Accurate Estimation Technique for Adjacent Frequency Component

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Abstract. A spectrum correction algorithm is proposed in this paper to estimate frequency component of a dual-frequency signal accurately, this method is a kind of algebraic algorithm base on the model of dual-frequency or multi-frequency spectrum, and it is a real-time algorithm. The principle of the algorithm is explained in this paper, and also simulated with sampled signal, the result illustrate the effectiveness of the algorithm in parameter estimation, components in 1 frequency resolution can be distinguished in a high precision. The result shows that this algorithm is effective in estimating signal parameters, and it has a good application in engineering.

Introduction

In signal measure and parameter estimation, the ordinary method is using Discrete Fourier Transform (DFT) to estimate frequency, phase, and amplitude of each frequency component. The estimation error caused by spectrum leakage and fence effect is unavoidable because of signal truncation, especially to the short-time signal. Spectrum correction is very necessary in frequency estimation of measured signal for improve estimation accuracy, and there are some kinds of spectrum correction algorithm for single frequency signal. To the signal of dual-frequency or multi-frequency, the single frequency spectrum correction algorithms are still efficient if the frequency components separated more than 2 frequency resolution by estimating each frequency component one by one. The frequency resolution is relative with signal sampled length, short-time sampled signal provide big frequency resolution, then the frequency components are adjacent in one or two frequency resolution, the single frequency spectrum correction algorithms are unable to each frequency components.

This paper propose a spectrum correction algorithm for adjacent dual-frequency component in 1 or 2 frequency resolution, this algorithm provide analytic solution of each frequency and phase, and it is convenient for calculation, without iteration.

Theory of Dual-Frequency Spectrum Correction

Consider a signal constructed with L frequency components:

$$x_0(t) = \sum_{i=1}^{L} A_i \cos(2\pi f_i t + \varphi_i) \tag{1}$$

The spectrum in positive frequency domain is:

$$X_0(\omega) = \sum_{i=1}^{L} \frac{1}{2} A_i e^{j\varphi_i} \delta(\omega - \omega_i) \tag{2}$$

It can be also expressed with the spectral line l:

$$X_0(l) = \sum_{i=1}^{L} \frac{1}{2} A_i e^{j\varphi_i} \delta(l - l_i) \tag{3}$$

The signal is sampled and truncated which equivalent to a discrete window in spectrum analysis, suppose that the spectrum of a N point discrete window is $e^{-j\Omega(\frac{N-1}{2})}W(\Omega)$, or $e^{-j(\frac{N-1}{N})\pi l}W(l)$ which is expressed with spectral line $l = \frac{\Omega N}{2\pi}$. The spectrum of windowed signal is a convolution of original spectrum $X_0(l)$ and window spectrum $e^{-j(\frac{N-1}{N})\pi l}W(l)$:

$$X(l) = \sum_{i=1}^{L} \frac{1}{2} A_i e^{j\varphi_i} e^{-j(\frac{N-1}{N})(l-l_i)\pi} W(l-l_i) \tag{4}$$

Suppose $l_i = k_i + \delta_i$, k_i is a positive integer, and $|\delta_i| < 1$ is the spectrum correcting value, then the discrete spectrum is

$$X(k) = \sum_{i=1}^{L} \frac{1}{2} A_i e^{j(\varphi_i + \frac{N-1}{N}\delta_i \pi)} e^{-j(\frac{N-1}{N})(k-k_i)\pi} W(k-k_i-\delta_i) \tag{5}$$

set $$Z_i = \frac{1}{2} A_i e^{j(\varphi_i + \frac{N-1}{N}\delta_i \pi)} \tag{6}$$

For convenience of calculation, suppose that $e^{-j(\frac{N-1}{N})(k-k_i)\pi} \approx e^{-j(k-k_i)\pi} = \cos[(k-k_i)\pi]$, then

$$X(k) = \sum_{i=1}^{L} Z_i \cos[(k-k_i)\pi] W(k-k_i-\delta_i) \tag{7}$$

For the dual-frequency signal containing two spectrum peaks, $L = 2$, the peak spectral line k_1 and k_2 can be found in amplitude spectrum, Z_1 and Z_2 are complex according to formula(6), then the formula(7) has 6 unknown variable: $real(Z_1)$, $real(Z_2)$, $image(Z_1)$, $image(Z_2)$, δ_1 and δ_2, there could be 6 equations from 3 complex spectral lines and the spectrum $W(l)$ of window to solve Z_1, Z_2, δ_1 and δ_2, then the number of spectral line $l_1 = k_1 + \delta_1$ and $l_2 = k_2 + \delta_2$, which corresponding the frequency components, can be solved.

Suppose ϕ_1 and ϕ_2 are the argument of complex number Z_1 and Z_2, then the phase estimating result are $\varphi_1 = \phi_1 - \frac{N-1}{N}\delta_1 \pi$ and $\varphi_2 = \phi_2 - \frac{N-1}{N}\delta_2 \pi$.

Spectrum Correction Algorithm

Consider that m is the interval of the two peaks of dual-frequency spectral lines, the situation of $m = 1$ is discussed in this paper, there is only one peak in the spectrum because of adjacent dual-frequency, then $X(k_1 + 1) = X(k_2)$, $X(k_2 - 1) = X(k_1)$, to the spectral lines near the peak, formula(7) can be expanded as:

$$X(k_1) = Z_1 W(\delta_1) - Z_2 W(1+\delta_2) \tag{8-1}$$

$$X(k_1 - 1) = -Z_1 W(1+\delta_1) + Z_2 W(2+\delta_2) \tag{8-2}$$

$$X(k_2) = -Z_1 W(1-\delta_1) + Z_2 W(\delta_2) \tag{8-3}$$

$$X(k_2 + 1) = Z_1 W(2-\delta_1) - Z_2 W(1-\delta_2) \tag{8-4}$$

Substitute the spectrum of window, take rectangular window as an example, the spectrum of a N-point rectangular window is $W_R(e^{j\Omega}) = e^{-j(\frac{N-1}{2})} \frac{\sin(\Omega N/2)}{\sin(\Omega/2)}$, It can be also expressed as $W_R(l) = e^{-j(\frac{N-1}{N})\pi l} W(l)$ in spectrum line l, where $W(l) = \frac{\sin(\pi l)}{\sin(\pi l/N)}$.

Actually it is difficult to derive algebraic equations and solve δ_1 and δ_2 analytic by the spectrum of window given above, take $W(l)=\dfrac{\sin(\pi l)}{\pi l/N}$ as a approximation of window spectrum, then there will be:

$$Z_1=\frac{\pi}{N}[\frac{\delta_1(\delta_1-1)(\delta_2+1)X(k_1)-\delta_1(\delta_1-1)\delta_2X(k_2)}{(\delta_1-\delta_2-1)\sin(\pi\delta_1)}] \tag{9-1}$$

$$Z_2=\frac{\pi}{N}[\frac{-\delta_1\delta_2(\delta_2+1)X(k_1)+(\delta_1-1)\delta_2(\delta_2+1)X(k_2)}{(\delta_1-\delta_2-1)\sin(\pi\delta_2)}] \tag{9-2}$$

and

$$X(k_1-1)=\frac{2\delta_1(\delta_2+1)}{(\delta_1+1)(\delta_2+2)}X(k_1)+\frac{-\delta_2(\delta_1-1)}{(\delta_1+1)(\delta_2+2)}X(k_2) \tag{10-1}$$

$$X(k_2+1)=\frac{-\delta_1(\delta_2+1)}{(\delta_1-2)(\delta_2-1)}X(k_1)+\frac{2\delta_2(\delta_1-1)}{(\delta_1-2)(\delta_2-1)}X(k_2) \tag{10-2}$$

Separate formula (10-1) to real part and image part, set $A=\dfrac{2\delta_1(\delta_2+1)}{(\delta_1+1)(\delta_2+2)}$ and $B=\dfrac{-\delta_2(\delta_1-1)}{(\delta_1+1)(\delta_2+2)}$, then

$$(\delta_1+1)(\delta_2+2)=\frac{-2}{A+B-1} \tag{11-1}$$

$$(\delta_1+1)+(\delta_2+2)=\frac{2A+B-3}{A+B-1} \tag{11-2}$$

Therefore, δ_1+1 and δ_2+2 can be considered as the solution of equation

$$(A+B-1)\delta^2+(-2A-B+3)\delta-2=0 \tag{12}$$

A and B can be solved by the following formula:

$$\begin{bmatrix}A\\B\end{bmatrix}=\begin{bmatrix}\mathrm{Re}(k_1) & \mathrm{Re}(k_2)\\ \mathrm{Im}(k_1) & \mathrm{Im}(k_2)\end{bmatrix}^{-1}\begin{bmatrix}\mathrm{Re}(k_1-1)\\ \mathrm{Im}(k_1-1)\end{bmatrix} \tag{13}$$

$\mathrm{Re}(k)$ and $\mathrm{Im}(k)$ are the real part and image part of $X(k)$.

Then δ_1 and δ_2 can be solved by substitute A and B in equation (12).

Another group of δ_1 and δ_2 can be solved from formula (10-2), take the average value of two groups as the frequency correcting result.

To the situation of $m=2$ or other numbers, the frequency correcting result can be also solved by this algorithm.

Simulations

Verification by simulated signal:

Verify the algorithm by simulated dual-frequency signal, which is generated by Matlab program.

According to the real situation, sampling points $N=1024$, sampling rate is 51200Hz, then the signal length is 20*ms*, and frequency resolution is $\Delta f=50Hz$.

Table 1 lay out 4 group correcting result of dual-frequency signal in various frequency and phase without noise, the signal amplitude are normalized, and the dual-frequency are in 2 frequency resolutions.

Table 1 Frequency and Phase Correcting Result of a Signal without Noise

Amplitude	Frequency/Hz	Frequency Correcting Result/Hz	Phase/deg	Phase Correcting Result/deg
1	468	467.9604	30	30.2104
0.5	505	505.3673	45	44.8928
1	3726	3726.0015	0	0.1279
1	3742	3742.0680	45	45.0984
0.5	16355	16354.9494	60	60.2924
0.8	16378	16377.9966	45	44.9470
1	21425	21424.9151	60	60.4037
0.8	21506	21505.9496	90	90.2920

The simulation result shows that the accuracy of frequency correction could reach 1/10 of frequency resolution for both low frequency and high frequency.

The real signal contain noise, simulation is carried for noised signal, table 2 lay out correcting result of dual-frequency signal in various signal noise ratio(SNR).

Table 2 Frequency and Phase Correcting Result of a Signal in different SNR

Amplitude	Frequency /Hz	Phase/deg	SNR	Frequency Correcting Result /Hz	Phase Correcting Result /deg
1	4628	45	0dB	4629.715	46.7757
			6dB	4628.149	44.0322
			10dB	4628.113	44.3372
1	4582	90	0dB	4576.757	102.8087
			6dB	4579.865	93.5553
			10dB	4581.175	93.1802

The simulation result shows that the frequency correcting result has high accuracy with 10dB SNR, and the accuracy drop down with 6dB SNR or less.

Verification by real sampled signal:

The algorithm is also verified by sampled underwater electromagnetic signal, the signal frequency are $500Hz$ and $520Hz$, sample points $N = 2048$, sample rate is $100kHz$, then the frequency resolution is $\Delta f = 48.8281Hz$.

Table 3 shows the frequency correcting result of 3 sections of sampled signal.

Table 3 Frequency Correcting Result of a Sampled Signal

Frequency /Hz	Frequency Correcting Result /Hz		
500	499.5682	499.9674	498.5840
520	523.3040	526.3696	524.8773

Obviously this algorithm is efficient for the dual-frequency signal which frequency components are less than one frequency resolution, and the frequency correcting accuracy could reach 1/10 of frequency resolution.

Factors affecting the accuracy of frequency and phase estimation are mainly the following:

Firstly, the algorithm is based on the ideal dual-frequency signal model in frequency domain, the noise and other factors, which will have a bad effect on the frequency estimation, are not considered in the model.

Secondly, when the amplitude difference of the two frequency components is large, the small spectrum peak is easily submerged, causing the wrong results. There could be some methods that can help to identify the spectrum peak, such as the location window locked spectrum by prior knowledge, and so on.

Conclusion

In the case of short-time measured signal, the frequency resolution is large, the algorithm has a high accuracy of frequency estimation. The algorithm is fast and convenient in calculation, without iteration, and efficient for dual-frequency components less than one frequency resolution, the algorithm has a good prospect of engineering application.

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