Application of Synchrosqueezing Wavelet Transform In Detection of Harmonic and Interharmonic

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Abstract. To analyze harmonic and interharmonic accurately is very important for power system. In this paper, a novel time-frequency analysis method ,namely SST(synchrosqueezing wavelet transform) ,is applied to detect harmonic and interharmonic. Firstly, the SST algorithm is used to extract a series of intrinsic mode type functions (IMTs), then the Hilbert transform is applied to each IMT to calculate instantaneous amplitude and instantaneous frequency.This method can accurately detect the amplitude and frequency of dynamic harmonic, and the starting and ending time of disturbance in the noisy interference background .The simulation results show that the proposed method is better than the traditional HHT method, and verify the validity of the proposed method.

Introduction

With the promotion of flexible AC power transmission and the smart grid, more and more power electronic equipments are put into operation, which improves the flexibility and reliability of power grid. However, lots of harmonics are caused, which affects power quality of power grid. In order to improve power quality of power grid,the harmonic in the power system must be compensated. So how to accurately detect harmonic components in power grid is a hot research direction.

At present,the methods of detecting harmonic components in power grid include Fourier analysis method[1],Wavelet Transform[2],HHT[3] and so on. Fourier analysis method can only deal with stationary signals, and it can only analyze the integer harmonics, although it can be used to suppress the fence effect and spectrum leakage by interpolation and window, but it can introduce false components. Harmonic and interharmonic can be detected by using wavelet transform method, but the high frequency part has wider frequency band, resulting in its low resolution. According to the features of wavelet transform method, frequency aliasing will be caused, and it's difficult to choose the wavelet base, constructing the algorithm is very complex. Although HHT [4] can handle nonstationary and nonlinear signals, false components can be introduced by Emprical Mode Decomposition and the method can cause frequency aliasing, and the HT transform can not explain the negative frequency after HT transform, and the HHT method is very sensitive to noises.

Synchrosqueezing Wavelet Transform (SST) [5] is a nonlinear time-frequency reassignment algorithm based on continuous wavelet transform. After SST algorithm do continuous wavelet transform to noisy signals ,the wavelet coefficients with small amplitude are discarded, so the harmonic detection is reduced by the effect of noise, when the signal is mixed with strong noise, SST can gain clear time-frequency curves and approximatively invariable decomposition results[6,7]. At the same time, time-frequency graphs after wavelet transform will be compressed by SST in the frequency domain, and there is no cross term between time-frequency curves, it is beneficial to solve the problem of mode mixing in the traditional analysis method. The SST algorithm has been applied to climatic analysis[6] ,mechanical fault diagnosis[7],signal denoising[8], and achieved good results. In this paper, SST is introduced into harmonic and interharmonic analysis in noisy circumstances, firstly, the SST algorithm is applied to decompose the harmonic and interharmonic signal mixed with noise into a series of intrinsic mode type functions (IMTs), then extracts the harmonic,so as to realize automatic extraction to harmonics. The harmonic amplitude and frequency are accurately detected by using the Hilbert transform to extracted harmonics, starting and ending time problems of frequency and amplitude caused by harmonic disturbances are solved. The simulation experiments show that the method in this paper can accurately extract each harmonic component in the noisy interference background,and it basically eliminates mode mixing, the precision of harmonic detection gained by the proposed method is better than traditional HHT method.

The basic theory of SST

The main theory regarding to SST is presented by Daubechies et al in paper[5]: Pick a wavelet function $\psi \in C^1$,its Fourier transform function is supported in $[1-\Delta,1+\Delta]$,with $\Delta < \frac{d}{1+d}$. Continuous wavelet transform of ψ is $W_f(a,b)$, with the condition of threshold $\tilde{\varepsilon}$ and accuracy δ, the function $S^{\delta}_{f,\tilde{\varepsilon}}(b,\omega)$ obtained by synchrosqueezing $W_f(a,b)$.

$$S^{\delta}_{f,\tilde{\varepsilon}}(b,\omega)=\int_{A_{\tilde{\varepsilon},f}(b)} W_f(a,b)\frac{1}{\delta}h(\frac{\omega-\omega_f(a,b)}{\delta})a^{-\frac{3}{2}}da. \tag{1}$$

where $A_{\tilde{\varepsilon},f}(b) := \left\{a\in R_+; |W_f(a,b)|>\tilde{\varepsilon}\right\}$, if ε is small enough, the following conditions are true.

1) Define $Z_k = \left\{(a,b): \left|a\phi_k'(b)-1\right| < \Delta\right\}$,when $(a,b)\in Z_k$ and $\left|W_f(a,b)\right| > \tilde{\varepsilon}$ are both valid, $\omega_f(a,b) = \frac{-i}{W_f(a,b)} \times \frac{\partial(W_f(a,b))}{\partial b}$ is instantaneous frequency of the signal, then:

$$\left|\omega_f(a,b)-\phi_k'(b)\right| \le \tilde{\varepsilon}. \tag{2}$$

2)There is a constant c ,for $b\in R$, we can get:

$$\left|\lim_{\delta\to 0}\left(\frac{1}{R_\psi}\int_{\left\{\eta:\left|\eta-\phi_k'(b)\right|\le\tilde{\varepsilon}\right\}} S^{\delta}_{f,\tilde{\varepsilon}}(b,\eta)d\eta\right) - A_k(b)e^{2\pi i\phi_k(b)}\right| \le c\tilde{\varepsilon}. \tag{3}$$

where $R_\psi = \frac{1}{2}\int_0^{+\infty}\overline{\hat{\psi}}(\xi)\frac{d\xi}{\xi}$.

The theorem indicates that SST can extract the component $f_k(t)$ from the signal with extremely high accuracy, we can know it from formula(1) ,if $f \in \mathrm{A}_{\varepsilon,d}$,squeezed $S^{\delta}_{f,\tilde{\varepsilon}}$ is mainly around the narrowband which centres on instantaneous frequency $\omega = \phi_k{}'(b)$ of the k-th component, the formula(2) indicates that instantaneous frequency $\omega = \phi_k{}'(b)$ gained by the wavelet coefficient calculation can approximate instantaneous frequency $\phi'(b)$ of signals with extremely high accuracy, then by conducting integration to the value $S^{\delta}_{f,\tilde{\varepsilon}}$ of synchronously squeezed transformation which is around $\phi_k{}'(t)$, reestablish the k-th component $f_k(t)$, the formula(3) indicates that Synchrosqueezing Wavelet Transform value $\phi_k{}'(t)$'s integral values which are around the k-th instantaneous frequency curves $\omega = \phi_k{}'(b)$ can reestablish the k-th component $f_k(t)$ of f.

The harmonic and interharmonic detecting based on SST and Hilbert transformation

There are four steps when conducting detection to time-varying harmonics. The concrete steps are as follows:

1) Conduct continuous wavelet transform to power signal f which includes harmonics: continuous wavelet transform is:

$$W_f(a,b)=\int_{-\infty}^{+\infty} f(t)a^{-\frac{1}{2}}\overline{\psi\left(\frac{t-b}{a}\right)}dt\ . \tag{4}$$

2) Get discrete synchronism extrusion $T_f(\omega_l,b)$ of the time frequency plane:Assume sample interval is Δt ,with $n_v=32$, $n_a=Ln_v$,according to Shannon's sampling theorem, frequency domain of signals can be determined in $[\frac{1}{n\Delta t},\frac{1}{2\Delta t}]$ by sample interval Δt , ,and $\{\omega_l\}_{l=0}^{\infty},\omega_{l+1}>\omega_l$ is called frequency division.Divide frequency domain of power signals into different frequency domain $W_l=[\frac{\omega_l+\omega_{l-1}}{2},\frac{\omega_l+\omega_{l+1}}{2}]$, so the signal's Synchrosqueezing Wavelet Transform at center frequency ω_l is:

$$T_f(\omega_l,b)=\sum_{a_k:\omega_f(a,b)\in W_l} W_f(a,b)a_k^{-\frac{3}{2}}(\Delta a)_k\ . \tag{5}$$

3) The signal extraction of each harmonic and fundamental wave component: f_k can be reestablished according to discrete formula (6):

$$f_k(t_m)=\frac{2}{R_\psi}\mathrm{Re}(\sum_{l\in L_k(t_m)}\tilde{T}_{\tilde{f}}(\omega_l,t_m))\ . \tag{6}$$

where $L_k(t_m)$ stands for the subscript collection around the narrowband of f_k curve.

4) The calculation of harmonic parameters: Conduct Hilbert transform to component f_k :

$$y(t)=\frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{f_k(t)}{t-\tau}d\tau\ . \tag{7}$$

We can gain its Hilbert inverse transform by interchanging y and f_k in formula (7), where:

$$A(t)=\sqrt{f_k^2(t)+y^2(t)}\ ,\theta(t)=\arctan[y(t)/f_k(t)]\ ,\ \omega_d=\frac{1}{2\pi}\frac{d\theta(t)}{dt}\ . \tag{8}$$

Thus, the instantaneous amplitude of the signal is $A(t)$, the instantaneous frequency is ω_d

The analysis and detecting to harmonic and interharmonic signals

Interharmonic signals

The actual electric arc furnace signal is composed of fundamental wave and interharmonic , the frequency is 50HZ, 25HZ and 125HZ, respectively, and it contains 5% white noise.The signals' sampling frequency is 4096.The SST mode extraction is shown in figure 1, EMD decomposition is shown in figure 2, the components of EMD include 10 items,only the first 6 items are listed.

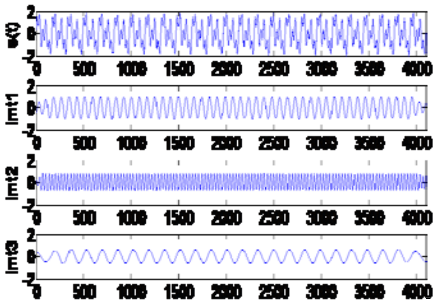

Figure.1 The SST results of arc furnace signal

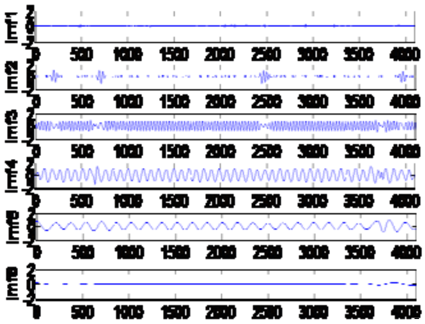

Figure.2 The EMD result of arc furnace signal

As is shown in figure 2, the mode mixing phenomenon occured after EMD decomposition, it is easy for the second IMF to mistakenly judge that there are transient disturbances in the electric arc furnace signals. However, we can know in figure 1, the components 1,2,3 extracted by SST respectively correspond to the components whose frequencies are 50Hz,125Hz and 25Hz,the components extracted by SST can clearly correspond to interharmonic and fundamental wave components. Because of the accurate extraction of SST, the instantaneous frequency and the amplitude error is small by using Hilbert transform,, the results are shown in figure 3 and 4.

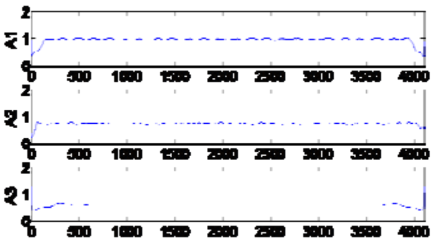

Figure.3 The instaneous amplitude of the SST

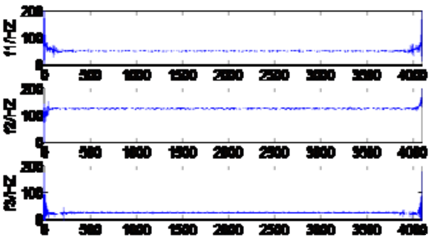

Figure.4 The instaneous frequency of SST

As is shown in figure 3 and 4, the instaneous frequency and amplitude of components gained by SST method is close to actual value,. The results of detection are shown in table 1,in the noisy environment, the frequency relative error of fundamental wave is 0.6% ,the frequencies of interharmonics have no errors.

Table1. Results of detection

each component	component 1	component 2	component 3
frequency	49.997	125	25
amplitude	0.9965	0.7461	0.6452

The harmonic signal

Assume the harmonic signal is:

$$s(t)=\begin{cases}\sin(100\pi t)+0.6\sin(300\pi t), 0<t<1s \\ 0.4\sin(500\pi t), 0.4s<t\le 0.6s \\ 0.3\sin(700\pi t), 0.6s<t\le 0.8s\end{cases}$$

The sampling time and sampling frequency are the same as the previous example, signal contains 5% white noise.The extraction figure of SST is shown in figure 5, EMD decomposition figure is shown in figure 6.

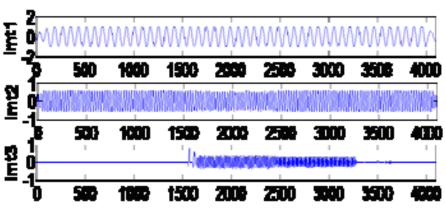

Figure.5 SST decomposition result of noisy signal

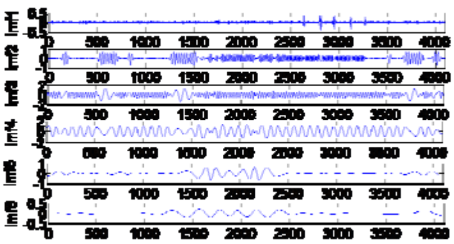

Figure.6 EMD decomposition result of noisy signal

As is shown in figure 6, the mode aliasing occured after EMD decomposition, its components lose its original physical significance, it's difficult to identify harmonics.The figure 5 show the first one is the fundamental wave, the second is a third harmonic, and the third one is a time-varying harmonic. SST can accurately separate fundamental wave and harmonics, it's easy to identify fundamental wave or harmonics , and it can accurately find transient harmonics in the noisy environment and accurately detect the amplitude and frequency of signals. The results of detecting are shown in table 2.

Table2. Results of detection

each component	imt1/actual value	imt2/ actual value	Imt3/ actual value
frequency	49.992/50	150.216/150	250.245/250(0.4-0.6s); 349.865.265/350(0.6-0.8s)
amplitude	0.983/1	0.587/0.6	0.354/0.4(0.4-0.6s); 0.288/0.3(0.6-0.8s)

Conclusions

1) The SST algorithm proposed in this paper can be used to improve the frequency aliasing phenomenon, and to suppress the noise, and can accurately extract the components of harmonic and interharmonic, which is better than the conventional time-frequency detection tools in the anti-mode-mixing capacity and anti-noise performance of HHT.

2) The SST method squeezes wavelet scalogram after wavelet transform, there are less components through SST to extract harmonics from signal compared to HHT, it is easy to identify harmonics and interharmonics corresponding to extractive components, which can achieve the goal of detection.

Acknowledgements

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