Synchronization Control of General Delayed Complex Networks via LMI Approach

Haiyi Sun
College of Science
Shenyang JianZhu University
Shenyang, China
shy_xx@163.com

Zhongtang Chen
College of Science
Shenyang JianZhu University
Shenyang, China
Chen630609@163.com

Ning Li
Institute of Systems Science
Northeastern University
Shenyang, China
Lining80@163.com(corresponding author)

Hongmei Yan
College of Science
Shenyang JianZhu University
Shenyang, China
yan1204shanyu@163.com

Abstract—This paper introduces a general model of complex dynamical networks with coupling delays and delays in the dynamical nodes. The synchronization control of the complex dynamical networks with coupling delays and delays in the dynamical nodes has been studied. Via the theory of Lyapunov-Krasovskii stability and linear matrix inequalities (LMIs) technique, we design the linear feedback synchronization controller for the coupling coefficients known by constructing appropriate Lyapunov functions, which can be solved easily by the LMI toolbox in MATLAB. The controller is very useful for understanding the mechanism of synchronization in complex dynamical networks with coupling delays and delays in the dynamical nodes. It should be pointed out that the node dynamic need not satisfy the very strong and the matrix is not assumed to be symmetric or irreducible. Moreover, the resulting for network synchronization are expressed in simple forms that can be readily applied in practical situations. The numerical example of the synchronization control problem has been illustrated how to this theorem can be applied to judge and achieve synchronization in complex networks with coupling delays and delays in the dynamical nodes.

Keywords- complex networks; coupling delays; delayed nodes; synchronization control; linear matrix inequalities (LMIs)

I. INTRODUCTION

In the last few years, complex networks have been extensively investigated across many fields of science and engineering[1-8]. A complex network is a set of coupling nodes interconnected by edges, and its every node is a dynamical system. There have been a rich body of literature on analyzing complex networks, and one of the most significant dynamical behaviors of complex networks that has been widely investigated is the synchronization motion of its dynamical elements[1-4]. The synchronization in complex networks not only can well explain many natural phenomena, but also has many potential applications in image processing, secure communication, etc., which has been a favorite topic for research in complex networks [8].

In practice, the information transmission within complex networks is in general not instantaneous since the signals traveling speed is limited, and this is very common in biological and physical networks [8-10].This fact gives rise to the time delays that may cause undesirable dynamic network behaviors such as oscillation and instability. Therefore, time delays should be modeled in order to simulating more realistic networks.

In this paper, we first introduce a general model of complex dynamical network with coupling delays and delays in the dynamical nodes. Then we further study the synchronization control of this model. Based on the theory of asymptotic stability of linear time-delay systems and Lyapunov method combined with linear matrix inequality technique, the complex networks with linear feedback controllers are considered for the case where the coupling coefficients are known. It should be pointed out that the node dynamic need not satisfy the very strong and the matrix is not assumed to be symmetric or irreducible.

The rest of the paper is organized as follows. In Section 2, the model of a general complex dynamical network with coupling delays and delays in the dynamical nodes is presented and some preliminaries are also given. In Section 3, the linear feedback synchronization controllers for the coupling coefficients known are designed. The numerical example for verifying the theoretical result is given in Section 4. Finally, conclusions are presented in Section 5.

II. MODEL DESCRIPTION AND PRELIMINARIES

The control complex networks with coupling delays and delays in the dynamical nodes can be described as follows:

$$\begin{aligned}\dot{x}_i(t) &= f(x_i(t), x_i(t-\tau_1)) + \sum_{j=1}^{N} a_{ij}\Gamma_1 x_j(t) \\ &\quad + \sum_{j=1}^{N} b_{ij}\Gamma_2 x_j(t-\tau_2) + u_i(t), \\ &\quad (i = 1, 2, \cdots, N)\end{aligned} \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \cdots, x_{in}(t))^{\mathrm{T}} \in R^n$ is the state and input variable of node i at time t, $u_i(t) \in R^n$ is the input variables of node i. $f: R^n \times R^n \to R^n$ is a continuous and differentiable function, τ_1 and $\tau_2 > 0$ are the time delay of coupling delays and delays in the dynamical nodes, respectively, which are arbitrary but bounded, i.e., $\tau_1, \tau_2 \in (0, h]$, where h is a positive constant. $A = (a_{ij}) \in R^{N\times N}$ and $B = (b_{ij}) \in R^{N\times N}$ are the coupling matrices with zero-sum rows, which represent the coupling strength and the underlying topology for non-delayed configuration and delayed one τ_2 at time t, respectively, $a_{ij} \ge 0$, $b_{ij} \ge 0$ for $i \ne j$, a_{ij}, b_{ij} are defined as follows: if there is a connection from node j to node $i (i \ne j)$ $a_{ij} > 0, b_{ij} > 0$, otherwise $a_{ij} = 0, b_{ij} = 0\ (i \ne j)$, and $\Gamma_1, \Gamma_2 \in R^{n\times n}$ are positive diagonal matrices which describe the individual couplings between node i and j for non-delayed configuration and delayed one τ_2 at time t respectively.

When the delayed dynamical network (1) achieves synchronization, namely, the states $x_1(t) \to x_2(t) \to \cdots \to x_N(t) \to s(t)$, as $t \to \infty$, where $s(t) \in R^n$ is a solution of an isolate node, i.e.

$$\dot{s}(t) = f(s(t), s(t-\tau_1)). \tag{2}$$

$s(t)$ can be an equilibrium point, a nontrivial periodic orbit, or even a chaotic orbit. Let $C([-h,0], R^n)$ be the Banach space of continuous functions mapping the interval $[-h, 0]$ into R^n with the norm $\|\phi\| = \sup_{-h\le\theta\le 0}\|\phi(\theta)\|$, where $\|\cdot\|$ is the Euclidean norm. The rigorous mathematical definition of synchronization for delayed dynamical network (1) is introduced as follows.

Definition 1. Let $x_i(t;t_0;\phi), i = 1, 2, \cdots, N$ be a solution of delayed dynamical network (1), where $\phi = (\phi_1^{\mathrm{T}}, \phi_2^{\mathrm{T}}, \cdots \phi_N^{\mathrm{T}})^{\mathrm{T}}$, $\phi_i = \phi_i(\theta) \in C([-h,0], R^n)$ are initial conditions. If there is a nonempty subset $\Lambda \subseteq R^n$, such that ϕ_i take values in Λ and $x_i(t;t_0;\phi) \in R^n$ for all $t \ge t_0$ and

$$\lim_{t\to\infty}\| x_i(t;t_0;\phi) - s(t;t_0;s_0)\| = 0, \quad i = 1, 2, \cdots, N \tag{3}$$

where $s(t;t_0;s_0)$ is a solution of the system (2) with $s_0 \in R^n$, then the delayed dynamical network (1) is said to realize synchronization, and $\Lambda \times \Lambda \times \cdots \times \Lambda$ is called the region of synchrony of the delayed dynamical network (1).

Define the error vector by

$$e_i(t) = x_i(t) - s(t), \quad i = 1, 2, \cdots, N \tag{4}$$

Notice that in (1) $\sum_{j=1}^{N} a_{ij} = 0$ and $\sum_{j=1}^{N} b_{ij} = 0$, then the error system can be described by

$$\begin{aligned}\dot{e}_i(t) &= f(x_i(t), x_i(t-\tau_1)) - f(s(t), s(t-\tau_1)) \\ &\quad + \sum_{j=1}^{N} a_{ij}\Gamma_1 e_j(t) + \sum_{j=1}^{N} b_{ij}\Gamma_2 e_j(t-\tau_2) + u_i(t).\end{aligned} \tag{5}$$

Then the synchronization problem of the dynamical network (1) is equivalent to the problem of stabilization of the error dynamical system (5).

Hypothesis 1. (H1) Let $S = \mathrm{diag}(\beta_1, \beta_2, \cdots, \beta_N)$ be a positive-definite diagonal matrix. The nonlinear vector-valued continuous function, $f: R^n \times R^n \times R^+ \to R^n$ satisfied the semi-lipschiz condition:

$$\begin{aligned}&(x_i(t) - s(t))^{\mathrm{T}} S(f(x_i(t), x_i(t-\tau_1)) - f(s(t), s(t-\tau_1))) \\ &\le \gamma_1(t)\| x_i(t) - s(t)\|^2 + \gamma_2(t)\| x_i(t-\tau_1) - s(t-\tau_1)\|^2.\end{aligned} \tag{6}$$

where $i = 1, 2, \cdots, N$, $\gamma_1(t)$ and $\gamma_2(t)$ are unknown time-varying nonzero parameters with unknown bounds, that is $\gamma_1(t) \in [\underline{\gamma_1}, \overline{\gamma_1}]$ and $\gamma_2(t) \in [\underline{\gamma_2}, \overline{\gamma_2}]$. $\underline{\gamma_1}, \overline{\gamma_1}$, $\underline{\gamma_2}, \overline{\gamma_2}$, are unknown constants.

We define $\gamma_1 = \max(\underline{\gamma_1}, \overline{\gamma_1})$ and $\gamma_2 = \max(\underline{\gamma_2}, \overline{\gamma_2})$.

Note that Hypothesis 1 is less conservative than general uniformly Lipschitz condition. For example, all linear and piecewise linear functions satisfy this condition. In addition, if $\frac{\partial f_i}{\partial x_j}(i, j = 1, 2, \cdots, n)$ are bounded and Γ_0 is positive definite, the above condition is satisfied. So, it includes many well-known systems, such as the Lorenz system, Chen system, Lü system, recurrent neural networks, Chua's circuit, and so on.

Lemma 1. For any vectors $x, y \in R^m$ positive definite matrix $Q \in R^{m\times m}$, the following matrix inequality holds:

$$2x^{\mathrm{T}}y \le x^{\mathrm{T}}Qx + y^{\mathrm{T}}Q^{-1}y$$

If not specified otherwise, inequality $Q > 0$ ($Q < 0, Q \ge 0, Q \le 0$) means Q is a positive (or negative, or semi-positive, or semi-negative) definite matrix, where Q is a square matrix.

III. SYNCHRONIZATION CONTROL OF THE GENERAL DELAYED COMPLEX DYNAMICAL NETWORKS

In this section, we study the synchronization of the general complex dynamical networks with couplings delays and delays in the dynamical nodes by designing linear controllers for each node.

Let $F = (f(x_1(t), x_1(t-\tau_1)) - f(s(t), s(t-\tau_1)), \cdots, f(x_N(t), x_N(t-\tau_1)) - f(s(t), s(t-\tau_1)))$, $\bar{A} = A \otimes \Gamma_1$, $\bar{B} = B \otimes \Gamma_2$, $e(t) = (e_1^{\mathrm{T}}(t), e_2^{\mathrm{T}}(t), \cdots, e_N^{\mathrm{T}}(t))^{\mathrm{T}}$, $u(t) = (u_1^{\mathrm{T}}(t), u_2^{\mathrm{T}}(t), \cdots, u_N^{\mathrm{T}}(t))^{\mathrm{T}}$.

With the Kronecker product \` $\otimes$' for matrices, system (5) can be recast into

$$\dot{e}(t) = I_{nN}F + \bar{A}e_j(t) + \bar{B}e_j(t-\tau_2) + u(t), \tag{7}$$

Based on a Lyapunov-Krasovskii function, we detrude the synchronization control criteria of the delayed dynamical network (1).

Theorem 1. Consider the complex networks (1) with no input variables $\tau_1, \tau_2 \in (0,h]$ and no input variables. For given scalars $\rho_l\,(l=2,3,4,5)$, if there exist matrices $\hat{P}>0$, $\hat{Q}_1>0$, $\hat{Q}_2>0$, $\hat{R}_1>0$, $\hat{R}_2>0$, $\hat{S}>0$, , block-diagonal nonsingular matrices *X*, *Y* and any matrices $\hat{N}_i$ $(i=1,\cdots,5)$ of appropriate dimensions such that the following LMI holds:

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} & \Xi_{17} & \Xi_{18} & \Xi_{19} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & \Xi_{25} & \Xi_{26} & \Xi_{27} & \Xi_{28} & \Xi_{29} \\ * & * & \Xi_{33} & \Xi_{34} & \Xi_{35} & \Xi_{36} & \Xi_{37} & \Xi_{38} & \Xi_{39} \\ * & * & * & \Xi_{44} & \Xi_{45} & \Xi_{46} & \Xi_{47} & \Xi_{48} & \Xi_{49} \\ * & * & * & * & \Xi_{55} & \Xi_{56} & \Xi_{57} & \Xi_{58} & \Xi_{59} \\ * & * & * & * & * & \Xi_{66} & \Xi_{67} & \Xi_{68} & \Xi_{69} \\ * & * & * & * & * & * & \Xi_{77} & \Xi_{78} & \Xi_{79} \\ * & * & * & * & * & * & * & \Xi_{88} & \Xi_{89} \\ * & * & * & * & * & * & * & * & \Xi_{99} \end{bmatrix} < 0. \tag{8}$$

$\Xi_{11} = \hat{R}_1 + \hat{R}_2 + \hat{N}_1 + \hat{N}_1^{\mathrm{T}} + \hat{T}_1 + \hat{T}_1^{\mathrm{T}} + X\bar{A} + \bar{A}^{\mathrm{T}}X^{\mathrm{T}} - Y - Y^{\mathrm{T}}$,

$\Xi_{12} = -\hat{N}_1 + \hat{N}_2^{\mathrm{T}} + \hat{T}_2^{\mathrm{T}} + \rho_2 X\bar{A}^{\mathrm{T}} - \rho_2 Y^{\mathrm{T}}$

$\Xi_{13} = \hat{N}_3^{\mathrm{T}} - X + \rho_3 X\bar{A}^{\mathrm{T}} + \hat{P} + \hat{T}_3^{\mathrm{T}}$,

$\Xi_{15} = \hat{N}_5^{\mathrm{T}} + \hat{T}_5^{\mathrm{T}} - \hat{T}_1 + \rho_5 X\bar{A}^{\mathrm{T}} - \rho_5 Y^{\mathrm{T}} + \bar{B}X$, $\Xi_{16} = h\hat{N}_1$,

$\Xi_{17} = h\hat{T}_1$, $\Xi_{18} = \Xi_{29} = I$, $\Xi_{22} = -\hat{R}_1 - \hat{N}_2 - \hat{N}_2^{\mathrm{T}}$,

$\Xi_{23} = -\hat{N}_3^{\mathrm{T}} - \rho_2 X$, $\quad \Xi_{24} = -\hat{N}_4^{\mathrm{T}} + \rho_2 X$,

$\Xi_{25} = -\hat{N}_5^{\mathrm{T}} - \hat{T}_2 + \rho_2 \bar{B}X^{\mathrm{T}}$, $\Xi_{26} = h\hat{N}_2$, $\quad \Xi_{27} = h\hat{T}_2$,

$\Xi_{33} = h\hat{Q}_1 + h\hat{Q}_2 - \rho_3 X - \rho_3 X^{\mathrm{T}}$, $\Xi_{34} = \rho_3 X^{\mathrm{T}} - \rho_4 X$,,

$\Xi_{35} = -\hat{T}_3 - \rho_5 X + \rho_3 \bar{B}X^{\mathrm{T}}$, $\Xi_{36} = h\hat{N}_3$, $\quad \Xi_{37} = h\hat{T}_3$,

$\Xi_{44} = \rho_4 X + \rho_4 X^{\mathrm{T}}$, $\Xi_{45} = -\hat{T}_4 + \rho_5 X + \rho_4 \bar{B}X^{\mathrm{T}}$,

$\Xi_{46} = h\hat{N}_4$, $\quad \Xi_{47} = h\hat{T}_4$,

$\Xi_{55} = -\hat{R}_2 - \hat{T}_5 - \hat{T}_5^{\mathrm{T}} + \rho_5 \bar{B}X^{\mathrm{T}} + \rho_5 X\bar{B}$, $\Xi_{56} = h\hat{N}_5$,

$\Xi_{57} = h\hat{T}_5$, $\quad \Xi_{66} = -h\hat{Q}_1$, $\quad \Xi_{77} = -h\hat{Q}_2$, $\quad \bar{S} = S \otimes I_n$,

$\Xi_{19} = \Xi_{28} = \Xi_{38} = \Xi_{39} = \Xi_{48} = \Xi_{49} = \Xi_{58} = \Xi_{59} = \Xi_{67}$

$= \Xi_{68} = \Xi_{69} = \Xi_{78} = \Xi_{79} = \Xi_{89} = \Xi_{98} = 0.$

$\Xi_{88} = \gamma_1^{-1} I$, $\quad \Xi_{99} = \gamma_2^{-1} I$.

where $\bar{A}$ and $\bar{B}$ are given by (7). Then under the controller

$$u_i = -k_i e_i(t) \tag{9}$$

Let $K = \mathrm{diag}\{k_1, k_2, \cdots, k_N\}$ with $K \otimes I_n = YX^{-1}$, then the synchronization is achieved. $*$ denotes the elements below the main diagonal of a symmetric block matrix.

Proof: Selecting a Lyapunov-Krasovskii function of the form

$$\begin{aligned} V(t) &= e^{\mathrm{T}}(t)Pe(t) + \sum_{i=1}^{2}\int_{t-\tau_i}^{t} e^{\mathrm{T}}(s)R_i e(s)\,\mathrm{d}s + \\ &\sum_{i=1}^{2}\int_{t-\tau_i}^{t}\int_{s}^{t} \dot{e}^{\mathrm{T}}(s)Q_i\dot{e}(s)\,\mathrm{d}v\,\mathrm{d}s. \end{aligned} \tag{10}$$

Taking the time derivative of $V(t)$ along the trajectory of (7) yield that

$$\begin{aligned} \dot{V}(t) &= 2e^{\mathrm{T}}(t)P\dot{e}(t) - \sum_{i=1}^{2}\int_{t-\tau_i}^{t} \dot{e}^{\mathrm{T}}(s)Q_i\dot{e}(s)\mathrm{d}s \\ &+ \sum_{i=1}^{2}\tau_i\dot{e}^{\mathrm{T}}(t)Q_i\dot{e}(t) - \sum_{i=1}^{2}e^{\mathrm{T}}(t-\tau_i)R_i e(t-\tau_i) \\ &+ \sum_{i=1}^{2}e^{\mathrm{T}}(t)R_i e(t) + 2\xi^{\mathrm{T}}N[e(t) - e(t-\tau_1) \\ &- \int_{t-\tau_1}^{t} \dot{e}^{\mathrm{T}}(s)\mathrm{d}s] + 2\xi^{\mathrm{T}}T[e(t) - e(t-\tau_2) \\ &- \int_{t-\tau_2}^{t} \dot{e}^{\mathrm{T}}(s)\mathrm{d}s] + 2\xi^{\mathrm{T}}M[F + \bar{A}e(t) \\ &+ \bar{B}e(t-\tau_2) - \dot{e}(t)] \end{aligned} \tag{11}$$

From Lemma 1, it follows that

$$-2\xi^{\mathrm{T}}N\int_{t-\tau_1}^{t} \dot{e}^{\mathrm{T}}(s)ds \le h\xi^{\mathrm{T}}NQ_1^{-1}N^{\mathrm{T}}\xi + \int_{t-\tau_1}^{t} \dot{e}^{\mathrm{T}}(s)Q_1\dot{e}(s)\mathrm{d}s, \tag{12}$$

$$-2\xi^{\mathrm{T}}T\int_{t-\tau_2}^{t} \dot{e}^{\mathrm{T}}(s)ds \le h\xi^{\mathrm{T}}TQ_2^{-1}T^{\mathrm{T}}\xi + \int_{t-\tau_2}^{t} \dot{e}^{\mathrm{T}}(s)Q_2\dot{e}(s)\mathrm{d}s, \tag{13}$$

Then, combining (10})-(11) and Hypothesis 1, we have

$$\begin{aligned} \dot{V}(t) \le\ & 2e^{\mathrm{T}}(t)P\dot{e}(t) + \sum_{i=1}^{2}[h\dot{e}^{\mathrm{T}}(t)Q_i\dot{e}(t) + e^{\mathrm{T}}(t)R_i e(t) \\ & -e^{\mathrm{T}}(t-\tau_i)R_i e(t-\tau_i)] + h\xi^{\mathrm{T}}NQ_1^{-1}N^{\mathrm{T}}\xi \\ & +h\xi^{\mathrm{T}}TQ_2^{-1}T^{\mathrm{T}}\xi + 2\xi^{\mathrm{T}}N[e(t) - e(t-\tau_1)] \\ & +2\xi^{\mathrm{T}}M[F + \bar{A}e(t) + \bar{B}e(t-\tau_2) - \dot{e}(t)] \\ & +2\xi^{\mathrm{T}}T[e(t) - e(t-\tau_2)] + \gamma_1 e^{\mathrm{T}}(t)e(t) \\ & +\gamma_2 e^{\mathrm{T}}(t-\tau_1)e(t-\tau_1) - e^{\mathrm{T}}(t)\bar{S}F \\ =\ & \xi^{\mathrm{T}}\Omega\xi \end{aligned} \tag{14}$$

where

$$\Omega = \begin{bmatrix} \Pi_{11}+\gamma_1 I & \Pi_{12} & \Pi_{13} & \Pi_{14} & \Pi_{15} \\ * & \Pi_{22}+\gamma_2 I & \Pi_{23} & \Pi_{24} & \Pi_{25} \\ * & * & \Pi_{33} & \Pi_{34} & \Pi_{35} \\ * & * & * & \Pi_{44} & \Pi_{45} \\ * & * & * & * & \Pi_{55} \end{bmatrix} + hNQ_1^{-1}N^{\mathrm{T}} + hTQ_2^{-1}T^{\mathrm{T}}, \tag{15}$$

where

$\Pi_{11} = R_1 + R_2 + N_1 + N_1^{\mathrm{T}} + T_1 + T_1^{\mathrm{T}} + M_1\bar{A} + \bar{A}^{\mathrm{T}}M_1^{\mathrm{T}} - K - K^{\mathrm{T}}$, $\quad \Pi_{12} = -N_1 + N_2^{\mathrm{T}} + T_2^{\mathrm{T}} + \bar{A}^{\mathrm{T}}M_2^{\mathrm{T}}$,

$\Pi_{13} = N_3^{\mathrm{T}} - M_1 + \bar{A}^{\mathrm{T}}M_3^{\mathrm{T}} + P + T_3^{\mathrm{T}}$,

$\Pi_{15} = N_5^{\mathrm{T}} + T_5^{\mathrm{T}} - T_1 + \bar{A}^{\mathrm{T}}M_5^{\mathrm{T}} + M_1\bar{B}$, $\Pi_{16} = hN_1$,

$\Pi_{17} = hT_1$, $\Pi_{18} = \Pi_{29} = I$, $\Pi_{22} = -R_1 - N_2 - N_2^{\mathrm{T}}$,

$\Pi_{23} = -N_3^{\mathrm{T}} - M_2$, $\quad \Pi_{24} = -N_4^{\mathrm{T}} + M_2$,

$\Pi_{25} = -N_5^{\mathrm{T}} - T_2 + M_2\bar{B}$, $\Pi_{26} = hN_2$, $\quad \Pi_{27} = hT_2$,

$\Pi_{33} = hQ_1 + hQ_2 - M_3 - M_3^{\mathrm{T}}$, $\Pi_{34} = M_3 - M_4^{\mathrm{T}}$,,
$\Pi_{35} = -T_3 - M_5^{\mathrm{T}} + M_3\bar{B}$ $\Pi_{36} = hN_3$, $\Pi_{37} = hT_3$,
$\Pi_{44} = M_4 + M_4^{\mathrm{T}}$, $\Pi_{45} = -T_4 + M_5^{\mathrm{T}} + M_4\bar{B}$,
$\Pi_{46} = hN_4$, $\Pi_{47} = hT_4$,
$\Pi_{55} = -R_2 - T_5 - T_5^{\mathrm{T}} + M_5\bar{B} + \bar{B}^{\mathrm{T}}M_5^{\mathrm{T}}$,

By Schur complement and $\Omega < 0$, we can get the follow matrix inequality.

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} & \Pi_{15} & \Pi_{16} & \Pi_{17} & \Pi_{18} & \Pi_{19} \\ * & \Pi_{22} & \Pi_{23} & \Pi_{24} & \Pi_{25} & \Pi_{26} & \Pi_{27} & \Pi_{28} & \Pi_{29} \\ * & * & \Pi_{33} & \Pi_{34} & \Pi_{35} & \Pi_{36} & \Pi_{37} & \Pi_{38} & \Pi_{39} \\ * & * & * & \Pi_{44} & \Pi_{45} & \Pi_{46} & \Pi_{47} & \Pi_{48} & \Pi_{49} \\ * & * & * & * & \Pi_{55} & \Pi_{56} & \Pi_{57} & \Pi_{58} & \Pi_{59} \\ * & * & * & * & * & \Pi_{66} & \Pi_{67} & \Pi_{68} & \Pi_{69} \\ * & * & * & * & * & * & \Pi_{77} & \Pi_{78} & \Pi_{79} \\ * & * & * & * & * & * & * & \Pi_{88} & \Pi_{89} \\ * & * & * & * & * & * & * & * & \Pi_{99} \end{bmatrix} < 0. \quad (16)$$

where $\Pi_{56} = hN_5$, $\Pi_{57} = hT_5$, $\Pi_{66} = -hQ_1$,
$\Pi_{77} = -hQ_2$, $\bar{S} = S \otimes I_n$, $\Pi_{88} = \gamma_1^{-1}I$, $\Pi_{99} = \gamma_2^{-1}I$,
$\Pi_{19} = \Pi_{28} = \Pi_{38} = \Pi_{39} = \Pi_{48} = \Pi_{49} = \Pi_{58} = \Pi_{59} = \Pi_{67} =$
$\Pi_{68} = \Pi_{69} = \Pi_{78} = \Pi_{79} = \Pi_{89} = \Pi_{98} = 0.$
others $\Pi_{ij}(i, j = 1,\cdots,5)$ are given by (15).

Define $M_2 = \rho_2 M_1, M_3 = \rho_3 M_1, M_4 = \rho_4 M_1$. Obviously, it implies M_1 is nonsingular. Pre- and post-multiplying both sides of (16) with $diag(X, X, X, X, X, X, X, I, I)$ and its transpose, where $X = M_1^{-1}$, and introducing new variables

$$\hat{P} = XPX^{\mathrm{T}}, \hat{Q}_i = XQ_iX^{\mathrm{T}}, \hat{R}_i = XR_iX^{\mathrm{T}}(i = 1,2),\ Y = \bar{K}X^{\mathrm{T}}$$
$$\hat{S} = X\bar{S}X^{\mathrm{T}}, \hat{N}_j = XN_jX^{\mathrm{T}}, \hat{T}_j = XT_jX^{\mathrm{T}}(j = 1,2,3,4,5),$$

we can obtain (8). Therefore, we can complete the proof.

Remark 1. Base on LMI method, we construct a more general Lyapunov function to analyze the synchronization problem of the complex dynamical networks with couplings delays and delays in the dynamical nodes. The new delay-dependent conditions presented in Theorem 1 are formulated in the form of LMI, which can be solved by the LMI toolbox in Matlab.

IV. NUMERICAL SIMULATIONS

Example 1. We show that a delayed network with $N = 5$ nodes described by (1). Consider a delayed electromechanical device network as the node dynamical system. It is composed of an electrical part (Duffing oscillator) coupled to a mechanical part governed by a linear oscillator. The coupling between both parts is realized through the electromagnetic force due to a permanent magnet. It creates a Laplace force in the mechanical part and the Lenz electromotive voltage in the electrical part. The electrical part of the system consists of a resistor R, an inductor L, a condenser C and a sinusoidal voltage source $e(t)$ all connected in series. The mechanical part is composed of a mobile beam which can move along the $\vec{z}$-axis on both sides. The rod T which has the similar motion is bound to a mobile beam with a spring. A single delayed dynamical equation is described by the following form[11]:

$$f = \begin{cases} 35(x_{i2}(t) - x_{i1}(t)), \\ -7x_{i1}(t) - x_{i1}(t)x_{i3}(t-\tau_1) + 28x_{i2}(t), \\ x_{i1}(t)x_{i2}(t-\tau_1) - 3x_{i3}(t-\tau_1). \end{cases} \quad (17)$$

The asymmetric coupling matrices as A and B are random and satisfied with the coupling condition, $\tau_1, \tau_2 \in (0,1.2]$. The individual couplings matrices are $\Gamma_1 = \Gamma_2 = diag\{1,1,1\}$. Similar to [12], obviously, Hypothesis1 holds.

Applying Theorem 1 with, $\rho_2 = \rho_3 = 0.2, \rho_4 = 2$, $\gamma_1 = \gamma_2 = 4$, $\tau_1 = 0.6$, $\tau_2 = 1.2$ and solving the LMI (8) by using LMI toolbox of Matlab, it is found the linear feedback gain $K = diag\{4.8466, 4.8017, 4.6102, 4.2442, 3.8236\}$.The synchronous error e_i is shown in Fig .1. For this simulation, the initial values of states are $x_i(0) = (2,2,3)^{\mathrm{T}}$ and $s(0) = (1,1,1)^{\mathrm{T}}$.

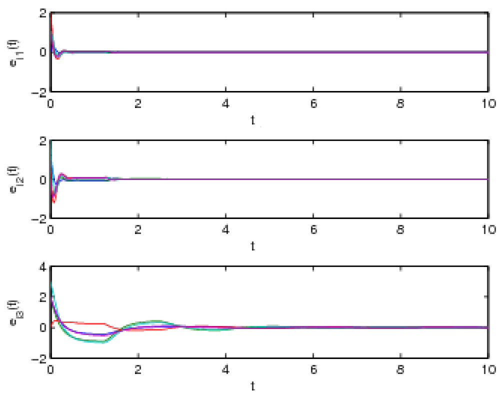

Figure 1. Synchronization errors $e_{i1}, e_{i2}, e_{i3} (i = 1,2,\cdots,5)$ of networks (1).

V. CONCLUSION

The synchronization control of a general complex dynamical network with coupling delays and delays in the dynamical nodes which represents a realistic form of networks has been studied in this paper. By constructing appropriate Lyapunov functions, the linear feedback synchronization controllers are derived. These controllers are very useful for understanding the mechanism of synchronization in complex dynamical networks with coupling delays and delays in the dynamical nodes. Moreover, the resulting for network synchronization is expressed in simple forms that can be readily applied in practical situations. Finally, the effectiveness of these synchronization criteria is verified by numerical simulations.

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