

Modeling and Simulation of the Silicon Micro-machined Gyroscope Mode-Coupling System

Qirui Li^{1, a}, Weihong Jin^{1, b}, Haimei Xu^{1, c}

¹Chongqing Communication Institute, Chongqing 400035, China

^a18680950214@163.com, ^b168106572@qq.com, ^cxuhaimei@uestc.edu.cn

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Abstract: This paper focuses on the mode-coupling characteristics of gyroscope, and tries to rebuild a more perfect system model of silicon micro-machined gyroscope. It analyses the factors causing the coupling motions between drive-mode and detect-mode of gyroscope system. The mathematical model is deduced out based on the dynamic model and dynamic equations of gyroscope system. And the simulation resulted by MATLAB is in good agreement with the experiment results, which verifies the correctness of the mathematical model of silicon micro-machined gyroscope mode-coupling system.

Introduction

Silicon micro-machined gyroscope can be widely used in military and civilian fields for its smaller size, lower power consumption and lower price [1]. To improve its controlling and measuring accuracy has always been one of the main research contents in silicon micro-machined gyroscope's development.

System model is the foundation of system controlling and measuring, and hence the accuracy of gyroscope's mathematical model has a great influence on the performance of silicon micro-machined gyroscope system. Building a more accurate mathematical model of silicon micro-machined gyroscope is therefore a way to improve gyroscope system's controlling and measuring accuracy.

The drive-mode and detect-mode of silicon micro-machined gyroscope are coupled [2]. However, they are usually considered as two full decoupling modes, while the mathematical model of gyroscope is equivalent to a second order system model approximately, regardless of non-ideal factors such as mode-coupling. To improve the accuracy and adequacy of gyroscope system model, this paper tries to rebuild a more accurate mathematical model of silicon micro-machined gyroscope concerning the factors of drive-mode and detect-mode coupling, which can provide forceful support for the future studies on the silicon micro-machined gyroscope mode-coupling system.

Idealized Mathematical Model of Silicon Micro-machined Gyroscope System



a. Idealized equivalent dynamic model

b. Non-idealized equivalent dynamic model

Fig.1 Equivalent Dynamic Model

Equivalent Dynamic Model. Ideally, silicon micro-machined gyroscope system is equivalent to a second order dynamic system as shown in Fig.1.a, where x -direction represents *drive-mode* direction, y -direction represents *detect-mode* direction, and z -direction represents gyroscope's *angular velocity*

measuring direction. D_x , D_y and k_x , k_y indicate the damping coefficients and elasticity coefficients in *drive-mode* direction and *detect-mode* direction, respectively. And Ω_z indicates the angular velocity in z -direction. In the ideal case, the moving mode in x -direction of the mass m and its moving mode in y -direction are considered as full decoupled, which means that either one of the movements in *drive-mode* or *detect-mode* direction won't affect the movement in another direction.

Equivalent Dynamic Equations. Suppose that a driving force F_d is imposed on the mass m in the x -direction, and the equivalent dynamic system (gyroscope) rotates around z -direction. In the reference xyz -axes system, the mass m is subjected by *Driving Force*, *Damping Force*, *Elasticity Force* and *Coriolis force* in the xy -plane. Based on the above force analysis and *Newton's Second Law*, the dynamic equations are derived out, and the idealized mathematical model of silicon micro-machined gyroscope system is often written as the following simultaneous differential equations[3]:

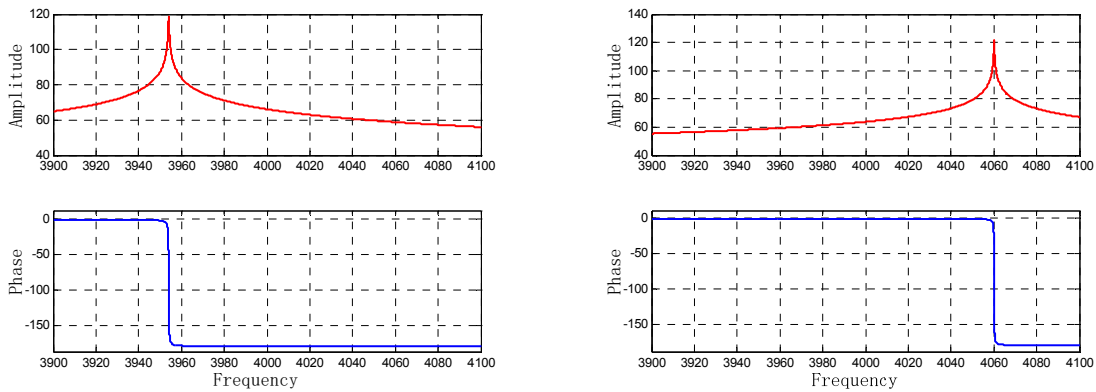
$$\begin{cases} m\ddot{x} + D_x\dot{x} + k_x x = F_d \\ m\ddot{y} + D_y\dot{y} + k_y y = -2m\Omega_z\dot{x} \end{cases} \quad (1)$$

Analysis of Silicon Micro-machined Gyroscope's Mode-Coupling Characteristics

Frequency Domain Analysis. According to Equation(1), supposing to impose driving force F_{dx} in the *drive-mode* direction and assuming $\Omega_z=0$, the system transfer function of gyroscope's drive-mode is derived by *Laplace Transformation* as Equation(2). In the same way, the system transfer function of the detect-mode is written as Equation(3). Plug the gyroscope parameters into the transfer functions, the frequency response curves are obtained by MATLAB as shown in Fig.2.

$$G_x(s) = \frac{X(s)}{F_{dx}(s)} = \frac{1}{ms^2 + D_x s + k_x} \quad (2)$$

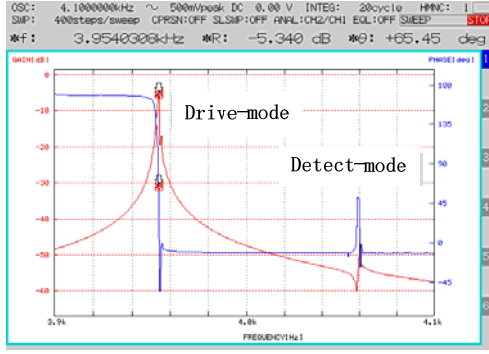
$$G_y(s) = \frac{X(s)}{F_{dy}(s)} = \frac{1}{ms^2 + D_y s + k_y} \quad (3)$$



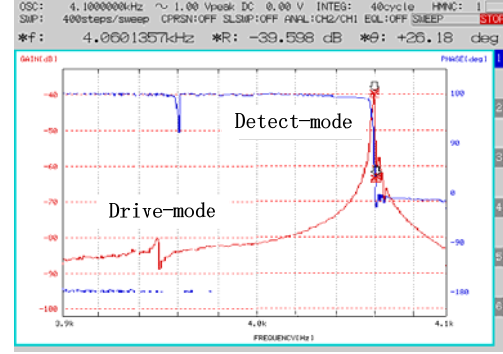
(a) Frequency response curves of $G_x(s)$

(b) Frequency response curves of $G_y(s)$

Fig.2 Frequency Response Curves of $G_x(s)$ and $G_y(s)$



(a) Measurement result of drive-mode



(b) Measurement result of detect-mode

Fig.3 Frequency Response Measurement Results

Frequency Response Measurement. However, the results of the frequency response measuring experiment for a certain real silicon micro-machined gyroscope system is shown as Fig.3. It indicates that the frequency response curves of drive-mode has a step change at the resonance frequency of detect-mode, while the curves of detect-mode also has a step change at the resonance frequency of drive-mode, which implies that the drive-mode and detect-mode of the gyroscope system are coupling.

Mode-Coupling Characteristic Analysis. Silicon micro-machined gyroscope is one kind of vibrational sensors. It is based on the vibrational motions of gyroscope's silicon micro-machined vibrational structure to sense the angular velocity. Ideally, the vibrational motions of gyroscope's drive-mode and detect-mode are completely orthogonal, while the vibrational motions are coupling accurately[2]. There are three main sides of non-ideal factors causing the coupling motions between drive-mode and detect-mode: ①Structural asymmetry. Because of mismachining tolerance and material defect, gyroscope's silicon micro-machined vibrational structure is asymmetric, which arouses the asymmetry of rotational inertia and non-idealized movements. ②Unbalanced driving force. The driving forces are imposed on the vibrational structure through driving electrodes. Due to structural and circuit errors, the driving forces imposed on the structure are unbalanced, which causes non-idealized dynamic coupling movements. ③Principium factor. In the design process of silicon micro-machined gyroscope system, some approximate deposits in principle lead to the neglect of the coupling characteristics of the system itself, including the coupling motions between drive-mode and detect-mode.

Mathematical Model Deduction for the Mode-Coupling System

Considering the mode-coupling characteristics of the silicon micro-machined gyroscope system, its equivalent dynamic model is shown as Fig.1.b, which identifies the motions in drive-mode direction and detect-mode direction are non-orthogonal. Assuming the resultant forces in the x -direction and y -direction are respectively F_x , F_y , Equation (4) and (5) are written as following, based on *Newton's Second Law*. In the equations, D_{xx} and k_{xx} are the components of drive-mode's damping and elasticity coefficients in x -axis, ΔD_y and Δk_y are the components of detect-mode's damping and elasticity coefficients in x -axis, D_{yy} and k_{yy} are the components of detect-mode's damping and elasticity coefficients in y -axis, ΔD_x and Δk_x are the components of drive-mode's damping and elasticity coefficients in y -axis, a_x and a_y indicate the absolute accelerations of the mass m in x -direction and y -direction.

$$ma_x = F_x - k_{xx}x - D_{xx}\dot{x} - \Delta k_y y - \Delta D_y \dot{y} \quad (4)$$

$$ma_y = F_y - k_{yy}y - D_{yy}\dot{y} - \Delta k_x x - \Delta D_x \dot{x} \quad (5)$$

Refer to the deduction process of *Coriolis* acceleration, the absolute acceleration \mathbf{a} of the mass m is shown as Equation (6) [4]. \mathbf{r} is the absolute vector of the mass m in the inertial reference frame, Ω indicates the angular velocity of sensor system.

$$\mathbf{a} = [a_x \quad a_y \quad a_z]^T = \frac{\partial^2 \mathbf{r}}{\partial t^2} + 2\Omega \times \frac{\partial \mathbf{r}}{\partial t} + \frac{\partial \Omega}{\partial t} \times \mathbf{r} + \Omega \times \Omega \times \mathbf{r} \quad (6)$$

Given $\mathbf{r} = [x \quad y \quad z]^T$, $\Omega = [\Omega_x \quad \Omega_y \quad \Omega_z]^T$ then Equation (6) is rewritten as following:

$$\begin{aligned} \mathbf{a} &= \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} + 2 \begin{bmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{bmatrix} \times \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} + \begin{bmatrix} \dot{\Omega}_x \\ \dot{\Omega}_y \\ \dot{\Omega}_z \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{bmatrix} \times \begin{bmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} \\ &= \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} + \begin{bmatrix} 0 & -2\Omega_z & 2\Omega_y \\ 2\Omega_z & 0 & -2\Omega_x \\ -2\Omega_y & 2\Omega_x & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} + \begin{bmatrix} -(\Omega_y^2 + \Omega_z^2) & \Omega_x \Omega_y - \dot{\Omega}_z & \Omega_x \Omega_z + \dot{\Omega}_y \\ \Omega_x \Omega_y + \dot{\Omega}_z & -(\Omega_x^2 + \Omega_z^2) & \Omega_y \Omega_z - \dot{\Omega}_x \\ \Omega_x \Omega_z - \dot{\Omega}_y & \Omega_y \Omega_z + \dot{\Omega}_x & -(\Omega_x^2 + \Omega_y^2) \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \end{aligned} \quad (7)$$

So the components in x-axis and y-axis of the absolute acceleration \mathbf{a} are written as:

$$a_x = \ddot{x} - 2\Omega_z \dot{y} + 2\Omega_y \dot{z} - (\Omega_y^2 + \Omega_z^2)x + (\Omega_x \Omega_y - \dot{\Omega}_z)y + (\Omega_x \Omega_z + \dot{\Omega}_y)z \quad (8)$$

$$a_y = \ddot{y} + 2\Omega_z \dot{x} - 2\Omega_x \dot{z} + (\Omega_x \Omega_y + \dot{\Omega}_z)x - (\Omega_x^2 + \Omega_z^2)y + (\Omega_y \Omega_z - \dot{\Omega}_x)z \quad (9)$$

The motion along z-axis is not taken into account by the equivalent dynamic model, so it's considered that:

$$z = \dot{z} = 0 \quad (10)$$

Take Equation (8)(9)(10) into Equation (4)(5), then they convert to:

$$m\ddot{x} + D_{xx}\dot{x} + (k_{xx} - m\Omega_y^2 - m\Omega_z^2)x = F_x - (-2m\Omega_z + \Delta D_y)\dot{y} - (m\Omega_x \Omega_y - m\dot{\Omega}_z + \Delta k_y)y \quad (1)$$

$$m\ddot{y} + D_{yy}\dot{y} + (k_{yy} - m\Omega_x^2 - m\Omega_z^2)y = F_y - (2m\Omega_z + \Delta D_x)\dot{x} - (m\Omega_x \Omega_y + m\dot{\Omega}_z + \Delta k_x)x \quad (1)$$

Combining Equation (11) and (12), the matrix equation, which presents the mathematical model of silicon micro-machined gyroscope mode-coupling system, is obtained as following:

$$\begin{aligned} &\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} + \begin{bmatrix} D_{xx} & \Delta D_y \\ \Delta D_x & D_{yy} \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} k_{xx} - m\Omega_y^2 - m\Omega_z^2 & m\Omega_x \Omega_y - m\dot{\Omega}_z + \Delta k_y \\ m\Omega_x \Omega_y + m\dot{\Omega}_z + \Delta k_x & k_{yy} - m\Omega_x^2 - m\Omega_z^2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \\ &= \begin{bmatrix} 0 & 2m\Omega_z \\ -2m\Omega_z & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} F_x \\ F_y \end{bmatrix} \end{aligned} \quad (1)$$

Assuming the above equation is equal to:

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 & 2m\Omega_z \\ -2m\Omega_z & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} F_x \\ F_y \end{bmatrix} \quad (1)$$

Then, the equations are derived by *Laplace Transformation* as following:

$$X(s) = \frac{(2m\Omega_z - D_{12})s - k_{12}}{ms^2 + D_{11}s + k_{11}} Y(s) + \frac{1}{ms^2 + D_{11}s + k_{11}} F_x(s) = G_{xy}(s)Y(s) + G_{F_x}(s)F_x(s) \quad (15)$$

$$Y(s) = \frac{(-2m\Omega_z - D_{21})s - k_{21}}{ms^2 + D_{22}s + k_{22}} X(s) + \frac{1}{ms^2 + D_{22}s + k_{22}} F_y(s) = G_{yx}(s)Y(s) + G_{F_y}(s)F_y(s) \quad (16)$$

According to Equation (15) and (16), the controlling structure diagram of silicon micro-machined gyroscope mode-coupling system is shown as Fig.4.

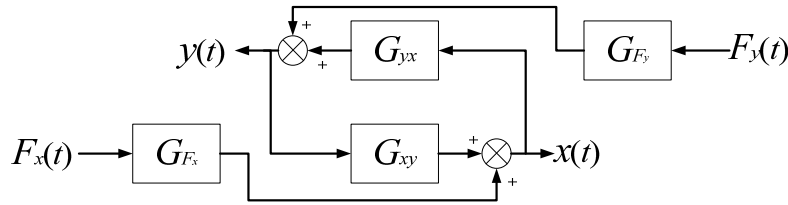


Fig.4 Controlling Structure Diagram of Mode-Coupling System

Therefore, depending on the mathematical model and controlling structure diagram, it is represented that silicon micro-machined gyroscope system is a multiple input multiple output (MIMO) system.

Simulating Verification by MATLAB

Based on Equation (15) and (16), the equations are obtained as following:

$$X(s) = \frac{G_{F_x}(s)}{1 - G_{xy}(s) \cdot G_{yx}(s)} F_x(s) + \frac{G_{xy}(s) \cdot G_{F_y}(s)}{1 - G_{xy}(s) \cdot G_{yx}(s)} F_y(s) \quad (17)$$

$$Y(s) = \frac{G_{yx}(s) \cdot G_{F_x}(s)}{1 - G_{xy}(s) \cdot G_{yx}(s)} F_x(s) + \frac{G_{F_y}(s)}{1 - G_{xy}(s) \cdot G_{yx}(s)} F_y(s) \quad (18)$$

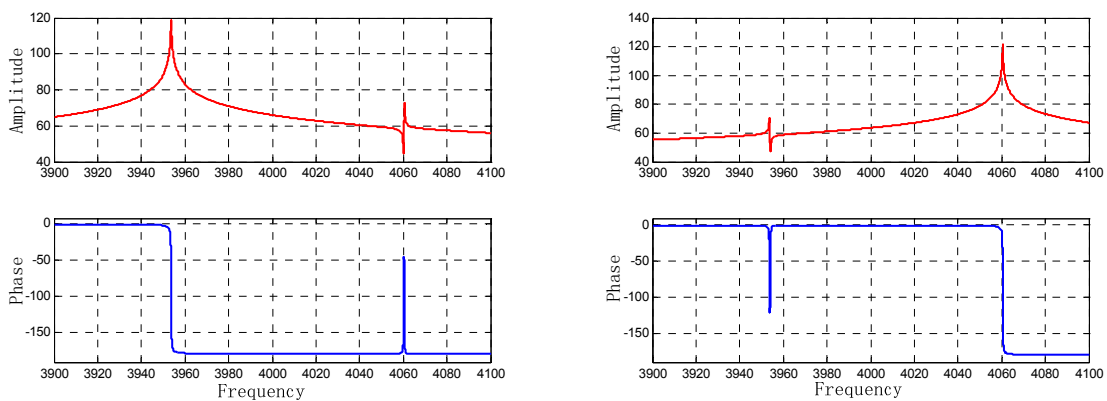
During the experiment of frequency response measurement to gyroscope's drive-mode, $F_y=0$ and $\Omega_z=0$, thus the transfer function of drive-mode is written as:

$$G_X(s) = \frac{X(s)}{F_x(s)} = \frac{G_{F_x}(s)}{1 - G_{xy}(s) \cdot G_{yx}(s)} = \frac{ms^2 + D_{22}s + k_{22}}{(ms^2 + D_{11}s + k_{11})(ms^2 + D_{22}s + k_{22}) - (D_{12}s + k_{12})(D_{21}s + k_{21})} \quad (19)$$

During the experiment of frequency response measurement to gyroscope's detect-mode, $F_x=0$ and $\Omega_z=0$, thus the transfer function of detect-mode is written as:

$$G_Y(s) = \frac{Y(s)}{F_y(s)} = \frac{G_{F_y}(s)}{1 - G_{xy}(s) \cdot G_{yx}(s)} = \frac{ms^2 + D_{11}s + k_{11}}{(ms^2 + D_{11}s + k_{11})(ms^2 + D_{22}s + k_{22}) - (D_{12}s + k_{12})(D_{21}s + k_{21})} \quad (20)$$

Plug the gyroscope parameters into the transfer functions (19) and (20), the frequency response curves are obtained by MATLAB as shown in Fig.5.



(a) Frequency response curves of $G_X(s)$

(b) Frequency response curves of $G_Y(s)$

Fig.5 Frequency Response Curves of $G_X(s)$ and $G_Y(s)$

Compared with Fig.3, the characteristics of the frequency response curves in Fig.5 fit the results of the frequency response measuring experiment well, which verifies the correctness of the above mathematical model of silicon micro-machined gyroscope mode-coupling system.

Summary

Comparing between the idealized frequency response curves with the experiment results of frequency response measurement, the mode-coupling characteristics of silicon micro-machined gyroscope system are discussed in this paper. The non-ideal factors are analyzed from three main perspectives: structural asymmetry, unbalanced driving force, and principium factor. According to the analysis and the non-idealized dynamic model, paper deduced the dynamic equations and transfer functions of mode-coupling gyroscope system, and obtained the mathematical model of silicon micro-machined gyroscope mode-coupling system. Through the simulating experiment by MATLAB, the validity of the mathematical model was discussed, which fit the experimental results well.

Acknowledgements

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