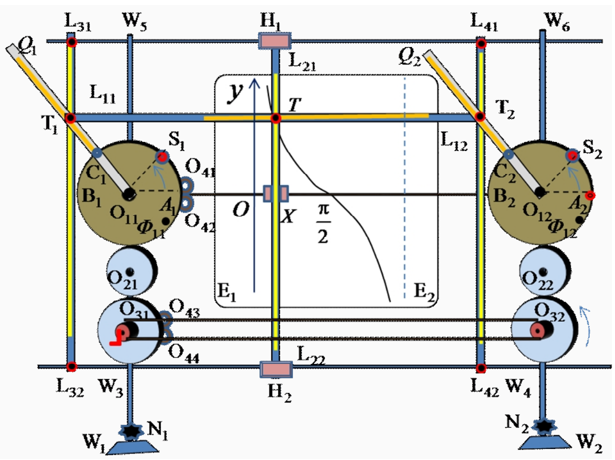

Fig.4. y = cotx images depicting the process

Model application.

System model image also can draw more complex function image, demonstrate and draw the image of tangent functions of such forms as the initial phase, variable period and amplitude.

Description of image of function $y = a \cdot tanx,(-\pi/2 < x < \pi/2, a > 0)$,System model under description of the initial state of the curve $f(x) = tanx,(0 \le x < \pi/2)$ moves the longitudinal poles $L_{31}L_{32}$ and $L_{41}L_{42}$ to turn right or left at the same time, thus arriving at the points of D_1 and D_2 fixed respectively with $O_{11}D_1 = O_{12}D_2 = aR$. Other preparation and operation is exactly same as the description of curve $y = tanx,(-\pi/2 < x < \pi/2)$. Eventually, brush Y can draw the image of function $y=a\cdot tanx,(-\pi/2<x<\pi/2,a>0)$ on the drawing board E_1E_2.

Description of image of function $y = tan(x+\varphi),(-\pi/2 < x < \pi/2, \varphi \ne 0)$. System model under description of the initial state of the curve $f(x) = tanx,(0 \le x < \pi/2)$ adjusts the initial points S_1 and S_2 of the wheel disc ⊙O_1 and ⊙O_2. If $\varphi > 0$, beginning with $O_{11}A_1$ and O_{12} A_2 respectively, discs ⊙O_{11} and ⊙O_{12} are rotated counterclockwise to make $\angle S_1 O_{11} A_1 = \varphi$ and $\angle S_2 O_{12} A_2 = \varphi$. If $\varphi < 0$, beginning with $O_{11}A_1$ and $O_{12}A_2$, discs ⊙O_{11} and ⊙O_{12} are rotated clockwise to make $\angle A_1 O_{11} S_1 = \angle A_2 O_{12} S_2 = \varphi$. Other preparation and operating is exactly same as the description of curve $y = tan\,x,(-\pi/2 < x < \pi/2)$. Eventually, brush Y draws the image of function $y = tan(x+\varphi),(-\pi/2 < x < \pi/2, \varphi \ne 0)$ on the drawing board E_1E_2.

Conclusion.

In the commonly used traditional manual tracing, computer software drawing and automatic plotter drawing, automatic plotter is superior to others in process performance, image accuracy and convenience in use and so on. By plane circular motion, this paper expounds the expansion of the circular arc and displacement mapping principle, designs a dynamic demonstration tangent and cotangent function and draws the curve of the system model. Besides, structure of the model is introduced in detail with the specific analyses of the main parameters in the system and the description of the working principle of the system. In combination with different forms of function, the specific method about drawing image is given.

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