Study on the Method of Extracting Phase Using the Fourier Transform

Zhan-rong Zhou*, Jun Xu, Xuan-ke Zhao, Xiao-fang Shen

Dept. of Physics Xi'an High Technology Studio Xi'an, China，710025

* zhouzhou76@163.com

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Abstract. A new method of extracting interference fringe phase is proposed. Firstly, the interference fringe was transformed from the spatial domain into the frequency domain using the Fourier transform. The positive spectrum carried the wave-front information. Secondly, the digital filters were design to filter the zero and negative spectrum through the weighted filter, and then the wrapped phase was obtained using the inverse Fourier transform. Finally, based on the analysis of various unwrapping algorithms, the idea of the correction after the unwrapping was given to solve the problem of error point. The research results show that the method is simple and applicative, and is proved effective for most of the patterns.

Introduction

The optical interference measurement has advantages of non-contact, full field, non – interference，and high precision. It plays a special role in the industry, the national defense, and the scientific research, etc [1~3]. In the optical measurement, the measured physical quantity information implied in the light of the phase, but the measured results are recorded in the form of interference fringes. How to extract the phase information from the interference fringe is a key step in obtaining measurements. According to the principle of light interference, a new method is proposed to extract the phase information based on the Fourier transform [4~6].

The extract phase principle of the Fourier transform

The foundation of phase extraction is the mathematical description of the interference fringe [7,8]. The gray level of most optical measuring carrier fringe pattern can be expressed as:

$$g(x,y) = a(x,y)\{1 + b(x,y)\cos[2p\,(f_{0x}x + f_{0y}y) + f(x,y)]\} \qquad (1)$$

In the formula (1): a(x, y) as the background, b(x, y) as the light wave amplitude, f (x, y) as wave phase. Usually the measured physical quantity represented by fringe pattern can be expressed as the function of f (x, y). If the function of f (x, y) can be got, the physical quantity results can be obtained. In general, a(x, y), b(x, y) and f (x, y) are slow changing signals compared with the spatial carrier fox and foy. Therefore the various spectral components of g(x, y) are separated from each other.

In order to obtain f (x, y), the formula (1) is written as a complex deformation:

$$g(x,y) = a(x,y) + c(x,y)\exp[\,\mathrm{j}2p\,(f_{0x}x + f_{0y}y)] + c^*(x,y)\exp[-\mathrm{j}2p\,(f_{0x}x + f_{0y}y)] \qquad (2)$$

The symbol '*' indicates complex conjugation

$$c(x,y) = \frac{1}{2}\{a(x,y)b(x,y)\exp[\,\mathrm{j}f(x,y)]\} \qquad (3)$$

$$c^*(x,y) = \frac{1}{2}\{a(x,y)b(x,y)\exp[-\mathrm{j}f(x,y)]\} \qquad (4)$$

The formula (4) was transformed by the Fourier transform:

$$G(f_x, f_y) = A(f_x, f_y) + C(f_x - f_{0x}, f_y - f_{0y}) + C^*(f_x + f_{0x}, f_y + f_{0y}) \qquad (5)$$

In the formula (5), the C and C* are the Fourier transform of the c and c* respectively. The $A(f_x, f_y)$ is the distribution function of the zero order spectrum from the background intensity transform. $C(f_x - f_{0x}, f_y - f_{0y})$ is the positive spectrum distribution function.

$C^*(f_x + f_{0x}, f_y + f_{0y})$ is the negative spectrum distribution function. (f_{0x}, f_{0y}) is the positive spectrum center, and $(-f_{0x}, -f_{0y})$ is the negative spectrum center. As long as the filter with appropriate center frequency spectrum and spectral width is designed, the positive spectrum can be separated from the magnitude spectrum, and then it carries on the inverse Fourier transform.

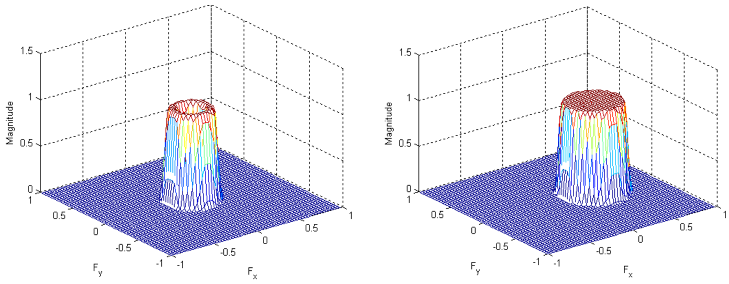
$$F^{-1}\{C(f_1, f_2)\} = c(x,y) = \frac{1}{2}a(x,y)b(x,y)\exp(jf(x,y)) \qquad (6)$$

The arctangent algorithm:

$$f(x,y) = \arctan\left\{\frac{\mathrm{Im}[c(x,y)]}{\mathrm{Re}[c(x,y)]}\right\} \qquad (7)$$

Processing steps of the Fourier transform

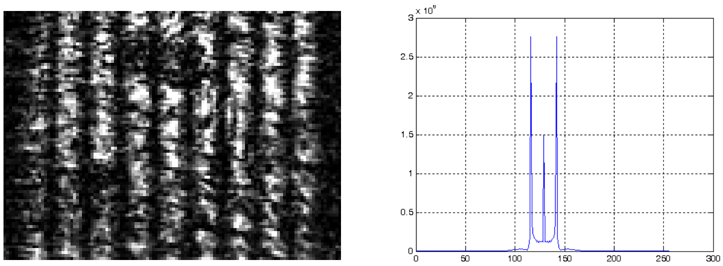
The filter design is a key step in extraction fringe phase using the Fourier transform. The filter design includes two aspects. One is to determine the position of the center and the width of the frequency domain. The other is to determine the filter shape. The formula (5) shows zero and ± 1 spectrum center coordinates (u_0, v_0), (u_1, v_1) and (u_2, v_2) must be in a straight line. The low frequency point on the line of (u_0, v_0) and (u_1, v_1) is found. The bandwidth of filter is the spectrum interval from the low frequency point to the center (u_1, v_1). The two 2-D filters were designed with the Hamming window function and the Hanning window function. As shown in Figure 1.



(a) the Filter 1 (b) the Filter 2

Figure.1.The design of two dimensional filters

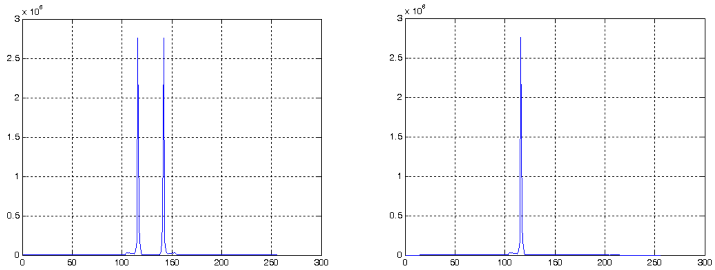
In figure 2: (a) is interference fringe. (b) is two-dimensional spectrum of the interference fringe by the Fourier transform.



(a) The interference fringe (b) 2-D spectrum

Figure2 .the interference fringe and the spectral distributions

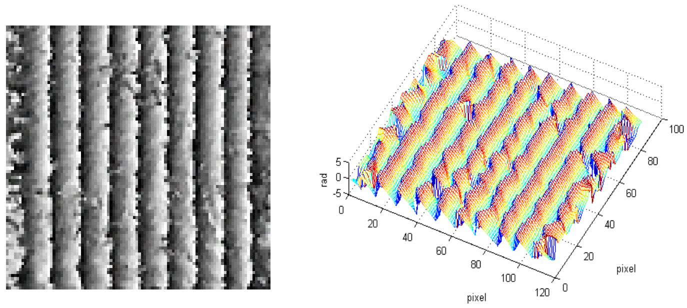
The zero and negative spectrum were filtered by the two filters. The results as shown in figure 3



（a）The first result （b）The second result

Figure.3. Extraction spectrum from the interference fringe

The results show that the proposed methods of determining center frequency and band width of the filter are effective. It can completely extract +1 spectrum by two two-dimensional filters. Using the Fourier inverse transform, the wrapped phase as shown in figure 4:



（a）2-D wrapped phase （b）3-D wrapped phase

Figure. 4 . The wrapped phase

The phase is beyond this range $[-p, p]$ by the Fourier transform. In order to obtain the actual phase, the wrapped phase must be unwrapped [7, 8].

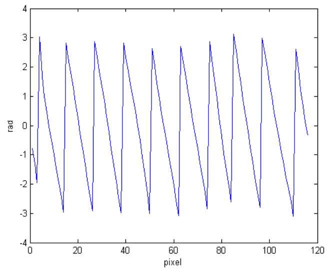
Mathematically, the phase unwrapping is not very difficult. The method is as the following: along the phase truncation direction of the data matrix row or column, two adjacent point phase value were compared. If the difference is less than p ，the phase value of the latter point plus $2p$. If the difference is bigger than p , the phase value of the latter point minus $2p$. However, there will always be some error points in the wrapped phase map of the actual interference fringe. As the error will be launched along the path of the direction of unwrapping, the error bands appear in the phase map After the cause of error bands were analyzed, a new phase unwrapping method is proposed. The new method steps are as follows:

Step1: the error points were highlighted by the wrapped phase difference of the adjacent points.

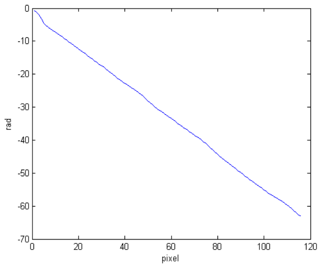
Step2: the phase unwrapping threshold was determined by the statistical features of the wrapped phase. The Phase was scanned along unwrapping path.

Step3: based on the characteristics of the smooth continuous phase in the interference field, the phase of error points were corrected along the scanning path.

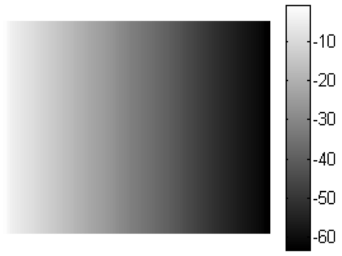
The results of unwrapping phase as shown in Figure 5: (a) one-dimensional wrapped phase. (b) one-dimensional unwrapping phase. (c) three-dimensional wrapped phase. (d) three-dimensional unwrapping phase.



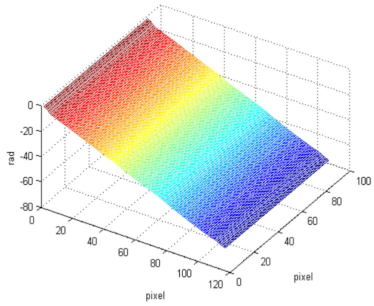
（a） 1-D wrapped phase



（b） 1-D unwrapping phase



（c） 2-D wrapped phase



（d） 3-D unwrapping phase

Figure.5. The results of phase unwrapping

Conclusion

Extracting the phase from the interference fringe was studied by the Fourier transform. Aiming at the difficulty of designed filter, the filter center frequency and filter bandwidth were focused. The two 2-dimensional digital filters were design to extract +1 spectrum from the interference fringe. The wrapped phase was obtained using the Fourier inverse transform. On the basis of the phase unwrapping algorithms, the ideas of the correction after the unwrap is proposed. It solves the difficulty of the common unwrap algorithm avoiding error point. The method was applied in the actual interference fringe, and obtained unwrapping phase.

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