

# Numerical Computation Method in Specific Harmonic Elimination Technology of Step Wave

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**Abstract.** Combining the specific harmonic elimination pulse width modulation technology of cascading H-bridge convertor, this paper discusses the function relationships between its parameters, and establishes the corresponding multiple nonlinear system of equations, and focuses on analysis of advantages and disadvantages of numerical computation methods used to solve this system of equations, such as Newton Raphson method, genetic algorithm and particle swarm optimization. Through comprehensive comparison, this paper finds out unknown parameters in improved particle swarm optimization with switching angle initial selection rule used to solve the specific harmonic elimination technology of step wave of cascading H-bridge convertor. This method is of good practical engineering value in preparing the control strategy of cascading H-bridge convertor.

## Introduction

On the high-voltage and high-power occasions, cascading multilevel converter has been more and more widely applied. Cascading multilevel converter consists of several converter modules in series to achieve high-voltage and multi-level output. Its basic system structure is shown in Figure 1.

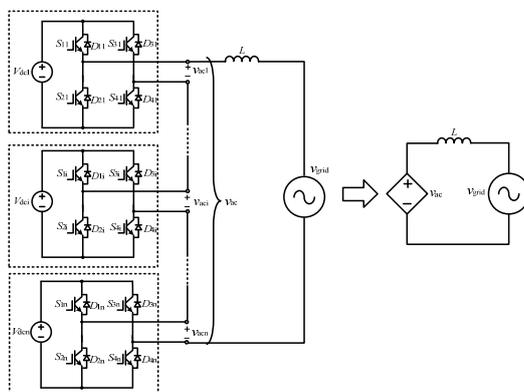


Fig. 1 Grid-connected system structure and equivalent circuit based on cascading H-bridge convertor

In the cascade H-bridge converter system, for  $i$  H-bridge converter unit, when  $(S_{1i}, S_{3i})$  or  $(S_{2i}, S_{4i})$  opens, 0 level is output, namely the output voltage  $V_{acfi}$  is 0; when  $(S_{1i}, S_{4i})$  opens, 1 level is output, namely the output voltage  $V_{acfi}$  is  $V_{dci}$ ; when  $(S_{2i}, S_{3i})$  opens, -1 level is output, namely the output voltage  $V_{acfi}$  is  $-V_{dci}$ ; the overall output level number of cascading H-bridge convertor can be  $(2n + 1)$ .

The output voltage waveform of cascading H-bridge multi-level converter in engineering practice is generally shown in Figure 2. In the theoretical analysis, the waveform can be simplified as shown in Figure 3.

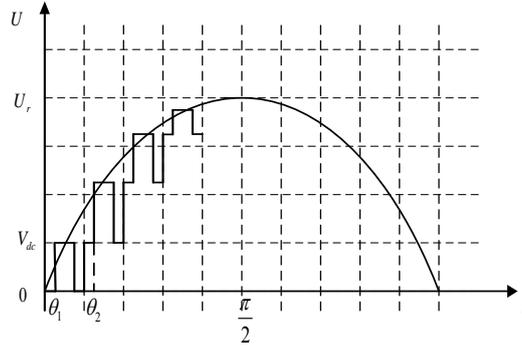


Fig. 2 Output voltage wave in engineering practice

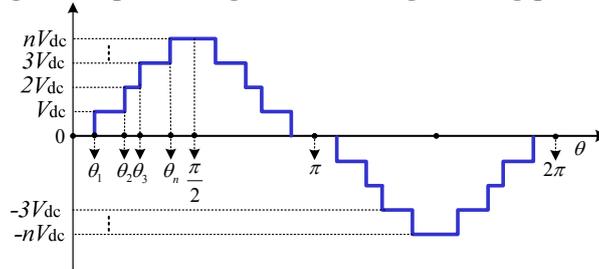


Fig. 3 Simplified output voltage wave

Do harmonic expansion for step wave in Figure 3 and obtain the general nonlinear system of equations of multi-level inverter SHEPMW:

$$V_i = \frac{4V_{dc}}{m} [\cos(s\theta_1) + \cos(s\theta_2) + \dots + \cos(s\theta_n)] \quad (2)$$

Wherein:

$$0 \leq \alpha_1 < \alpha_2 < \dots < \alpha_{2n} \leq \frac{\pi}{2}$$

For cascading convertor system of n H-bridge convertor units, when meeting the expected foundational wave voltage amplitude, the number of specific harmonic waves that can be cancelled is (n-1). A group  $\theta_i (i = 1, \dots, n)$  can be found out to make the foundational wave weight amplitude  $V_{1m}$  without low-order harmonic waves.

Define the modulation ratio  $m = V_{1m} / (nV_{dc})$ . Based on above constrains, system of nonlinear algebraic equations about  $\theta_i (i = 1, \dots, n)$  can be obtained.

$$\begin{cases} \cos(\theta_1) + \cos(\theta_2) + \dots + \cos(\theta_n) = \frac{\pi V_{1m}}{4V_{dc}} = \frac{\pi m}{4} \\ \cos(5\theta_1) + \cos(5\theta_2) + \dots + \cos(5\theta_n) = 0 \\ \cos(7\theta_1) + \cos(7\theta_2) + \dots + \cos(7\theta_n) = 0 \\ \dots \end{cases} \quad (3)$$

For specific cascading module number n, only if modulation ratio m is within a certain range, the system of nonlinear equations has solutions. For solution, a group of initial values  $\theta_i^0 (i = 1, \dots, n)$  is needed. The selection of initial value has direct impacts on solving convergence of the system of equations.

Meanwhile, the waveform quality of output voltage  $V_{out}$  based on specific harmonic elimination technology of step wave can be described by the Total Harmonics Distortion (THD), shown as Eq.4:

$$THD = \sqrt{\frac{\sum_{k=2,3,\dots}^{\infty} V_k^2}{V_1^2}} \times 100\% = \sqrt{\frac{V_2^2 + V_3^2 + V_{11}^2 + \dots + V_{2n}^2}{V_1^2}} \times 100\% \quad (4)$$

### Common Numerical Solutions of System of Nonlinear Multivariable Equations

For the system of nonlinear equations (shown as equation (5) below), there has been no analytical methods to extract its root, while in the engineering computation, the numerical solution is used for such questions. This paper will compare the commonly-used numerical solutions for the system of nonlinear multivariable equations.

$$\begin{cases} f_1(x_1, x_2, \dots, x_n) = 0 \\ f_2(x_1, x_2, \dots, x_n) = 0 \\ \dots \\ f_n(x_1, x_2, \dots, x_n) = 0 \end{cases} \quad (5)$$

Newton-Raphson method is the most common method to solve nonlinear equations and system of equations on the engineering. In solving the low-order equations, its effect is relatively good. However, in solving high-order equations, the order of equations is higher, so it will take longer time in the preparation and operation of the program. Therefore, it is not applicable to solution of the system of nonlinear multivariable equations in the specific harmonic elimination technology of step wave.

Genetic algorithm is an evolutionary algorithm, which encodes the problem parameter as chromosomes, and then uses an iterative approach for operations such as selection, crossover and mutation to exchange the information on chromosome in population, and finally to generate the chromosome meeting the optimized target. Apply the optimized genetic algorithm in solving the system of nonlinear equations, and randomly generate initial population, which can simplify calculation. Genetic algorithm has no problems in preparation and operation of the program unlike Newton-Raphson method. However, in solving the system of high-order nonlinear equations, the local convergence is easy to occur, leading to inaccurate results.

PSO (particle swarm optimization) is an iterative optimization algorithm. The system is initialized as a group of random solutions, through iterative search for the optimal value. The basic principle of PSO is through mutual cooperation between individuals in the whole as well as information sharing between them to find the optimal solution. Its advantages are that the algorithm is simple, the parameter is concise and easy to achieve, there is no complex search adjustment, speed is fast and there is no requirements for the continuity of optimization problems. Therefore, PSO is the optimal method to solve the system of nonlinear multivariable equations of simple H-bridge converter among above three methods.

### Analysis of Complex H-bridge Converter

Complex H-bridge converter means H-bridge converter with more cascading modules. At this moment, though the system of nonlinear multivariable equations similar to simple H-bridge converter can be established, when the PSO is used to solve initial value, there are certain problems. It is mainly expressed that in the optimization process, the impact of initial value on the convergence of target function gradually intensifies with the increase of order of target function. Therefore, it is necessary to improve PSO and determine the initial value of switching angle before iteration.

When  $n=15$ , with the increase of order of equation, other conditions remain unchanged. At this moment, there is:

$$\begin{cases} f_1(\theta_1, \dots, \theta_n) = \cos(\theta_1) + \dots + \cos(\theta_n) - 0.2125m - 0 \\ f_2(\theta_1, \dots, \theta_n) = \cos(\beta_2\theta_1) + \dots + \cos(\beta_2\theta_n) - 0 \\ \dots \\ f_{15}(\theta_1, \dots, \theta_n) = \cos(\beta_{15}\theta_1) + \dots + \cos(\beta_{15}\theta_n) = 0 \end{cases} \quad (6)$$

Wherein:  $\beta_n = [1, 5, 7, 11, 13, 17, 19, 23, 25, 29, 31, 35, 37, 41, 43]$

Convert Eq.6 into

$$\begin{cases} m^2 = f_1^2 + f_2^2 + \dots + f_{15}^2 \\ m^2 = f_1^2 - f_2^2 - \dots - f_{15}^2 \end{cases} \quad (7)$$

To find more appropriate initial value of switching angle, this paper uses the waveform of output voltage  $V_{an}$  controlled by step wave SHEPWM in the engineering practice. Specifically speaking, divide the reference sinusoidal signals in this area into several equal parts in time. In each equal part,

use 2 levels similar to reference sinusoidal signal to be equivalent to reference signal, thus making the area consisting of reference sinusoidal signal and time axis equal to the area consisting of 2 levels and time axis. As only SHEPWM problem of 1/4 cycle symmetry is studied here, so only the PWM signal in 1/4 cycle needs to be obtained. Other PWM signals in a cycle can be obtained by symmetry. To make PWM signals generated conform to the sine law, it is necessary to ensure that the PWM waveform is at the highest level at  $\pi/2$ . Therefore, 0 to  $\pi/2$  is divided into n equal parts to ensure the phase voltage PWM waveform is rising at the last jump change in 1/4 cycle.

Firstly, divide  $0 - \pi/2$  into n equal parts. Set the length of each part is  $T_f$  (shown as Figure 4), then the reference signal period  $T = 4nT_f$ .

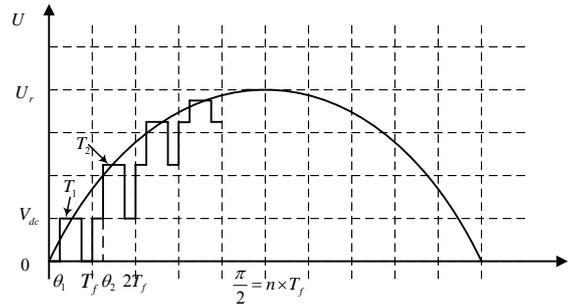


Fig. 4 Reference waveform in 1/4 cycle and corresponding SHEPWM initial value

Assume the reference signal  $i$  ( $i=1,2,\dots,n$ ) is at the level from  $(i-1)V_{dc}$  and  $iV_{dc}$ , then the reference signal in this section can be equivalent by the convex wave with central pulse width of  $T$ , height of  $iV_{dc}$  and pulse widths at both sides of  $T_f - T_i$ , and height of  $(i-1)V_{dc}$ . At this point, the relationship between  $T_i$  and  $T_f$ , and level  $V_{dc}$  shall meet:

$$T_i i V_{dc} + (T_f - T_i)(i-1)V_{dc} = \int_{(i-1)T_f}^{iT_f} U_r \sin(\omega t) dt \quad (8)$$

Thus, there is:

$$T_i = \frac{\int_{(i-1)T_f}^{iT_f} U_r \sin(\omega t) dt}{V_{dc}} - (i-1)T_f$$

$$= \frac{U_r}{\omega V_{dc}} [\cos \omega(i-1)T_f - \cos \omega iT_f] - (i-1)T_f$$

Wherein:  $U_r=1$ ,  $\omega = \frac{2\pi}{T} = \frac{2\pi}{4nT_f} = \frac{2\pi}{4n(\frac{\pi}{2})} = 1$ .

Take data in Eq.8 and obtain

$$T_i = \frac{1}{V_{dc}} \left[ \cos(i-1)\frac{\pi}{2n} - \cos i\frac{\pi}{2n} \right] - (i-1)\frac{\pi}{2n} \quad (9)$$

With the increase of the number of levels of step waves output by the inverter, the output waveform gradually tends to a sine wave. Improved particle swarm algorithm is applied to the case there are more output levels. Therefore, it can be considered that the output waveform is very close to the sine wave (reference signal), and the peak value of output voltage is approximately equal to the peak value of sine wave. Namely:

$$nV_{dc} = U_r = 1 \quad (10)$$

Therefore, Equation (9) can be equivalent to

$$T_i = n \left[ \cos(i-1)\frac{\pi}{2n} - \cos i\frac{\pi}{2n} \right] - (i-1)\frac{\pi}{2n} \quad (11)$$

Wherein:  $n=15$

$$T_i = 15 \left[ \cos(i-1)\frac{\pi}{30} - \cos i\frac{\pi}{30} \right] - (i-1)\frac{\pi}{30} \quad (12)$$

Wherein:

$$i=1,2,\dots,15$$

Therefore:  $T=4nT_f$

Therefore:

$$T_f = \frac{T}{4n} = \frac{\frac{2\pi}{60}}{4n} = \frac{2\pi}{60 \cdot 4n} = \frac{\pi}{30} \quad (13)$$

Use PSO to obtain:

$$\theta_i = (i-1)T_f + \frac{\theta_1 - T_f}{2} = (i-1)\frac{\pi}{30} + \frac{\theta_1 - T_f}{2} \quad (14)$$

Combine Eq.12 and Eq.13 to obtain the initial values of switching angles ( $\theta_i$ ), as shown in Table 1.

Table 1 Initial values of switching angles

i	1	2	3	4	5	6	7	8
$\theta_i(\text{rad})$	0.0113	0.0866	0.1633	0.2423	0.3243	0.4102	0.5008	0.5968
i	9	10	11	12	13	14	15	
$\theta_i(\text{rad})$	0.6989	0.8077	0.9237	1.0473	1.1790	1.3190	1.4675	

## Conclusion

Determining control strategies of each modular unit of H-bridge converter is an important part of specific step wave harmonic elimination technology of cascading H-bridge converter. Solving the system of nonlinear multivariable equations of waveform output by H-bridge converter is the basis for preparing the control strategies. Therefore, studying the numerical solutions meeting engineering needs is of outstanding engineering practical significance.

This paper holds that simple H-bridge converter, Newton-Raphson method, genetic algorithm and PSO can be used for solutions. However, the calculation efficiency of Newton-Raphson method is lower, difficult to meet the needs of modern engineering practice. The genetic algorithm is easy to fall into local convergence. Therefore, PSO is the optimal method to solve such problems. In many solving processes, although there will be local convergence in PSO, in most of tests, the optimal solutions found by PSO are basically the same. The repeated iterations can be used for solution.

With advance in related technologies, the number of cascaded modules of H-bridge inverter module has gradually increased and the order of the system of nonlinear multivariable equations has gradually improved, which proposes higher requirements for computer algorithm to solve equations. The disadvantage of the original PSO that it will be affected by the initial value in solving the system of high-order nonlinear multivariable equations has gradually appeared. For such as problem, this paper gives an improved PSO with the selection rules of switching angle, which basically eliminates the impact of local convergence, and improves the computation accuracy of PSO.

## References

- [1] Fei Wanmin, Zhang Yanli, Wang Xuehua, Yuan Xinbo. Inclusion Relations of SHEPWM Solutions of Multi-level Inverter J. Proceedings of the CSEE, 2008,15: 61-66.
- [2] Ma Dongsheng, Lei Yongjun. Numerical Calculation Method. Beijing, China Machine Press, 2006.
- [3] Shi Feng, Wang Hui, Yu Lei, Hu Fei. Analysis of 30 Cases of Intelligent Algorithm, Beijing University of Aeronautics and Astronautics Press (2011).
- [4] Wen Jifang. Research on Inverter SHEPWM Based on Improved Particle Swarm Optimization D. Southwest Jiaotong University (2011).
- [5] Fei Wanmin, Ruan Xinbo, Zhang Yanli, Ju Rong. Research on Initial Value Problem of Specific Harmonic Elimination Pulse Width Modulation SHEPWM of Multi-level Inverter J. Proceedings of the CSEE (2007) 13: 87-92.