

Design of Overload Control for Target Missiles Based on Robust Adaptive Terminal Sliding-mode Control

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Keywords: robust adaptive control; terminal sliding-mode control; neural network; overload control

Abstract. Backstepping design, neural network technology, robust adaptive control and terminal sliding-mode control is combined. Firstly, neural networks are used to estimate the virtual control laws of the controlled system. Secondly, terminal sliding-mode control is utilized to improve the convergence speed and robustness of the controlled plant. Thirdly, robust adaptive control method is employed to estimate the unknown upper boundary of uncertainties. Lastly, backstepping design is incorporated to design the virtual control laws and actual control law of the controlled system. Based on Lyapunov stability theorem, all of error signals are bounded and exponentially converge to a bound neighbor of the origin. Simulation results show the effectiveness of the proposed control method.

Introduction

Compared with the attitude control system, the overload control method is simpler and more reliable. In this thesis, a robust adaptive terminal sliding mode control system is preed for the design of overload control system of target missile.

Overload control model of missile target missile

Taking the aerodynamic shape distributed in axial symmetry of missile target as an example, we studied the dynamics model of the pitch channel:

$$\begin{cases} \dot{\alpha} = \omega_z - \frac{1}{mV}(P \sin \alpha + Y - mg \cos \theta) \\ \dot{\omega}_z = \frac{M_z}{J_z} = \frac{m_z q S L}{J_z} \\ n_y = \frac{P \sin \alpha + Y}{mg} \end{cases} \quad (1)$$

In the formula, α is incidence; ω_z is the speed of pitch angle; δ_z is pitch rudder angle; n_y is normal overload; V is the flight speed; g is acceleration of gravity, the meanings of other symbols can be found in literature [1].

Linearizing the model we can get the linear dynamic model:

$$\begin{cases} \dot{\alpha} = \omega_z - a_{34}\alpha - a_{35}\delta_z + \Delta_\alpha \\ \dot{\omega}_z = a_{24}\alpha + a_{22}\omega_z + a_{25}\delta_z + \Delta_{\omega_z} \\ n_y = a_{34}V\alpha/g + a_{35}V\delta_z/g + \Delta_{n_y} \end{cases} \quad (2)$$

In the formula, a_{22} 、 a_{24} 、 a_{25} 、 a_{34} and a_{35} are all dynamic coefficients of the missile, Δ_α 、 Δ_{ω_z} and Δ_{n_y} are uncertainties in the linearizing process of the model.

Considering the dynamic characteristics of the rudder system:

$$\dot{\delta}_z = -w_{\delta_z} \delta_z + w_{\delta_z} u_{\delta_z} \quad (3)$$

In this formula, w_{δ_z} is the bandwidth of pitch control system; u_{δ_z} is control input.

In order to make the state of the system cover the performance of the missile directly, we transform the coordinate as follows: $[\alpha \ \omega_z \ \delta_z]^T \rightarrow [\alpha \ \omega_z \ n_y]^T$, combine (2) and (3) we can get :

$$\begin{bmatrix} \dot{\alpha} \\ \dot{\omega}_z \\ \dot{n}_y \end{bmatrix} = \begin{bmatrix} 0 & 1 & -\frac{g}{V} \\ a_{24} - \frac{a_{25}a_{34}}{a_{35}} & a_{22} & \frac{a_{25}g}{a_{35}V} \\ \frac{a_{34}w_{\delta_z}V}{g} & \frac{a_{34}V}{g} & -a_{34} - w_{\delta_z} \end{bmatrix} \begin{bmatrix} \alpha \\ \omega_z \\ n_y \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{a_{35}w_{\delta_z}V}{g} \end{bmatrix} u_{\delta_z} + \begin{bmatrix} \Delta_\alpha + \frac{\Delta_{n_y}g}{V} \\ \Delta_{\omega_z} - \frac{\Delta_{n_y}a_{25}g}{a_{35}V} \\ \frac{a_{34}V\Delta_\alpha}{g} + (a_{34} + w_{\delta_z})\Delta_{n_y} + \dot{\Delta}_{n_y} \end{bmatrix} \quad (4)$$

As for the pitch channel model of the missile target (4), control target is: under the effect of given normal overload order n_{yc} , design reasonable control law u_{δ_z} , making the actual normal overload of the missile n_y tracing n_{yc} , besides, incidence α and the speed of pitch angle ω_z all converge to ideal status.

In order to realize the above objectives, we transform (4) into standard triangle form, and define the reversible state transformation matrix T as follows:

$$T = \begin{bmatrix} 1 & \frac{a_{35}}{a_{25}} & 0 \\ \frac{a_{24}a_{35} - a_{34}}{a_{25}} & 1 + \frac{a_{22}a_{35}}{a_{25}} & 0 \\ \left(a_{24} - \frac{a_{25}a_{34}}{a_{35}} \right) \left(1 + \frac{a_{22}a_{35}}{a_{25}} \right) & \frac{a_{22}^2a_{35} + a_{24}a_{35}}{a_{25}} + a_{22} - a_{34} & \frac{g}{V} \left(\frac{a_{25}}{a_{35}} - \frac{a_{24}a_{35}}{a_{25}} + a_{22} + a_{34} \right) \end{bmatrix} \quad (5)$$

Applying the state transformation matrix T $\begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} = T \begin{bmatrix} \alpha \\ \omega_z \\ n_y \end{bmatrix}$, we can get:

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \\ \dot{\xi}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ b_1 & b_2 & b_3 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b_u \end{bmatrix} u_{\delta_z} + \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{bmatrix} \quad (6)$$

In the formula, the parameters are as follows:

$$b_1 = \frac{\left(\frac{a_{24}a_{35} - a_{34}}{a_{25}} \right) \left(a_{24} - \frac{a_{25}a_{34}}{a_{35}} \right) \left(1 + \frac{a_{22}a_{35}}{a_{25}} \right) + (a_{22} - b_3) \left(\frac{a_{24}a_{35} - a_{34}}{a_{25}} \right) \left(\frac{a_{22}^2a_{35} + a_{24}a_{35}}{a_{25}} + a_{22} - a_{34} \right)}{\frac{a_{35}}{a_{25}} \left(\frac{a_{24}a_{35} - a_{34}}{a_{25}} \right) - \left(1 + \frac{a_{22}a_{35}}{a_{25}} \right)}$$

$$+ \frac{a_{34} \left(\frac{a_{24}a_{35} - a_{34}}{a_{25}} \right) \left(\frac{a_{25}}{a_{35}} - \frac{a_{24}a_{35}}{a_{25}} + a_{34} + a_{22} \right) - \left(1 + \frac{a_{22}a_{35}}{a_{25}} \right) \left(\frac{a_{22}^2a_{35} + a_{24}a_{35}}{a_{25}} + a_{22} - a_{34} \right) \left(a_{24} - \frac{a_{25}a_{34}}{a_{35}} \right)}{\frac{a_{35}}{a_{25}} \left(\frac{a_{24}a_{35} - a_{34}}{a_{25}} \right) - \left(1 + \frac{a_{22}a_{35}}{a_{25}} \right)}$$

$$, \quad b_2 = \frac{\left(\frac{a_{22}^2a_{35} + a_{24}a_{35}}{a_{25}} + a_{22} - a_{34} \right) \left(\frac{a_{24}a_{35} - a_{34}}{a_{25}} \right) + \frac{a_{34}a_{35}w_{\delta_z}}{a_{25}} \left(\frac{a_{25}}{a_{35}} - \frac{a_{24}a_{35}}{a_{25}} + a_{34} + a_{22} \right)}{\frac{a_{35}}{a_{25}} \left(\frac{a_{24}a_{35} - a_{34}}{a_{25}} \right) - \left(1 + \frac{a_{22}a_{35}}{a_{25}} \right)}$$

$$\begin{aligned}
& + \frac{\left(a_{24} - \frac{a_{25}a_{34}}{a_{35}} \right) \left(1 + \frac{a_{22}a_{35}}{a_{25}} \right) \left(-\frac{a_{35}}{a_{25}}b_3 - 1 \right) - a_{22} \left(\frac{a_{22}^2a_{35} + a_{24}a_{35}}{a_{25}} + a_{22} - a_{34} \right)}{\frac{a_{35}}{a_{25}} \left(\frac{a_{24}a_{35}}{a_{25}} - a_{34} \right) - \left(1 + \frac{a_{22}a_{35}}{a_{25}} \right)} \\
& + \frac{\left(\frac{a_{22}^2a_{35} + a_{24}a_{35}}{a_{25}} + a_{22} - a_{34} \right) b_3 - a_{34} \left(\frac{a_{25}}{a_{35}} - \frac{a_{24}a_{35}}{a_{25}} + a_{34} + a_{22} \right)}{\frac{a_{35}}{a_{25}} \left(\frac{a_{24}a_{35}}{a_{25}} - a_{34} \right) - \left(1 + \frac{a_{22}a_{35}}{a_{25}} \right)}, \\
b_3 = & \frac{- \left(a_{24} - \frac{a_{25}a_{34}}{a_{35}} \right) \left(1 + \frac{a_{22}a_{35}}{a_{25}} \right) + \frac{a_{25}}{a_{35}} \left(\frac{a_{22}^2a_{35} + a_{24}a_{35}}{a_{25}} + a_{22} - a_{34} \right)}{\frac{a_{25}}{a_{35}} - \frac{a_{24}a_{35}}{a_{25}} + a_{34} + a_{22}} - (a_{34} + w_{\delta_z}) \\
b_u = & a_{35} w_{\delta_z} \left(\frac{a_{25}}{a_{35}} - \frac{a_{24}a_{35}}{a_{25}} + a_{34} + a_{22} \right), \quad \Delta_1 = \Delta_\alpha + \frac{a_{35}}{a_{25}} \Delta_{\omega_z}, \\
\Delta_2 = & \left(\frac{a_{24}a_{35}}{a_{25}} - a_{34} \right) \Delta_\alpha + \left(1 + \frac{a_{22}a_{35}}{a_{25}} \right) \Delta_{\omega_z} + \frac{g}{V} \left(\frac{a_{24}a_{35}}{a_{25}} - \frac{a_{25}}{a_{35}} - a_{34} - a_{22} \right) \Delta_{n_y}, \\
\Delta_3 = & \left(a_{24} + a_{34}^2 + \frac{(a_{22}a_{24} - a_{24}a_{34})a_{35}}{a_{25}} \right) \Delta_\alpha + \left(\frac{a_{22}^2a_{35} + a_{24}a_{35}}{a_{25}} + a_{22} - a_{34} \right) \Delta_{\omega_z} \\
& + \frac{g}{V} \left(\frac{(a_{34} - a_{22})a_{25}}{a_{35}} + \frac{(a_{22} - a_{34})a_{24}a_{35}}{a_{25}} + a_{34}^2 - a_{22}^2 + \frac{a_{25}w_{\delta_z}}{a_{35}} - \frac{a_{24}a_{35}w_{\delta_z}}{a_{25}} + a_{34}w_{\delta_z} + a_{22}w_{\delta_z} \right) \Delta_{n_y} \\
& + \frac{g}{V} \left(\frac{a_{25}}{a_{35}} - \frac{a_{24}a_{35}}{a_{25}} + a_{34} + a_{22} \right) \Delta_{n_y}.
\end{aligned}$$

Since the state transformation matrix T is reversible, the system (6) and the system (4) are equivalent transformation. based on system (6), we will design and analyze the overload control system of target missile pitch channel.

Design of Overload Control Based on Robust Adaptive Terminal Sliding-mode Control

Assumption 1 There exist unknown upper boundary r_1 , r_2 and r_3 , which enable the uncertainties of the system Δ_1 , Δ_2 and Δ_3 satisfy:

$$|\Delta_1| \leq r_1, \quad |\Delta_2| \leq r_2, \quad |\Delta_3| \leq r_3 \quad (7)$$

In the formula, $|\cdot|$ represents the absolute value of “ \cdot ”.

As for the pitch channel overload control model (6) of the missile after equivalent transformation, we adopt adaptive backstepping design and Terminal sliding-mode control design. Define the state error of under control target (6) as follows:

$$\begin{cases} \chi_1 = \xi_1 - \xi_{1d} \\ \chi_2 = \xi_2 - \xi_{2d} \\ \chi_3 = \xi_3 - \xi_{3d} \end{cases} \quad (8)$$

In the formula, ξ_{1d} , ξ_{2d} and ξ_{3d} are system expected state trajectory, ξ_{1d} is from the desired overload signal n_{yc} wave filtering, ξ_{2d} and ξ_{3d} are defined by (12) and (20) separately.

From (6) and (8), we can get the dynamic equation of state error:

$$\begin{cases} \dot{\chi}_1 = \dot{\xi}_1 - \dot{\xi}_{1d} = \xi_2 + \Delta_1 - \dot{\xi}_{1d} \\ \dot{\chi}_2 = \dot{\xi}_2 - \dot{\xi}_{2d} = \xi_3 + \Delta_2 - \dot{\xi}_{2d} \\ \dot{\chi}_3 = \dot{\xi}_3 - \dot{\xi}_{3d} = b_1\xi_1 + b_2\xi_2 + b_3\xi_3 + b_u u_{\delta_z} + \Delta_3 - \dot{\xi}_{3d} \end{cases} \quad (9)$$

Step 1: Considering the first formula of system (9), the form of RBF neural network is presented for the differential signal $\dot{\xi}_{1d}$ of the desired state. Because the RBF neural network has the ability of on-line approximation ^[2-3], we adopt pitch channel missile expected overload signal n_{yc} and state variables α, ω_z, n_y as the input signals of RBF neural network, apply arbitrary approximation ability of neural network to estimate signal $\dot{\xi}_{1d}$.

$$\text{Design the differential signal } \dot{\xi}_{1d} \text{ of desired state as: } \dot{\xi}_{1d} = W^{*T} \phi(n_{yc}, n_y, \alpha, \omega_z) + \varepsilon_{NN} \quad (10)$$

In the formula, optimal weight matrix $W^* \in R^{4 \times 1}$, Gauss function vector $\phi(n_{yc}, n_y, \alpha, \omega_z) \in R^{4 \times 1}$, network reconstruction error $\varepsilon_{NN} \in R$, and $|\varepsilon_{NN}| \leq r_{NN}$, r_{NN} is unknown upper boundary. We can integral differential signal $\dot{\xi}_{1d}$ to get the desired state ξ_{1d} .

Employ ξ_2 as the virtual control law of (9), thus there exists an ideal virtual control law:

$$\xi_2^* = \dot{\xi}_{1d} - \Delta_1 - k_1 \chi_1 = W^{*T} \phi(n_{yc}, n_y, \alpha, \omega_z) - \Delta_1 - k_1 \chi_1 + \varepsilon_{NN} \quad (11)$$

which enables $\dot{\chi}_1 = -k_1 \chi_1 + (\xi_2 - \xi_2^*)$, $k_1 > 0$ as design parameter, because ξ_2^* is inaccessible; we choose desired virtual control law as:

$$\xi_{2d} = \hat{W}^T \phi(n_{yc}, n_y, \alpha, \omega_z) - v_1 - k_1 \chi_1 \quad (12)$$

In the formula, \hat{W} is the estimated value of the optimal weight matrix W^* , v_1 is introduced robust item, whose form is as (17).

$$\text{We choose Lyapunov function as follows: } V_1 = \frac{1}{2} \chi_1^2 + \frac{1}{2} \text{tr}(\tilde{W}^T \Gamma_w^{-1} \tilde{W}) + \frac{1}{2} \tilde{r}_1^2 + \frac{1}{2} \tilde{r}_{NN}^2 \quad (13)$$

In the formula, $\tilde{W} = \hat{W} - W^*$, $\Gamma_w = \Gamma_w^T > 0$ is design parameter, \hat{r}_1 is the estimated value of the upper boundary r_1 , \tilde{r}_1 is estimated error $\tilde{r}_1 = \hat{r}_1 - r_1$, \hat{r}_{NN} is the estimated value of the upper boundary r_{NN} , \tilde{r}_{NN} is estimated error $\tilde{r}_{NN} = \hat{r}_{NN} - r_{NN}$.

According to derivation of V_1 , Apply (9), (10) and (12) to (13), we have:

$$\dot{V}_1 = -k_1 \chi_1^2 + \chi_1 \tilde{W}^T \phi(n_{yc}, n_y, \alpha, \omega_z) - \chi_1 \varepsilon_{NN} + \chi_1 \chi_2 - \chi_1 v_1 + \chi_1 \Delta_1 + \text{tr}(\tilde{W}^T \Gamma_w^{-1} \dot{\tilde{W}}) + \tilde{r}_1 \dot{\tilde{r}}_1 + \tilde{r}_{NN} \dot{\tilde{r}}_{NN} \quad (14)$$

Select the adaptive adjustive law of the of neural network parameters as:

$$\dot{\hat{W}} = -\Gamma_w \phi(n_{yc}, n_y, \alpha, \omega_z) \chi_1 - \Gamma_w \delta_w \hat{W} \quad (15)$$

In the formula, $\delta_w > 0$ is design parameter.

Combine (15) and (14), and take assumption1 into account, we have:

$$\begin{aligned} \dot{V}_1 = & -k_1 \chi_1^2 + \chi_1 \chi_2 - \delta_w \text{tr}(\tilde{W}^T \hat{W}) - \chi_1 \varepsilon_{NN} - \chi_1 v_1 + \chi_1 \Delta_1 + \tilde{r}_1 \dot{\tilde{r}}_1 + \tilde{r}_{NN} \dot{\tilde{r}}_{NN} \leq -k_1 \chi_1^2 + \chi_1 \chi_2 - \delta_w \text{tr}(\tilde{W}^T \hat{W}) \\ & + |\chi_1| r_{NN} + |\chi_1| r_1 - \chi_1 v_1 + \tilde{r}_1 \dot{\tilde{r}}_1 + \tilde{r}_{NN} \dot{\tilde{r}}_{NN} \end{aligned} \quad (16)$$

Select the adaptive adjustive laws of parameters \hat{r}_1, \hat{r}_{NN} and robust item v_1 as:

$$\begin{cases} \dot{\hat{r}}_1 = \sigma_1 \chi_1^2 - \eta_1 \hat{r}_1 \\ \dot{\hat{r}}_{NN} = \sigma_{NN} \chi_1^2 - \eta_{NN} \hat{r}_{NN} \\ v_1 = \hat{r}_1^2 \chi_1 + \hat{r}_{NN}^2 \chi_1 + \frac{\sigma_1^2 + \sigma_{NN}^2}{4} \chi_1 \end{cases} \quad (17)$$

In the formula, $\sigma_1 > 0, \eta_1 > 0, \sigma_{NN} > 0$ and $\eta_{NN} > 0$ are design parameters.

Combine (17) and (16), we have:

$$\dot{V}_1 \leq -k_1 \chi_1^2 + \chi_1 \chi_2 - \frac{\delta_w}{2} \|\tilde{W}\|^2 - \frac{1}{2} \eta_1 \tilde{r}_1^2 - \frac{1}{2} \eta_{NN} \tilde{r}_{NN}^2 + c_1 \quad (18)$$

In the formula, $c_1 = \frac{\delta_w}{2} \|W^*\|^2 + \frac{1}{2} \eta_1 r_1^2 + \frac{1}{2} \eta_{NN} r_{NN}^2 + \frac{r_1}{4\sigma_1} + \frac{r_{NN}}{4\sigma_{NN}}$.

Step 2, considering the second formula of system (9), select ξ_3 as the virtual control law of the second formula of (9), and there exists an ideal virtual control law:

$$\xi_3^* = \dot{\xi}_{2d} - \Delta_2 - k_2 \chi_2 \quad (19)$$

which enables $\dot{\chi}_2 = -k_2 \chi_2 + (\xi_3 - \xi_3^*)$, $k_2 > 0$ is design parameter, because ξ_3^* is inaccessible, we select desired virtual control law as: $\xi_{3d} = \dot{\xi}_{2d} - \chi_1 - k_2 \chi_2 + k_3 \chi_2^{q/p} - v_2$ (20)

In the formula, p and q are positive odd numbers and $p > q, k_3 > 0$ are design parameters, introduce v_2 into robust item, whose form is as (23).

$$\text{We select Lyapunov function as follows: } V_2 = V_1 + \frac{1}{2} \chi_2^2 + \frac{1}{2} \tilde{r}_2^2 \quad (21)$$

In the formula, \hat{r}_2 is the estimated value of upper boundary r_2 , \tilde{r}_2 is estimated error $\tilde{r}_2 = \hat{r}_2 - r_2$.

According to derivation of V_2 , and Combine (9), and take assumption 1 into account, we can get:

$$\dot{V}_2 = \dot{V}_1 + \chi_2 (\xi_3 + \Delta_2 - \dot{\xi}_{2d}) + \tilde{r}_2 \dot{\tilde{r}}_2 \leq \dot{V}_1 - k_2 \chi_2^2 - \chi_1 \chi_2 + \chi_2 (k_3 \chi_2^{q/p} + \chi_3) - \chi_2 v_2 + |\chi_2| r_2 + \tilde{r}_2 \dot{\tilde{r}}_2 \quad (22)$$

Select parameters \hat{r}_2 adaptive adjustive law and robust item v_2 as follows:

$$\begin{cases} \dot{\hat{r}}_2 = \sigma_2 \chi_2^2 - \eta_2 \hat{r}_2 \\ v_2 = \hat{r}_2^2 \chi_2 + \frac{\sigma_2^2}{4} \chi_2 \end{cases} \quad (23)$$

In the formula, $\sigma_2 > 0, \eta_2 > 0$ are design parameters.

Combine (23), we can have:

$$\dot{V}_2 \leq -k_1 \chi_1^2 - k_2 \chi_2^2 + \chi_2 (k_3 \chi_2^{q/p} + \chi_3) - \frac{\delta_w}{2} \|\tilde{W}\|^2 - \frac{1}{2} \eta_1 \tilde{r}_1^2 - \frac{1}{2} \eta_2 \tilde{r}_2^2 - \frac{1}{2} \eta_{NN} \tilde{r}_{NN}^2 + c_2 \quad (24)$$

In the formula, $c_2 = c_1 + \frac{1}{2} \eta_2 r_2^2 + \frac{r_2}{4\sigma_2}$.

Step 3, considering the third formula of system (9), u_{δ_c} is actual control law, and we adopt Terminal sliding-mode control design control law u_{δ_c} . Design system sliding surface as:

$$S = \chi_3 + k_3 \chi_2^{q/p} \quad (25)$$

Design system control law u_{δ_c} as:

$$u_{\delta_c} = -\frac{1}{b_u} \left(b_1 \xi_1 + b_2 \xi_2 + b_3 \xi_3 + v_3 - \dot{\xi}_{3d} + k_4 S + \chi_2 \right) + k_3 \frac{q}{p} \chi_2^{(q-p)/p} (\xi_3 - \dot{\xi}_{2d}) + k_5 \text{sgn}(S) \quad (26)$$

In the formula, v_3 is introduced robust item, whose form is as (32), $k_4 > 0, k_5 > 0$ are design parameters, $\text{sgn}(\cdot)$ represents sign function.

$$\text{Select Lyapunov function as follows: } V_3 = V_2 + \frac{1}{2} S^2 + \frac{1}{2} \tilde{r}_3^2 + \frac{1}{2} \tilde{r}_4^2 \quad (27)$$

In the formula, \hat{r}_3 is the estimated value of upper boundary r_3 , \tilde{r}_3 is estimated error $\tilde{r}_3 = \hat{r}_3 - r_3$, \hat{r}_4 is the estimated value of upper boundary $r_4 = \frac{k_3 q}{p} r_2$, \tilde{r}_4 is estimated error $\tilde{r}_4 = \hat{r}_4 - r_4$. According to derivation of V_3 , Combine (25), we can have:

$$\dot{V}_3 = \dot{V}_2 + S \left(\begin{array}{l} b_1 \xi_1 + b_2 \xi_2 + b_3 \xi_3 + b_u u_\delta + \Delta_3 - \dot{\xi}_{3d} \\ + k_3 \frac{q}{p} \chi_2^{(q-p)/p} (\xi_3 + \Delta_2 - \dot{\xi}_{2d}) \end{array} \right) + \tilde{r}_3 \dot{\tilde{r}}_3 + \tilde{r}_4 \dot{\tilde{r}}_4 \quad (28)$$

Combine (26) and (25), we can have:

$$\dot{V}_3 = \dot{V}_2 - k_4 S^2 - S \chi_2 + S \left(\Delta_3 + k_3 \frac{q}{p} \chi_2^{(q-p)/p} \Delta_2 - v_3 \right) - k_5 |S| + \tilde{r}_3 \dot{\tilde{r}}_3 + \tilde{r}_4 \dot{\tilde{r}}_4 \quad (29)$$

Define variable $\bar{\chi}_2$ as:
$$\bar{\chi}_2 = \begin{cases} \frac{1}{\chi_2} & 0 < |\chi_2| < 1 \\ \chi_2 & |\chi_2| \geq 1 \end{cases} \quad (30)$$

Considering variable $\bar{\chi}_2$ and assumption1, we can get:

$$\dot{V}_3 \leq \dot{V}_2 - k_4 S^2 - S \chi_2 + |S| r_3 + |S| |\bar{\chi}_2| r_4 - S v_3 + \tilde{r}_3 \dot{\tilde{r}}_3 + \tilde{r}_4 \dot{\tilde{r}}_4 \quad (31)$$

Select parameters \hat{r}_3 , \hat{r}_4 adaptive adjustive law and robust item v_3 as follows:

$$\begin{cases} \dot{\hat{r}}_3 = \sigma_3 S^2 - \eta_3 \hat{r}_3 \\ \dot{\hat{r}}_4 = \sigma_4 S^2 \bar{\chi}_2^2 - \eta_4 \hat{r}_4 \\ v_3 = \hat{r}_3^2 S + \hat{r}_4^2 S \bar{\chi}_2^2 + \frac{\sigma_3^2}{4} S + \frac{\sigma_4^2}{4} S \bar{\chi}_2^2 \end{cases} \quad (32)$$

In the formula, $\sigma_3 > 0$, $\eta_3 > 0$, $\sigma_4 > 0$ and $\eta_4 > 0$ are design parameters.

Combine (32) and (31), we can have:

$$\begin{aligned} \dot{V}_3 &\leq -k_1 \chi_1^2 - k_2 \chi_2^2 + \chi_2 (k_3 \chi_2^{q/p} + \chi_3) - \frac{\delta_W}{2} \|\tilde{W}\|^2 - \frac{1}{2} \eta_1 \tilde{r}_1^2 - \frac{1}{2} \eta_2 \tilde{r}_2^2 - \frac{1}{2} \eta_{NN} \tilde{r}_{NN}^2 + c_2 - k_4 S^2 \\ &\quad - S \chi_2 - \frac{1}{2} \eta_3 \tilde{r}_3^2 + \frac{1}{2} \eta_3 r_3^2 - \frac{1}{2} \eta_4 \tilde{r}_4^2 + \frac{1}{2} \eta_4 r_4^2 + \frac{r_3}{4\sigma_3} + \frac{r_4}{4\sigma_4} \\ &= -k_1 \chi_1^2 - k_2 \chi_2^2 - k_4 S^2 - \frac{\delta_W}{2} \|\tilde{W}\|^2 - \frac{1}{2} \eta_1 \tilde{r}_1^2 - \frac{1}{2} \eta_2 \tilde{r}_2^2 - \frac{1}{2} \eta_3 \tilde{r}_3^2 - \frac{1}{2} \eta_4 \tilde{r}_4^2 - \frac{1}{2} \eta_{NN} \tilde{r}_{NN}^2 + c_4 \leq -c_3 V_3 + c_4 \end{aligned} \quad (33)$$

In the formula,

$$c_3 = \min \left\{ 2k_1 \quad 2k_2 \quad 2k_4 \quad \frac{\delta_W}{\lambda_{\max}(\Gamma_W^{-1})} \quad \eta_1 \quad \eta_2 \quad \eta_3 \quad \eta_4 \quad \eta_{NN} \right\}, \quad \lambda_{\max}(\cdot) \text{ represents the maximum characteristic}$$

root of matrix “·”. $c_4 = c_2 + \frac{1}{2} \eta_3 r_3^2 + \frac{1}{2} \eta_4 r_4^2 + \frac{r_3}{4\sigma_3} + \frac{r_4}{4\sigma_4}$.

To sum up, we can conclude::

Theorem 1 Considering system (6), under the premise of assumption 1, virtual control law and control law take the forms of (12), (20) and (26), RBF neural network adjustive parameter law takes the form of (17), the adjustive laws of adaptive parameters \hat{r}_1 , \hat{r}_{NN} , \hat{r}_2 , \hat{r}_3 and \hat{r}_4 take the forms of (17), (23) and (32), the error o system state tracing χ_1, χ_2 and χ_3 , estimated error of neural network parameters, and estimated error of adaptive parameters all are bounded and exponentially converge to a bound neighbor of the origin: $\Omega = \{ \chi_1, \chi_2, \chi_3, \tilde{W}, \tilde{r}_1, \tilde{r}_2, \tilde{r}_3, \tilde{r}_4, \tilde{r}_{NN} \mid V_3 \leq 2c_4 / c_3 \}$ (34)

From (34) we can conclude that if we adjust $k_1, k_2, k_4, \delta_W, \eta_1, \eta_2, \eta_3, \eta_4$ and η_{NN} we can adjust convergence rate and the size of the domain of convergence.

System (6) and system (4) belong to equivalent system, control law (26) can also be applied to system (4). In the actual operating process, we can measure signal n_y by the pitch acceleration speed of missile target; we get the signal $\dot{\omega}_z$ of the speed of pitch angle by velocity measuring gyroscope; we can use accelerometer output signal n_y and pitch rudder angle signal δ_z to get

incidence α of the missile, calculation formula is $\alpha = \frac{g}{Va_{34}}n_y - \frac{a_{35}}{a_{34}}\delta_z$. Apply transformation of matrix T to the available signal vector $[\alpha \ \omega_z \ n_y]^T$, thus we can have the necessary state vector $[\xi_1 \ \xi_2 \ \xi_3]^T$ in control law (26).

Adopt the design model elaborated in this paper, the design schematic diagram of the target missile pitch channel overload control system^[4-5] is shown in Fig 1.

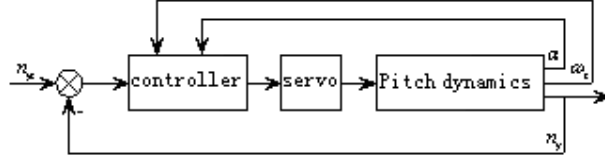


Fig 1. The design schematic diagram of the target missile pitch channel overload control system

Simulation analysis

The pitch channel dynamics model is shown in (2) and (3), and we assume the uncertainties in this model as: $\Delta_\alpha = 0.2\sin(5t)$, $\Delta_{\omega_z} = 0.07\cos(4t)$, $\Delta_{n_y} = 0.95\sin(3t)$.

According to the design requirements in theorem 1, we select parameters: $k_1 = 8$, $k_2 = 10$,

$$k_3 = 2.8, \quad k_4 = 6, \quad k_5 = 1.2, \quad \Gamma_w = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.6 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix},$$

$$\phi(n_{yc}, n_y, \alpha, \omega_z) = \left[\exp\left(\frac{-\|n_{yc} - 0.6\|^2}{3}\right) \exp\left(\frac{-\|n_y - 0.6\|^2}{3}\right) \exp\left(\frac{-\|\alpha - 0.3\|^2}{2}\right) \exp\left(\frac{-\|\omega_z - 0.1\|^2}{0.5}\right) \right]^T,$$

$p = 7$, $q = 5$, $\delta_w = 0.01$, $\sigma_1 = 0.5$, $\sigma_2 = 0.3$, $\sigma_3 = 0.1$, $\sigma_4 = 0.6$, $\sigma_{NN} = 0.5$, $\eta_1 = 1$, $\eta_2 = 1$, $\eta_3 = 1.2$, $\eta_4 = 2$, $\eta_{NN} = 1$. At the same time, in order to weaken the phenomenon in sliding-mode control, we operate continuous treatment to signal function $\text{sgn}(S)$, replace $\text{sgn}(S)$ with continuous function $S/(|S| + \varepsilon)$, $\varepsilon = 0.003$. Employing the controller designed above, we get the simulation results as shown in Fig 2~Fig 5.

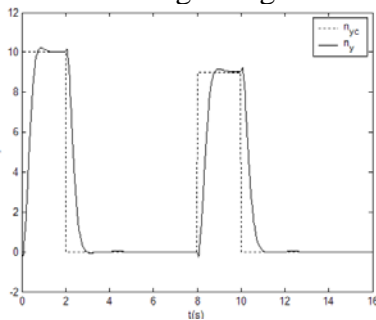


Fig 2 Normal overload n_y simulation curve

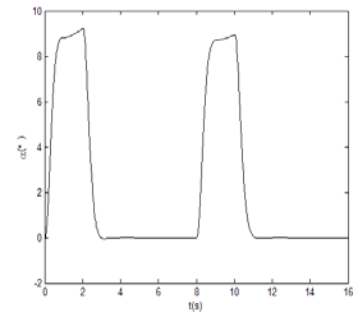


Fig 3 Incidence α simulation curve

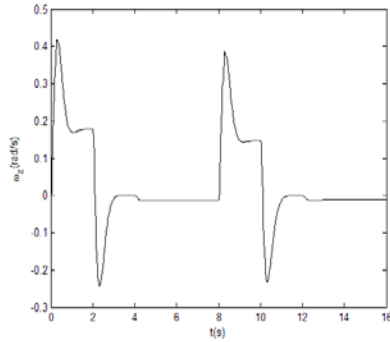


Fig 4 Pitch angle speed ω_z simulation curve

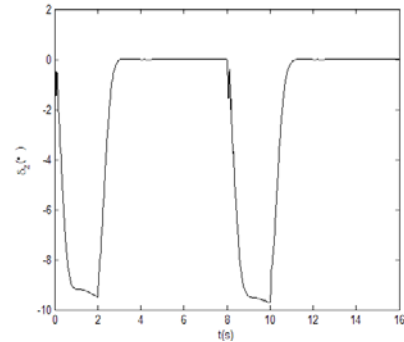


Fig 5 Pitching Rudder δ_z simulation curve

In simulation fig 2, the dotted line is normal overload order signal n_{yc} , the solid line is the actual normal overload signal n_y of the missile target. According to the simulation result, we can know that when there exist uncertainties in the pitching channel model, the control method proposed in this thesis can still control n_y and trace n_{yc} in a good condition and the incidence α and pitch angle speed ω_z signal are in reasonable range, and there is no buffeting phenomenon in pitching rudder δ_z . Therefore, it shows that the design of pitching channel control system based on the method proposed in this paper has good dynamic quality, tracing performance and robustness.

Conclusion

This thesis employs neural network to estimate the virtual control law of the controlled system; it adopts Terminal sliding-mode control to enhance the convergence rate and robustness; it uses robust adaptive function method to estimate the unknown upper boundary of the uncertainties in the system; it adopts back stepping design method to deduct the virtual control law and actual control law of the system. The simulation results show that the designed pitching channel control system has good dynamic quality, tracing performance and robustness.

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