Positioning Technology Based On The Shadow of The Sun

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Abstract: Positioning technology based on the sun's shadow is a method through the analysis of the sun's shadow change of object to determine the location of object. We can use our own shadow to positioning ourselves when there is no GPS. But how to estimate it? First of all, we need to use secondary polynomial fitting to fit the shadow's length of different time and then we can know the moment that shadow is shortest, which corresponds to Beijing time (the time in place where the east longitude 120 degrees).And we also learn that the shortest shadow's length of the time is 12 o 'clock in the noon local time. So we can measure the longitude according to the longitude of the difference. For the latitude, this method use exhaustive technique, combining with different times of the azimuth that we known, a measurement of the different latitude of the sun's azimuth change, as a benchmark, comparing with the given data by MATLAB. Finally, the latitude that change laws are same is what we need.

The basic assumptions:

1. The earth's rotation speed is constant.
2. Ignore the dust in the air and the carbon dioxide to the sun's rays scattering and reflection.
3. Be measured objects are perpendicular to the surface.

Symbol description:

Table 1 Notation

Symbol	Meaning
h	Solar altitude
δ	Declination of sun
A	Solar azimuth
T	Solar hour angle
ϕ	Latitude
λ	Longitude
t_b	The observation Beijing time
t_0	Local time
N	Days accumulated value

Analysis

This paper mainly studies how to determine latitude and longitude of the straight rod under the condition of the exact date and length of rod's shadow that we only know.

For longitude, we know the exact date when we measured, so we can infer longitude of rod's location, as long as we can determine the time when shadow is shortest, combining with geographical knowledge. For latitude, because of rod's length unknown, solar altitude is unknown. Thus, we think to iterate rod's length and we can obtain solar altitude. Finally, for each latitude value, we will get shadow length of corresponding period of time, combining with latitude solving formula, comparing with the given data, and what similarity is high is what we want.

Method

Longitude:

●Measurement of shadow's length and data fitting

First, we need to measure the length of the shadow in a certain period of time. For example:

Table 2 the straight-bar shadow length value. Date: April 18, 2015

time	14:42	14:45	14:48	14:51	14:54	14:57	15:00
shadow	1.149	1.182	1.215	1.249	1.283	1.318	1.353
time	15:03	15:06	15:09	15:12	15:15	15:18	15:21
shadow	1.389	1.426	1.463	1.501	1.540	1.580	1.620
time	15:24	15:27	15:30	15:33	15:36	15:39	15:42
shadow	1.661	1.703	1.746	1.790	1.835	1.881	1.928

●Data fitting to determine the shadow length changes over time

As far as we know, on the same day, with the increase of time, the shadow length get smaller before they are bigger, similar to the quadratic parabola, and shortest shadow appear on the noon. In order to know the time when the shortest shadow appears, we should get the whole curve of mathematical description. But we just know change of the shadow from 14:42 to 15:42. So we need first to fit given data to describe the whole curve and get time corresponding to lowest point.

We use MATLAB curve fitting tool to fit and result are as follow:

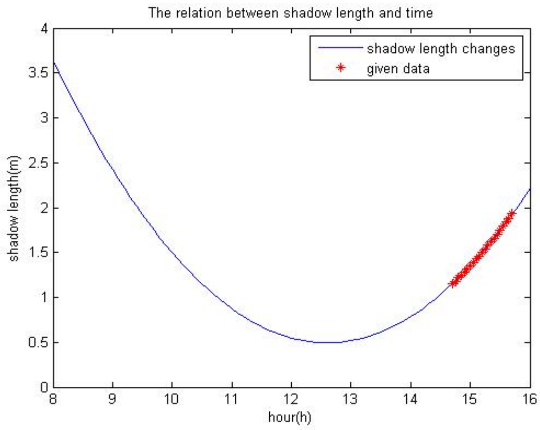

Figure 1 Curve fitting results

Fitting parameters are shown in the following table：

Table 3 Fitting parameters

Fitting types	Order number	SEE	R-square
Polynomial	two	1.6489e-05	1.0000

According to Figure 2 and Table 3, we can know, Curve fitting effect is better. Therefor, it can use to describe change of shadow length over the time. Mathematical expressions are as follows:

$$l = 0.1489 \times t^2 - 3.752 \times t + 24.13$$

When time is $t = -b/2a$, namely 12:36 on the noon, the straight rod's shadow is shortest. The shortest shadow is $l_{\min} = (4ac - b^2)/4a$, namely, 0.4942 meters.

●To estimate longitude

We know that one day the largest Angle of the sun, the shadow of the shortest time is 12 o'clock local time. We might as well to consider that this day (e.g. April 18, 2015) of the shortest shadow of time is t_0 $(t_0 = 12)$ local time. According to above results, we can get the time of shortest shadow (Beijing time) is 12:36, writing for t_b. Consequently, the time difference between local time and Beijing time Δt is:

$$\Delta t = t_b - t_0$$

In the same way, we consider that the longitude of the rods in place is λ_o.The difference between 120oE and λ_0 is:

$$\Delta\lambda = 120 - \lambda_0$$

According to the geographical knowledge, the relationship between longitude and time are as follows: ①time in same longitude is equal; ② The longitude 1 degree difference, time difference of 4 min. Longitude and time lag of logic relations can be expressed as follows:

$$\frac{\Delta t}{\Delta\lambda} = 4\,\text{min}/^\circ$$

Thus there are:

$$\frac{t_b - t_0}{120 - \lambda} = 4$$

and

$$\lambda = 120 - \frac{t_b - t_0}{4}$$

Finally, bring the above values to calculate the longitude where rod is. Result: 111° 0′ 51″ E.

Latitude

Because we only know shadow length and Beijing time when we measured, and we don't know straight rod length and the declination of sun, we think the exhaustive method is adopted. We can substitute the rod length and latitude position one by one into the model. Through that, we can find shadow length in different latitude and different rod length. Comparing with given data, latitude that similarity is higher is requested.

●The calculation of the declination of sun and solar hour angle

δ means the declination of sun, it can be found in the table of the declination, it can be also calculated by formula[1]:

$$\delta = [0.006918 - 0.39912\cos\theta_0 + 0.070257\sin\theta_0$$
$$-0.002697\cos 2\theta_0 + 0.000907\sin 2\theta_0 - 0.002697\cos 3\theta_0 + 0.001480\sin 3\theta_0] \times 180/\pi$$

In the formula,

$$\theta_0 = 360N/365$$

T means solar hour angle. It can be also calculated by formula:

$$T = (t_b - 12) \times 15^0$$

●The relationship between solar altitude and latitude

In a similar way, we can know the relationship between solar altitude h and latitude ϕ:

$$\sinh = \sin\phi\sin\delta + \cos\phi\cos\delta\cos T$$

●Use MATLAB to solve it

From the above analysis, we can know the relationship between and among solar altitude, latitude, declination of sun and solar hour angle. Declination of sun and solar hour angle can be obtained by exact date when we measured the shadow length. So we can test the latitude and rod length to exhaustion and get corresponding shadow. And then compared with the given data, latitude that shadow changes is consistent is requested. Here, we use MATLAB [2] to approximate latitude to seek the latitude that shadow changes calculated by iteration is equal to shadow known.

Results are as follows:

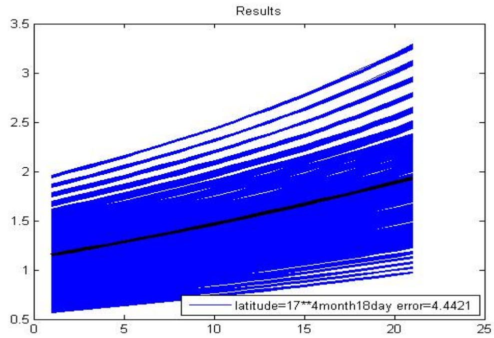

Figure 2 The iteration results

Results

What we finally determine is as follows: [3]

Longitude: 111°0′ 51″ E	Latitude: 17°N	Error=4.4421％

Summary

In conclusion, we know that, we can use the length of the shadow to position ourselves on the earth. It provides a method to locate our own latitude and longitude when there is no GPS. And by the text of the models, know its accuracy is accurate. So it's a feasible method.

References

[1] Guoqiang Zhou, Environmental Impact Assessment, Wuhan, China, 2011.

[2] Haibin Zhao, Application of MATLAB, Beijing, 2012.

[3] Information on http://www.gpaspg.com/maps.htm