The Travelling Wave Solutions of the nonlinear beam equations

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Abstract. This paper is concerned with the existence of exact traveling wave solutions of nonlinear evolution equation by the tanh-function method. The validity and reliability of this method is demonstrated by applying it to a variety of the nonlinear beam equations.

Introduction

It is well known that nonlinear phenomena are very important in a variety of scientific fields, especially in fluid mechanics, solid state physics, plasma physics, plasma waves, capillary-gravity waves, and chemical physics. Most of these phenomena are described by nonlinear partial differential equations. Analytical solutions of this problems are usually not available, especially when the nonlinear terms are involved. Therefore, finding its traveling solutions is of practical importance.

The methods of looking for exact traveling wave solutions of nonlinear evolution equations, has been tremendous development in recent decades, such as inverse scattering method[1], Hirota's bilinear technique[5], the Painlve expansion method[12].In the early nineties of last century, Huibin and Kelin[6] proposed a new method. The main idea of this method is taking hyperbolic tangent function of the power series as possible traveling wave solutions of the nonlinear evolution equations. In order to reduce the complex algebra computation, Malfiety[8–10] proposed the tanh-function method. Fan et al[3] proposed the extended hyperbolic tangent method, which replace the tanh-function by the solutions of Riccati equation. In[1,4,13], using the tanh function method, they got the exact form of traveling wave solutions of various types of evolution equations.

The Tanh-function Method

Let's consider the nonlinear partial differential equations

$$N(u,u_x,u_{xx},u_{xxx},...)=0, \tag{2.1}$$

Where $u(x,t)$ is the real function on R^2. At first, we assume the traveling wave solutions of (2.1) are the form of

$$u(x,t)=U(\omega)=U(c(x-vt)), \tag{2.2}$$

with the velocity v, and the constant c. Submitted (2.2) into (2.1), we can get the ODES abou ω

$$N(U,U',U'',U''',...)=0, \tag{2.3}$$

Second, we assume the possibly traveling wave solutions can be written

$$u(x,t)=U(\omega)=H(Y)=\sum_{i=0}^{K}a_iY^i, \tag{2.4}$$

where $Y=\tanh(\omega)=\dfrac{e^{\omega}-e^{-\omega}}{e^{\omega}+e^{-\omega}}$,the highest order K will be determined late. Then we can get

$$\frac{dY}{d\omega}=1-Y^2,\frac{dU}{d\omega}=(1-Y)H',\frac{d^2U}{d\omega^2}=(1-Y^2)(-2Y.H'+(1-Y^2)H''),$$

$$\frac{d^3U}{d\omega^3}=(1-Y^2)(6Y^2-2)H'-6Y(1-Y^2).H''+(1-Y^2)^2H'''),$$

Submitted above equations into (2.3)，we can get the ODES with Y

$$N(U,U',U'',U''',...)=0, \qquad (2.5)$$

where $H'=\frac{dH}{dY}$..To determine the parameter K we usually balance the nonlinear term and the highest order derivative term in equation (2.5).Then, we submitted (2.4) (with the determined K) into (2.5)，and get the polynomial equation with Y. Collecting all the coefficients of power of Y and letting the coefficients of each power of Y to be vanished, we can determined all the coefficient $a_1,a_2,...,a_K$.According to (2.4), we can get the traveling wave solutions of (2.1).

Application

Let us consider the traveling wave solutions of the nonlinear beam equations

$$u_{tt}=u_{xxxx}+u(1-u^2). \qquad (3.1)$$

Submitting (2.2)and (2.4)into(3.1),we can get

$$c^2\upsilon^2(1-Y^2)(-2YH')+(1-Y^2)H'')=c^4(1-Y^2)(8Y(2-3Y^2))H'$$

$$-2(4-13Y+3Y^4)H''+3(1-Y^2)(1-Y^2-2Y)H'''+(1-Y^2)^3H^{(4)})+H(1-H^2).$$

Balancing $Y^8H^{(4)}$ with H^4,we can get $K=2.$Thus

$$H(Y)=a_0+a_1Y+a_2Y^2.$$

Submitting it into above equation, we can get

$$c^2\upsilon^2(1-Y^2)(-2Y(a_1+2a_2Y)+2(1-Y^2)a_2)=c^4(1-Y^2)$$

$$(8Y(2-3Y^2)(a_1+2a_2Y)-4a_2(4-13Y+3Y^4))+(a_0+a_1Y+a_2Y^2)(1-(a_0+a_1Y+a_2Y^2)^2).$$

Comparing the coefficient of the same power of Y, we can get

$$\begin{cases} a_0-a_0^3-16c^4a_2-2c^2\upsilon^2a_2=0, \\ a_1+16c^4a_1+2c^2\upsilon^2a_1-3a_0^2a_1=0, \\ -3a_0a_1^2+a_2+100c^4a_2+8c^2\upsilon^2a_2-3a_0^2a_2=0, \\ -40c^2a_2-2c^2\upsilon^2a_1-a_1^3-6a_0a_1a_2=0, \\ -144c^4a_2-6c^2\upsilon^2a_2-3a_1^2a_2-3a_0a_2^2=0, \\ 24c^4a_1-3a_1a_2^2=0, \\ 60c^4a_2-a_2^3=0. \end{cases}$$

Then, we can get

$$a_1=0,a_0=-\frac{\sqrt{\frac{15}{2}}}{2},\upsilon=-\frac{\sqrt{3}}{2^{\frac{3}{4}}},a_2=\frac{\sqrt{\frac{15}{2}}}{2},c=-\frac{1}{2^{\frac{4}{5}}};$$

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Then, We can get

$$\begin{cases} u_1(x,t) = -\frac{\sqrt{\frac{15}{2}}}{2} + \frac{\sqrt{\frac{15}{2}}}{2}\tanh^2[\frac{1}{2^{\frac{5}{4}}}(x+\frac{\sqrt{3}}{2^{\frac{3}{4}}}t)], \\ u_2(x,t) = -\frac{\sqrt{\frac{15}{2}}}{2} + \frac{\sqrt{\frac{15}{2}}}{2}\tanh^2[\frac{1}{2^{\frac{5}{4}}}(x-\frac{\sqrt{3}}{2^{\frac{3}{4}}}t)], \\ u_3(x,t) = \frac{\sqrt{\frac{15}{2}}}{2} - \frac{\sqrt{\frac{15}{2}}}{2}\tanh^2[\frac{1}{2^{\frac{5}{4}}}(x+\frac{\sqrt{3}}{2^{\frac{3}{4}}}t)], \\ u_4(x,t) = \frac{\sqrt{\frac{15}{2}}}{2} - \frac{\sqrt{\frac{15}{2}}}{2}\tanh^2[\frac{1}{2^{\frac{5}{4}}}(x-\frac{\sqrt{3}}{2^{\frac{3}{4}}}t)], \end{cases} \tag{3.2}$$

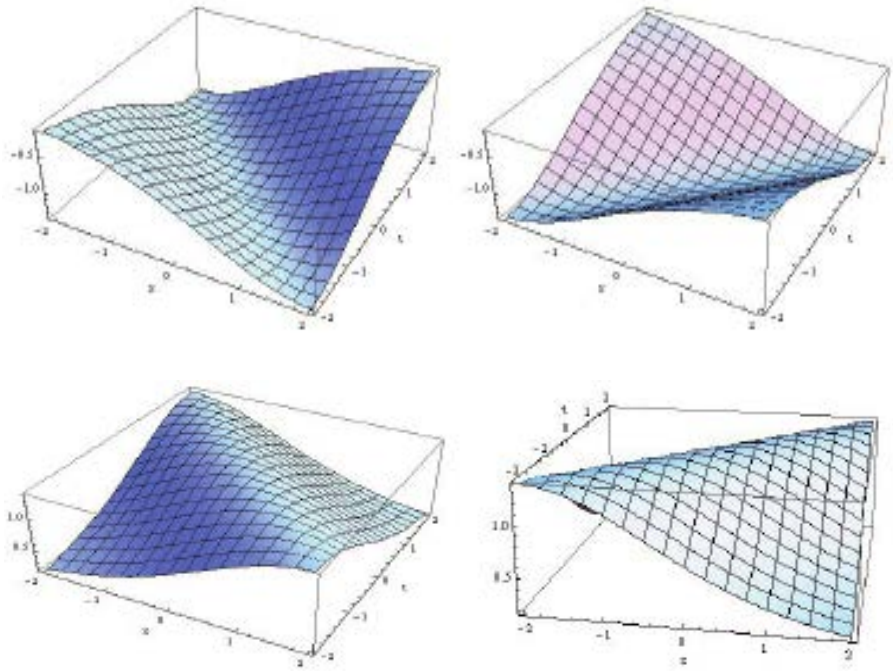

Fig. 1. the travelling wave solutions of (3.2), when $(x,t) \in [-2,2]\times[-2,2]$.

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