

Multifractal Description of Fractures in Tight Reservoir Based on Fracture Image

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Abstract: Tight oil and gas reservoir have a huge reserve all over the world. The shape and distribution of fractures in tight formation are the key contributors to high-efficient exploitation. An effective description of fractures is significant and fundamental for the study of fractures in tight reservoirs. A new effective description method is proposed after contrast and analysis of existing approaches. After conducting a binarization process to the fracture images, with the application of Matlab, multifractal theory is adopted to get the multifractal spectrum of different fractures. The research proposes a new method to describe fractures, which is to synthesize the multifractal parameters Δa , $\Delta f(a)$ and area density to describe the complex distributions and quantity features of fractures. The former parameters reflect distribution characteristics and later one shows quantity features. With this approach, a more accurate classification of tight reservoirs and a more specific development program can be conducted sequentially.

Introduction

More and more tight oil and gas reservoirs have been verified nowadays. As unconventional reservoirs, fractures are main flow channels in formations and its development condition will influence the reservoir development programming and result directly and significantly. In the systematic study of fractures, the degree of fracture maturity and distribution characteristic are the fundamental and essential aspects. Different reservoirs can be effectively differentiated and classified after the effective and accurate description of fractures. To similar reservoirs, a similar development program can be applied, which will significantly enhance the efficiency and accuracy of exploitation process. However, the existing fracture characterization methods are often unable to accurately and effectively distinguish the various fractures. By comparing conventional fracture description methods, single fractal description, and multifractal description method, a new approach is proposed which is to synthesize multifractal spectrum and fracture area density based on fracture image analysis. This has provided a more accurate and effective quantitative description of fracture system.

Conventional approaches of fracture description and its deficiency

Fracture density and its deficiency Linear fracture density and area fracture density are often used when it comes to planar fracture description. The definitions of these two densities are as eq1:

$$D_L = \frac{N}{L} \quad D_S = \sum_{i=1}^n L_i / A \quad (1)$$

However, the two parameters can only provide a general description of fractures' quantitative density, and no description of fractures' distribution characteristics at all. Two theoretical models A and B are showed in Fig. 1 for the further demonstration.

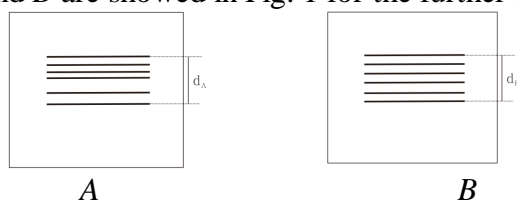


Fig. 1 Sketch map of theoretical fracture A,B

Fracture *A* and *B* have the same density parameters, however *A*'s distribution is totally different from *B*'s. To further describe fractures' distribution, a fractal^[1,2] method first proposed by Mandelbrot is applied in description of fractures.

Definition of fractal with a single dimension and its deficiency Conventional definition^[1] of fractal is that the individual and whole have a similar shape regardless of different viewing scales, including rigid self-similar fractal and statistical self-similar fractal. Through the observation of micro-fracture and macro-fracture, natural fractures conform to the fractal definition which means to describe fractures with fractal method is acceptable^[3].

The core descriptor of fractal is fractal dimension. In the study of fractures, box-counting dimension is often used for its practicality. The dimension is defined as follow in eq 2:

$$D = -\lim_{r \rightarrow 0} \frac{\ln N_r(A)}{\ln r} \quad (2)$$

In which, *A* - a fractal set; *D* - box-counting dimension; $N_r(A)$ - minimal number of boxes which are used to fill *A* completely; *r* - size of box;

Computation of *A, B*'s box-counting dimension With the help of Matlab, the calculation result is shown in Table 1 and Fig. 2:

Table 1 Calculation result for fracture *A, B* with box-counting method

<i>r</i>	$\ln r$	$N_r(A)$	$N_r(B)$	$\ln N_r(B)$	$\ln N_r(A)$
0.20	-1.60944	6	6	1.791759	1.791759
0.10	-2.30259	24	28	3.332205	3.178054
0.07	-2.70805	45	60	4.094345	3.806662
0.05	-2.99573	72	91	4.51086	4.276666
0.03	-3.4012	108	114	4.736198	4.682131
0.02	-3.91202	300	217	5.379897	5.703782

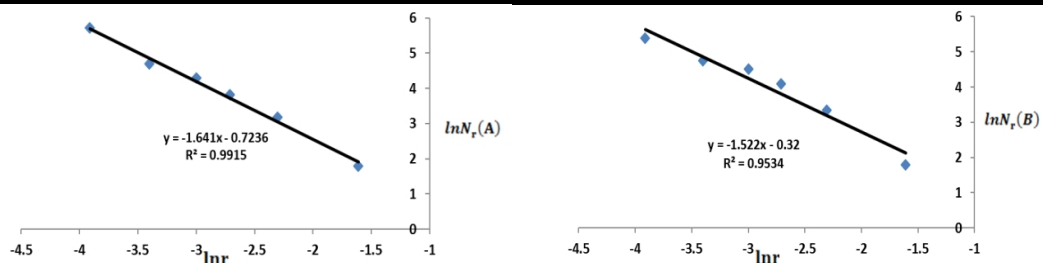


Fig. 2 Fitted curves of fracture *A* and *B*

From the result table, *A* and *B*'s box-counting dimensions are 1.641, 1.522 respectively, and correlation coefficients are 0.9915 and 0.9534 which means *A* and *B* are fractal images and conform to self-similar characteristics. Since *A*'s dimension is larger than *B*'s, it means *A*'s fracture distributions are more complex and non uniform than *B*'s. However when it comes to theoretical fracture *C* (shown in Fig. 3), the situation changes.

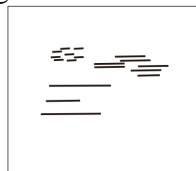


Fig. 3 Sketch map of stochastic fracture *C*

The box-counting dimension of *C* is 1.1831 and correlation coefficient is 0.9972, however it is obvious that *C* is much more dispersive than *A* and *B*. So the single fractal dimension cannot describe the full picture of fractures' distribution characteristics and degrees of development.

The causes of this phenomenon are:

- (1) The size of box cannot be infinitesimal, which means in one box, there may be one or more fractures. However when counting the boxes, they are treated equally. By doing so, some significant information of fractures' distribution and density have been ignored.

- (2) The fractures statistically conform to fractal, which means some individuals still vary when the scale changes, so single fractal dimension isn't enough to describe it ^[4].
So in order to describe the fractures distinctively and effectively, multifractal theory is applied.

Multifractal description of fractures

Definition of multifractal and methodology A simplified definition of multifractal^[1] is a fractal set composed by several subsets, and in different viewing scales, the dimensions of those subsets vary. A multifractal spectrum is used to characterise it. The spectrum can also be got by box-counting method ^[5] and some important formulas are required before getting a fracture's multifractal spectrum :

- (1) Firstly, define a probability distribution function in fracture system:

$$P_i(r) = n_i / \sum_{i=1}^N n_i \quad (3)$$

In which, n_i refers to the number of fracture pixels in single box i where the size of box is r , N refers to the total number of boxes.

- (2) Secondly, define a distribution function which is the summation of q th-power of probability distribution function:

$$c_q(r) = \sum P_i(r)^q = r^{t(q)} \quad (4)$$

In which q is named weight factor; $t(q)$ is named quantity index and can be derived from:

$$t(q) = \ln c_q(r) / \ln r (r \rightarrow 0). \quad (5)$$

- (3) Perform Legendre transformation on $t(q)$ to get multifractal spectrum $f(a)$:

$$a = dt(q) / dq = \lim_{r \rightarrow 0} \frac{\sum_{i=1}^N P_i(r)^q \ln P_i(r)}{\sum_{i=1}^N P_i(r)^q \ln r} \quad (6)$$

$$f(a) = \lim_{r \rightarrow 0} \frac{\sum_{i=1}^N P_i(r)^q \ln P_i(r)}{\sum_{i=1}^N P_i(r)^q \ln r} q - \lim_{r \rightarrow 0} \frac{\sum_{i=1}^N P_i(r)^q}{\ln r} \quad (7)$$

In which, a is named singularity exponent and it reflects the singular degree of fractal subsets. The larger a 's value is, the more singular the subset is, and the corresponding probability is smaller. $f(a)$ refers to the dimension of subsets which have the same a value. From the spectrum $f(a)$, different subsets of fractures can be shown and that meets the requirements to describe complex fractures.

The algorithm flow to get multifractal spectrum are shown as follow:

- (1) Input the fracture image and conduct a binarization process^[6];
- (2) Select different sizes of grids to fill the whole planar fracture image;
- (3) Select different q value, calculate function $c_q(r)$, fit the curves of $c_q(r)$ and r in a double \log axis, observe the curves' shape to find out whether it is linear or not, if it is linear then it conforms to multifractal and the slopes of curves are the values of $t(q)$.
- (4) Under a given r (small value), calculate multifractal spectrum $f(a)$ based on formula (3), (4).

The estimation of q 's value If the value of weight factor q can vary from $-\infty$ to $+\infty$, it will include all the subsets' influence. However with the limitation of computing capacity and practical engineering demand, the infinitary value is not a must. According to previous researches, it turns out that under certain circumstance, a varies a little with q 's change, which means a finite q can also satisfy the accuracy requirement. So the estimation procedure is shown as follow:

$$a = dt(q) / dq = \frac{d \ln \sum_{i=1}^N P_i(r)^q / \ln r}{dq} = \frac{\sum_{i=1}^N P_i(r)^q \ln p_i(r)}{\sum_{i=1}^N P_i(r)^q \ln r} \quad (8)$$

Get derivation of a :

$$\frac{da}{dq} = \frac{\sum_{i=1}^N \left\{ P_i(r)^q [\ln p_i(r)]^2 \right\} \sum_{i=1}^N P_i(r)^q - \left[\sum_{i=1}^N P_i(r)^q \ln p_i(r) \right]^2}{\left[\sum_{i=1}^N P_i(r)^q \right]^2 \ln r} \quad (9)$$

According to q 's function, the smaller the r is, which means the boxes are more intensive, and the larger range the q 's value varies, it can show the probability condition in the smaller boxes more comprehensively. So in order to estimate q , under a given small r , constrain $\frac{da}{dq}$ with x , and

$\frac{da}{dq} < x$, in which x refers to a small number approaching 0 and it depends on the practical accuracy demand. By doing so, q 's order of magnitude can be determined.

Results and discussion

In this calculation, the value range of q is $(-50, +50)$ and it meets the accuracy demand. Use Matlab programming to realize the whole computing process and the sizes of grids are $1/5, 1/10, 1/20, 1/40, 1/50, 1/100$ respectively. Four fracture models are calculated and its binary images are shown in Fig. 4

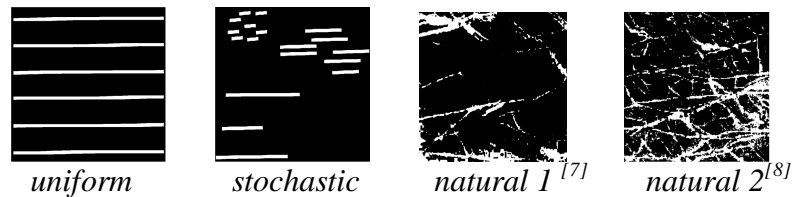


Fig. 4 Binary images of different fractures

Curves of $c_q(r)$ vs r in double log axis (shown in Fig. 5):

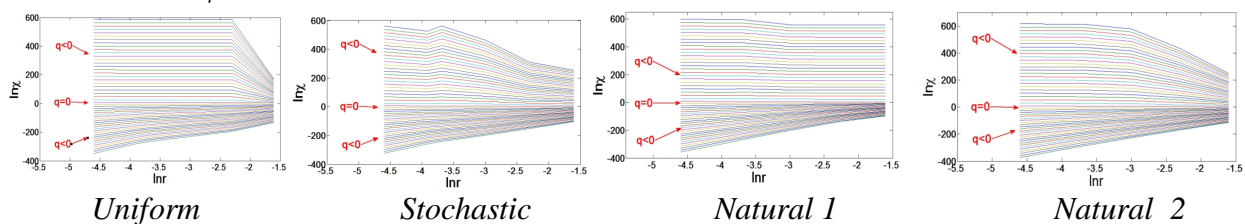


Fig. 5 $\ln c_q(r)$ vs $\ln r$ curves of different fractures

The curves show that, under the circumstance which is $q > 0$, $\ln c_q(r)$ and $\ln r$ have a good linear relationship, which means it conforms to self-similar characteristic, while when $q < 0$, $\ln c_q(r)$ and $\ln r$ are not in linear relationship which means it is not multifractal. It is mainly because the size of grid cannot be infinitesimal, while the size is becoming smaller and smaller, the small probability events hardly change. In fracture system, the small probability events refers to the error points or infinitesimal cracks. Those events don't influence the real distribution of fractures, but it have a great impact on function $c_q(r)$, because $q < 0$ which means it is very sensitive to small numbers. Since when $q < 0$, it cannot be described with multifractal and also in reservoir engineering, the developmental fractures are mainly concerned in the seepage process, the multifractal spectrum where $q > 0$ is got, shown in Fig. 6:

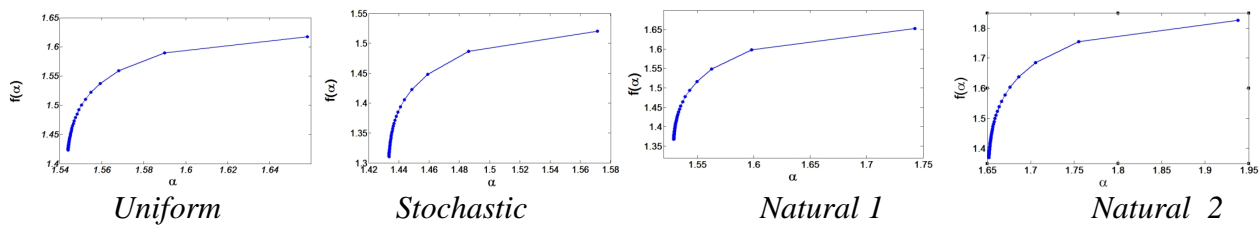
Fig. 6 Multifractal spectrums of different fractures ($q > 0$)**Calculation results (shown in Table 2):**

Table 2 The comparisons of multifractal parameters between different fractures

Type of fracture	D_0	D_1	Δa	$\Delta f(a)$	$f(a)_{\max}$	D_s
<i>Uniform</i>	1.62	1.59	0.114	0.19414	1.61725	0.1224
<i>Stochastic</i>	1.52	1.49	0.138	0.21015	1.52030	0.0736
<i>Natural 1</i>	1.65	1.59	0.214	0.28426	1.65246	0.1143
<i>Natural 2</i>	1.83	1.76	0.286	0.45641	1.82588	0.2010

In the table, D_0 refers to the dimension when $q=0$, the same dimension as single box-counting dimension; D_1 refers to dimension when $q=1$, and means information dimension; Δa refers to variation of a ; $\Delta f(a)$ refers to variation of $f(a)$; $f(a)_{\max}$ refers to maximum fractal dimension; D_s refers to area density.

Through the comparisons, the single dimension and information dimension cannot characterize the complexity of fractures, that also proves the necessity to introduce multifractal spectrum. The more complex the fractures' distribution is, the larger the values of Δa and $\Delta f(a)$ are, which means the value of Δa and $\Delta f(a)$ can characterize the non uniformity of fractures, while the maximum value can show some local complexity condition rather than the whole image. In addition to this, however Δa and $\Delta f(a)$ cannot show the quantities of fractures. As shown in the table, the area densities of *stochastic fracture* and *natural fracture 1* are smaller than *uniform fracture*, but the values of Δa and $\Delta f(a)$ are larger than later one, which indicates that the area density is a reasonable parameter to characterize the quantity features of fractures and it is an independent variable.

So to synthesize multifractal parameters and area density can provide a comprehensive description of fractures, the former one reflects the complexity of distribution, while the later one shows the quantity features.

Conclusions

Through comparing different description methods in theoretical and natural models, and applying multifractal theory with Matlab to get multifractal spectrums of different fractures, to draw conclusions:

- (1) Conventional approaches including density, single fractal dimension etc. cannot effectively describe the fracture's distribution characteristic and degree of maturity.
- (2) Standardize the procedures to get fracture's multifractal spectrum and provide an effective method to get the value range of weight factor q which is constrained by $da / dq < x$.
- (3) Synthesize multifractal parameters Δa , $\Delta f(a)$ and area density to characterize fractures, the former one reflects distribution condition, the larger the value is, the more complex the distribution is, and the later one shows quantity features, the larger the value is, the larger the number of fractures is.
- (4) With more effective and accurate quantitative descriptors, fractures and corresponding reservoirs can be classified more reasonably, and related operations, like reservoir appraisal, well distributions can be further conducted sequentially. Fracture description is a basic starting point to enhance efficiency and quality of reservoir exploitation.

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