FFT-based Fast-Response Quasi-Open Tracking Loop for GPS Receiver

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Abstract. This article discusses a new GNSS signal tracking architecture with a quasi-open tracking loop. Matched filtering is adopted for PRN code phase discrimination and FFT is used for frequency discrimination, which greatly shortens the duration of the pull-in step after acquisition. There is no loop filter any more. The discrimination results are fed back to code NCO and carrier NCO directly to modify the local signal in the next round. A Chirp-Z Transform (CZT) algorithm is considered for further improving the frequency resolution of FFT, which can also be computed using FFT. All the modules in this architecture are realized on hardware platform, showing characteristic advantages compared to traditional software tracking loop.

Introduction

In traditional tracking loop, carrier frequency and code phase of local signal is slowly and finely tuned by the discriminators and the loop filters. But this on the other hand reflects the drawbacks of traditional tracking loop that it takes relatively long time for the loop to recover from interference as well as the acquisition-to-tracking transition, i.e. pull-in stage. So it limits the flexibility of the loop.

Matched filtering and FFT algorithm is first put forward for fast acquisition [1]. But the parallel processing it is based on can also be applied in tracking [2]-[4]. It discriminates carrier frequency and code phase directly so that the modified quantity is elastic. Even if the frequency or code phase shifting is large, it still guarantees high adjusting speed. The whole structure can be realized on hardware, which saves the resources used for integrating a CPU on chip and accelerates the computing speed. Moreover, the tracking FFT module can multiplex the FFT module used in acquisition stage. So it won't take too many resources.

The main worry of the tracking structure in question is the frequency tracking precision because FFT is discrete. It is unpractical to do ultra long FFT, which takes too many resources and will cause the chip's area too large. The fact is that only the acquisition needs to search full spectrum. Tracking needs only searching the spectrum around the estimated frequency from acquisition. So spectrum refining methods should be introduced. There are several kinds of approaches to refine spectrum, such as some linear or nonlinear interpolation algorithms, Zoom-FFT and CZT [3], [5]. Simple interpolations may be too stiff with characteristics of frequency domain not been considered, and the algorithms' performances are difficult to assess. According to [5], CZT outperforms Zoom-FFT in not requiring filter and higher precision.

Tracking Structure

Acquisition. For better comprehension, it is necessary to introduce the acquisition method first, because tracking structure is built based on acquisition structure. In acquisition stage, matched filter and FFT is used for search in code phase dimension and frequency dimension respectively [1]. For GPS L1 signal, $10\,ms$ $128\,dots$ FFT is chosen to achieve a balance between performance and computing load. The acquisition process can be divided into following steps (Fig.1):

Suppose that the PRN number is already known (ignore the search in PRN dimension) and the sampling rate is f_s, local code NCO generates the $10\,ms$ corresponding PRN code, the length of which is $0.01 * f_s\,dots$.

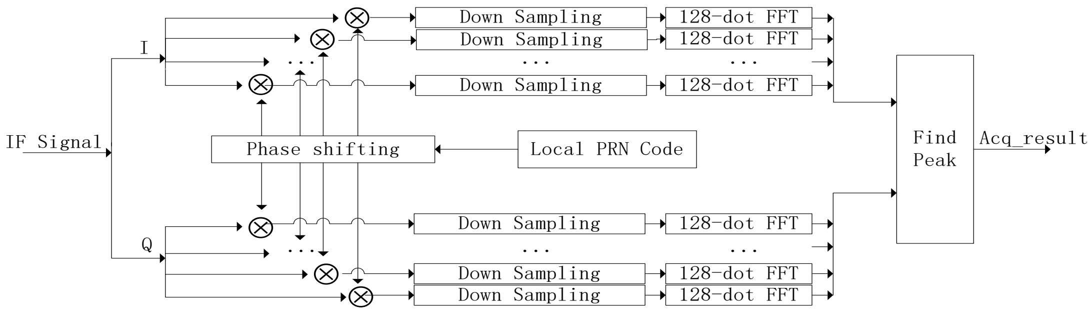

Fig. 1 Acquisition Structure

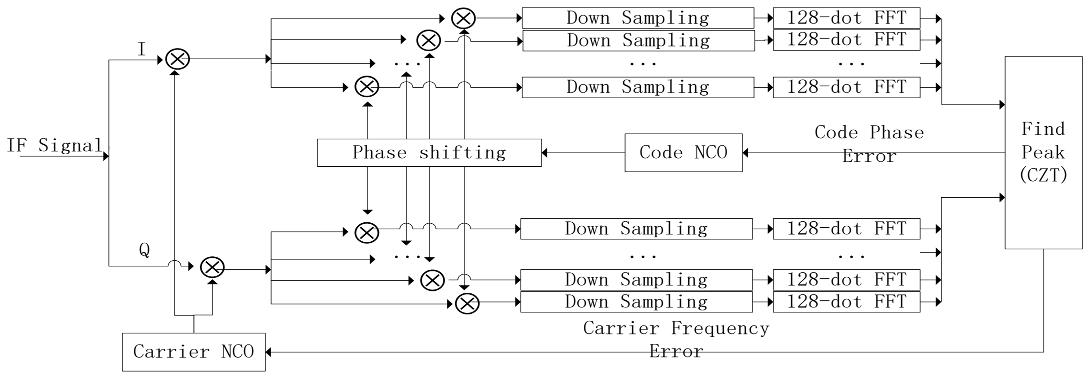

Fig. 2 Tracking Structure

1) The phase shifter shifts the local code's phase by $\frac{1}{2}chip$ at a time. At each phase the local code multiplies the received signal to get a result sequence $r(k,1{:}\,0.01*fs), k=1,2\,\ldots,2046$.
2) Piecewise summation is processed on these result sequences for down sampling, which also helps to improve SN_0. The sampling rate after down sampling is $12.8\,KHz$, which corresponds to the $\pm 6.4\,KHz$ frequency acquisition range. A 2046*128 array is built after down sampling:

$$s(k,n) = \sum_{i=\frac{0.01*fs}{128}*(n-1)+1}^{\frac{0.01*fs}{128}*n} r(k,i) \tag{1}$$

3) Do 128-point FFT on every row of the array above to get a 2046*128 array in frequency domain. The size of the frequency bin is $12.8\,KHz/128 = 100\,Hz$.
4) Find the location of the peak, the corresponding frequency and code phase of which is the acquisition results.

In practical application, the two adjacent $10\,ms$ will both be used in detection. Because if one $10\,ms$ is across a data bit, then the other will not be. The two correlation values are compared to select the larger one for detection.

Tracking. According to the acquisition (Fig. 1), the code phase error of the *acq_result* is $\pm\frac{1}{4}chip$ and the frequency error is $\pm 50\,Hz$. The result is still too rough for ranging so that the tracking stage follows. The tracking process is like a fine acquisition (Fig. 2).

In pull-in stage, for wider pull-in range, faster update rate and bit synchronization not realized, $1\,ms$ $128\,dots$ FFT is chosen to find relatively rough frequency estimation.

Then chirp-z transform algorithm is used to refine the frequency domain near the rough frequency.

For code phase pull-in, different from acquisition, only correlations near the rough code phase from acquisition rather than all code phase is computed based on sampling points, i.e. code phase shifting by 1 sample point at a time. If the IF is $4.123968\,MHz$ and the sampling rate is16.7667 MHz, then 1 code chip includes about 16 sampling points. The value of the IF and the sampling rate will be used all over the article. If correlations are made for every sampling point, then the code phase tracking resolution is $\pm\frac{1}{32}chip$, corresponding to $\pm 9m$ of pseudo-range. The pseudo-range resolution can be more refined with higher sampling rate.

Chirp-Z Transform Algorithm

Algorithm Description. The discrete Fourier transform (DFT) is the sampling Z transformation on the unit circle where the sampling points are uniformly-spaced, and the Chirp Z-transform (CZT), more generally, samples along spiral arcs in the Z-plane with alterable sampling space by changing the number of sampling points [5], defined as:

$$z_k = A \cdot W^{-k}, k = 0,1, \dots, M-1 \tag{2}$$

Where $A = A_0 e^{j\theta_0}$, is the start point of the sampling track with A_0 representing the distance between start point and the origin and θ_0 representing the angle of the start point; $W = W_0 e^{-j\varphi_0}$, is the ratio between two points with W_0 controlling the direction of the spiral arc and φ_0 being the phase difference of two adjacent points; M is the number of CZT points. The track's spiral arc bends to inside circle when $W_0 > 1$, while bends to outside circle when $W_0 < 1$. When $W_0 = 1$, the track is a circle arc.

With the property analyzed above, spectrum refining and peak detection is realized by taking out a specific spectrum section and doing the CZT. The steps are as follows [5, 6]:

1) Set $A_0 = W_0 = 1$, given sampling rate as f_s. Determine the spectrum section needed to be refined as $[f_1, f_2]$, and the value of M.
2) Map $[f_1, f_2]$ to unit circle as angle range$[\frac{2\pi f_1}{f_s}, \frac{2\pi f_2}{f_s}]$, and then start angle $\theta_0 = \frac{2\pi f_1}{f_s}$.
3) In the angle range M points need to be calculated, so

$$\varphi_0 = \left(\frac{2\pi f_2}{f_s} - \frac{2\pi f_1}{f_s}\right)/M \tag{3}$$

4) Calculate CZT according to (2).

Computing Method in Application. In practical engineering application, this calculation is implemented using FFT [5]. For the receiver structure in question, the sampling frequency is $128\,KHz$ after down sampling. The 128-point full spectrum FFT is calculated using the down sampled signal in the pull-in stage, with the frequency resolution being $1\,KHz$. Suppose the frequency corresponds to the peak is f_p, it is reasonable to set the CZT spectrum section as $[f_p - 500,\ f_p + 500]\,Hz$. The value of M depends on the requirements of the refining resolution. But remember, the bigger M, the more points of FFT and thus the more computing load. For example, if the resolution of $1.25\,Hz$ is required, M should be $1000/1.25 = 800$, which means 1024-point FFT should be computed ($1024 > 800 + 128 - 1$).

Parameter Selection. The frequency resolution influences the accuracy of the code phase estimation to eventually impact the pseudo-range accuracy. Low frequency resolution causes large frequency error. Hence the code doppler frequency error is large accordingly, which means there may be considerable difference between the length of $1\,ms$ local PRN code and $1\,ms$ received signal. Suppose Doppler frequency of the received signal is f_d, but local frequency estimation is $f_p + f_e$. Then code rate of received signal is:

$$f_{cr} = \frac{RF + f_d}{1540}\ Hz \tag{4}$$

Whereas code rate of local PRN code is:

$$f_{cl} = \frac{RF + f_d + f_e}{1540}\ Hz \tag{5}$$

When we calculate the number of sampling points corresponding to $1\,ms$ (1 code cycle) using local PRN code rate, we get:

$$dots_{1ms} = f_s * \frac{1540 * 1023}{RF + f_d + f_e}\ dots \tag{6}$$

But in the received signal such number of points corresponds to:

$$\frac{RF + f_d}{RF + f_d + f_e} code\ cycle < 1\ code\ cycle \tag{7}$$

So the correlation is done between 1 local code cycle and less than 1 received code cycle. Since RF is$1575.42\ MHz$, far larger than f_d andf_e, so this consideration can be ignored.

Beside the respect analyzed above, large left-over frequency after mixing with the local carrier will also decrease the overall value of the correlation according to (8)[7]:

$$\begin{aligned} I(n) &= aD(n)R(\tau)sinc(f_e T_{coh})cos\phi_e \\ Q(n) &= aD(n)R(\tau)sinc(f_e T_{coh})sin\phi_e \end{aligned} \tag{8}$$

Where $I,\ Q$ are coherent integration values from the in-phase and the quadra-phase paths; a is signal amplitude. $\mathrm{D(n)}$ is the modulated data bits. $R(\tau)$ is the autocorrelation of C/A code. The sinc function corresponds to the influence of the residual frequency error, which is determined by the product of frequency error and coherent integration time. With larger value of f_e and T_{coh} fixed, the integration is smaller. Hence reasonable value of M should be chosen to make a compromise between computing load and tracking precision.

Simulation Results

Diagrams for Detection. The acquisition and tracking is simulated on MATLAB. Doppler frequency is set to $219.945\ Hz$ ignoring dynamics. And code phase is set to $3.8\ chip$. $CN_0 = 40\ dB/Hz$. The acquisition process can be presented in Fig. 3.

The x-value of the peak is 3, which corresponds to the third frequency bin. The representative frequency of the third frequency bin is $200\ Hz$ (1^{st} bin is $0\ Hz$, 2^{nd} bin is $100\ Hz$). The y-value of the peak is 9, which corresponds to $4.5\ chip$ of code phase. The reason why the setting code phase is $3.8\ chip$ but the detection code phase is $4.5\ chip$ (the error is over $\pm\frac{1}{4}\ chip$) is the same as is described in III.C. We use nominal value of IF ($4.123968\ MHz$) and code frequency (1.023 MHz) to choose the two $10\ ms$ sequences. The 1^{st} $10\ ms$ sequence's start code phase is $3.8\ chip$. But due to the doppler frequency, it actually lasts for

$$\begin{aligned} \frac{IF * code\_length}{code\_freq * (IF + doppler)} * Num.ms &= \frac{4.123968MHz}{1.023MHz} \cdot \frac{1}{4123968 + 219.945} \cdot 1023 \cdot 10 \\ &= 0.009999467\ s \end{aligned} \tag{9}$$

Remaining

$$0.01s - 0.009999467\ s = 5.33 * 10^{-7}s \tag{10}$$

This $5.33 * 10^{-7}s$ is made up by the head of the 11th ms. So the 2^{nd} $10\ ms$ doesn't start from the start of the 11th ms, but the moment that has passed $5.33 * 10^{-7}s$ from the strat of the 11th ms. $5.33 * 10^{-7}s$ corresponds to

$$\frac{1023 * 5.33 * 10^{-7}}{\left(\frac{IF * code\_length}{code\_freq * (IF + doppler)}\right)} = 0.545\ chip \tag{11}$$

So the 2^{nd} $10\ ms$ start code phase is $3.8 + 0.545 = 4.345\ chip$. Also the 2^{nd} correlation value is larger than that of the 1^{st}. So the final code phase estimation is $4.5\ chip$. From the analysis above, we can not just set the code phase pull-in range as $\pm\frac{1}{4}\ chip$. it should be larger.

After acquisition, the estimated doppler frequency is $200\ Hz$. For bigger code phase pull-in range, the code phase shift is from $-2\ chip$ to $2\ chip$ centering on the code phase computed from the last 1 ms, so that is about 64 sampling points. Because the size of the frequency bin is 1 KHz, for high spectrum resolution, choose $M = 1000$ for CZT so that 1 CZT point corresponds to 1 Hz.

Fig. 4 shows the 64*128 array after FFT which represents the correlation value on every code phase and every frequency bin. The peak is obvious and the triangle area meets the autocorrelation of the gold PRN code. The peak indicates the first frequency bin and the modification of -1 in code phase. It means the frequency tracking error is less than 1 KHz and the code phase error is $-\frac{1}{16}chip$. So do CZT in $[0,1000\,Hz]$ to get Fig. 5. It shows that No.21 point has the biggest CZT results. Given that 1 point corresponds to 1 Hz, so the frequency error is 20 Hz. This result is quite accurate, because with the 200 Hz obtained from the acquisition stage, the total local frequency estimation is $200 + 20 = 220\,Hz$, leaving just 0.055 Hz of frequency error.

Long Period Simulation in Tracking. Fig. 6 and Fig. 7 present frequency tracking error and code phase error for 30 ms respectively. It is obvious to see, the two residual errors from acquisition are removed just in 1 ms. From the second ms, the errors start to be steady near zero.

The reason why the frequency error stays 0.055 Hz all the time is still the fence effect of FFT. Only that CZT samples the spectrum more densely to diminish the rest of error caused by the fence effect. Remember that the CZT frequency resolution is 1 Hz here.

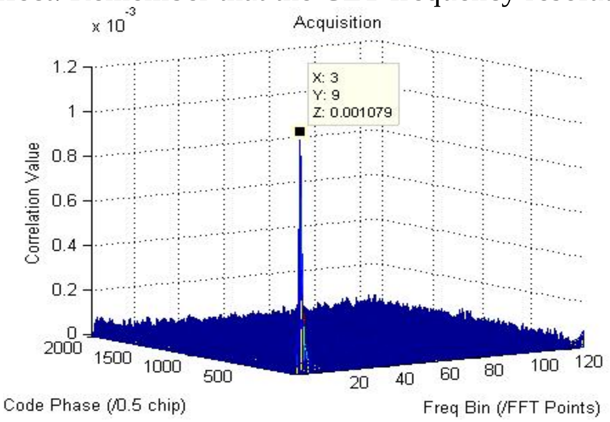

Fig. 3 Acquisition

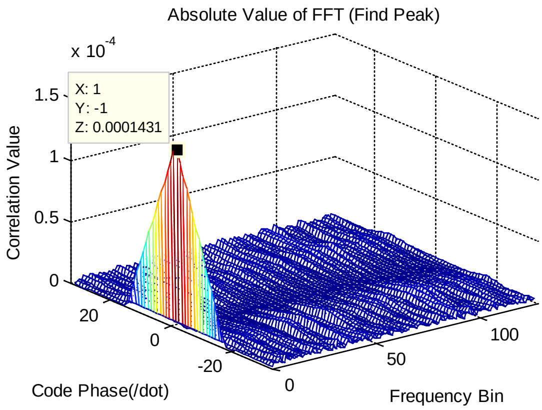

Fig. 4 Absolute value of FFT array

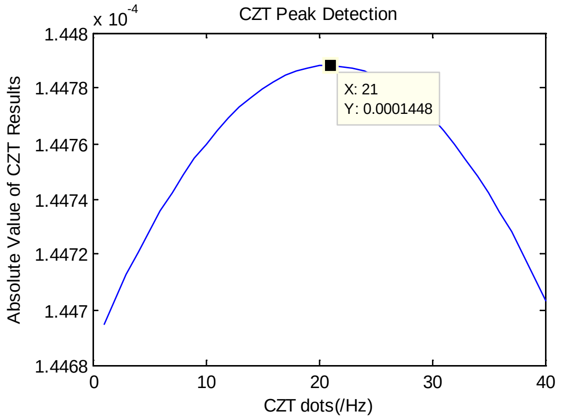

Fig. 5 CZT peak detection

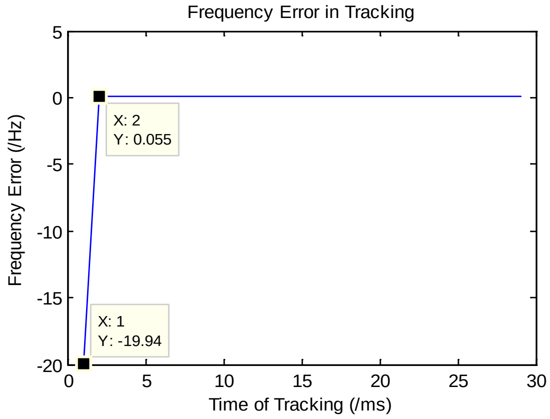

Fig. 6 Frequency error in tracking for 30 ms

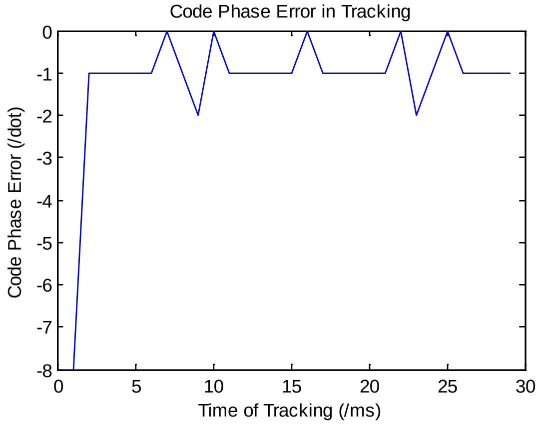

Fig. 7 Code phase error in tracking for $30\ ms$

Summary and Future Research

Primary analysis and simulation have shown that the tracking structure base on matched filter and FFT can achieve a faster reponse and will greatly shorten the pull-in time. The whole acquisition and tracking structure can be impremented in hardware with the FFT module multiplexed by acquisition,tracking and CZT. So there is potentiality of saving resources.

Beside the advantages mentioned above, there is research showing that this parallel correlate method improves tracking sensitivity [6]. In weak signal circumstances, SN_0 is too small to keep the traditional tracking loop locked. The main approach to improve sensitivity is extending the coherent integration time. But in traditional tracking loop, it will cause the loop update rate being too low to keep tracking in dynamic circumstances. Since the extension of the coherent integration time is irreplaceable for improving the sensitivity, the introduction of parallel correlation can save a lot of time from another respect. Matched filter and FFT are such methods to compute all the correlations in frequency dimension and the code phase dimension at the same time. Further research will be put on improving the sensitivity performance of this structure.

References

[1] Yuan Yu, “Research on Fast Algorithm of Spread Spectrum Code Acquisition Based on Matched Filter and FFT”, M.S. thesis, School of Electronics and Information Engineering, Harbin Institute of Technology, 2013.

[2] Jia Zhike, Wang Chi-shin, Fast fourier transform with down sampling based navigational satellite signal tracking, U.S. Patent 8,466,836, June 18, 2013.

[3] Krasner, Norman F, Method for open loop tracking GPS signals, U.S. Patent 6,633,255, October 14, 2003.

[4] Esther Anyaegbu, “A Frequency Domain Quasi-Open Loop Tracking Loop for GNSS Receivers”, Proceedings of the 19th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS 2006), Fort Worth, TX, September 2006, pp. 790-798.

[5] Men Aidong, Su Fei, *Digital Signal Processing*, 2nd ed., Beijing, China, 2011, pp. 124-128.

[6] Fan Xinhai, Zeng Xingxiang, Zhang Lixia, “Algorithm and Application of Spectrum Zoom Based on Chirp Z Transform”, *Journal of Academy of Armored Force Engineering*, vol. 26 No. 1, 2012.

[7] Xie Gang, *Principles of GPS and Receiver Design*, Beijing, China, 2011, pp. 380-381.