

# Study on Estimating Bidding Rivals' Average Tender Offer under the Condition of the Stochastic Compound Base Price

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**Abstract.** The chance of acceptance of the bid and bid winner's profit margins is to some extent determined by tender offer under the condition of the stochastic compound base price. Thus estimating bidding rivals' average tender offer becomes a key issue of the project bidders. Taking maximum probability of winning bid as the main object of studying, this article considers the standard form of the stochastic compound base price bid evaluation, and constructs an estimating model of bidding rivals' average tender offer through improving and integrating the existing model. Then method of probability theory is applied to solve the model and bidding rivals' average tender offer is obtained. The results of this study solve to a certain extent stochastic problem in theoretical model of optimal average tender offer, as well as provided a little enlightenment about estimating bidding rivals' average tender offer under the condition of the stochastic compound base price.

## Introduction

The formation of the price of project bidding is essentially a game process among bidders. Typically, the bid winner's profit margins is affected by the behavior of other competing bidders to a certain degree. While making a tender offer, one bidder should take into account bid evaluation methods according to tender document, as well as the response of other bidders.

Stochastic compound base amount method is often considered as an improvement of the lowest bid approach in bidding evaluation methods by setting a reasonable base price. Weight coefficient of base bid price and floating rate are randomly determined by tenderer or his representatives at tender opening. Moreover, the potential bidders' tender offer based on risk preference shows stochastic characterization. As a consequence, estimating bidding rivals' average tender offer accurately is the most urgent problem to be solved, as the bidding price is not only determined by the Budget Norm Standards, but other potential bidders' tender offer.

## Literature Review

Estimating tender offer is a rather widely explored topic in the literature. In the following, we review mainly two streams of literature related to our work: bidding decision-making model based on risk assessment and bidding decision-making model based on game theory.

The earlier studies mainly concentrate on systematical risk assessment. Fu, H.Y. (1995) explored a bidding decision-making model based on risk assessment to address the relationship between risk and benefit. Chua, D.K.H. & Li, D.Z. (2000) extended the previous work by introducing five types risk as critical factors, including policy, finance, market, technology and time value of money. Seung, H.H. & James Diekmann, E. (2001) formulated a model of international projects considering value risk, the probability of risk, risk variance and abstract risk. While the aforementioned studies mostly focused on finding risk factors to increase the bidders' profits, few studies refer to other bidding rivals' strategy, as well as maximum probability of acceptance bidding.

The second stream of research is about bidding decision-making model based on game theory. The majority of this literature concentrates on bidding rivals' strategy in the context of information asymmetry. Bostleman, R.L. (1969) firstly applied game theory to the course of project bidding by combining rival tender history information and specific environment. Hao, L.P. (2002) then studied a fuzzy theory to incomplete information static game model, discussing maximum probability of acceptance bid-ding and increasing the bidders' profits. Fan, J.Q. (2008) built dual game model and Multi-player game model based on incomplete information static game theory and focused on the relationship between the number of bidders and bid outcomes. Jiang, G.H. (2009) formulated a matrix game model and presented finite tendering price when given certain step size, thus determined the final tender offer under the method of the reasonable lowest evaluated bid price. Although the prior literature include other rivals' behavior, these studies have limited knowledge of bid-ding rivals' strategy.

The main contributions of this paper are summarized as follows. First, we analyze the behavior features of the bidding game and makes a comprehensive discussion about competitor analysis. Second, we considers the standard form of the stochastic compound base price bid evaluation, and constructs an estimating model of bidding rivals' average tender offer through improving and integrating the existing model. Third, the results of this study solve to a certain extent stochastic problem in theoretical model of optimal average tender offer, as well as provided a little enlightenment about estimating bidding rivals' average tender offer under the condition of the stochastic compound base price.

## Model Assumptions

Tendering offer under the condition of the stochastic compound base price, meeting the necessary conditions (i.e., players, strategy space, payoff structures) to make one game possible, has attribute of an in-complete information static game model

## Players

We first consider all participants include one tenderee and tenders under the condition of the stochastic compound base price. And the tenderee implement an open tendering to determine the winner of a certain project through tenders who have passed the prequalification and desired to achieve profit maximization. We assume that in the bidding process, the tenders valuation  $V_i (i=1,2, \dots, n)$  is identically and independently from other tenders. For tractability we further assume  $V_i \sim U[0,1]$  with p.d.f.  $f(x)$  and c.d.f.  $F(x)$  Notice that we do not consider risk preference in this paper and regard all players as risk-neutral participants. And this assumption is commonly used in tendering offer literature.

## Strategy Space

We first consider all tendering strategies symmetrical, which means the optimal function of each tender are identical. When an ad hoc tender being considered, it is concluded that certain tendering price is strictly more than that of others for the reason that all tenders valuation vary with their own costs. In generalized, the strategy set of each player is not a constant set which is independent of the strategy that others choose. We have the following symmetrical equilibrium strategy function  $b_i=b_i^*$ , strategy set  $s_i \in [0,1]$ , strategy space  $S=(s_1, s_2, s_3, \dots, s_n)$

## Payoff Structures

The following hypothesis are that tendering price be differential and strictly monotonously increasing. Given winning rules under the condition of the stochastic compound base price, it was proved that tenders whose tendering price is the nearest to the compound base price win a bid. Thus we have the following payoff function:

$$u_i(b_i, b_j, v) = \begin{cases} b_i - v_i, & \text{if } H - b_i < |H - b_j| \text{ and } (i \neq j) \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

Where  $H$  represents the mean value of all the tendering price. This captures the intuitive nature of tendering price respect to others tendering strategy. We do not consider  $H - b_i = |H - b_i|$  for the reason that it is rare that two tendering offer price is identical.

## MODEL ANALYSIS

In this section, we built the pricing model based on the expected profit maximization.

$$Eu_i = (b_i - c_i) \prod_{j=1, j \neq i}^{n+1} \text{prob} \left[ \left( H - b_i < |H - b_j| \right) \middle| \left( H > b_i \right) \right] \quad (2)$$

For Equation 2, we have

$$\prod_{j=1, j \neq i}^{n+1} \text{prob} \left[ \left( H - b_i < |H - b_j| \right) \middle| \left( H > b_i \right) \right] = \frac{\text{prob} \left( H - b_i < |H - b_j|, b_i < H \right)}{\text{prob}(b_i < H)} \quad (3)$$

When  $b_j < b_i$ , we can obtain  $b_j < b_i < H$ , this case do not stand for it's impossible all tendering price lower than the their average value.

When  $b_j < 2H - b_i$ , we can obtain

$$\begin{aligned} \text{prob}(b_j < 2H - b_i) &= \text{prob}(b^*(v_j) < 2H - b_i) = \text{prob}\left(v_j < b^{*-1}(2H - b_i)\right) \\ &= \text{prob}\left(v_j < \phi(2H - b_i)\right) = \text{prob}\left(v_j < \int_X^{\phi(2H - b_i)} f(x)dx\right) \end{aligned} \quad (4)$$

Where  $\phi(2H - b_i) = b^*(2H - b_i)$  based on  $b^*(v_i)$  can be differential and strictly monotonously increasing.

$$\begin{aligned} \prod_{j=1, j \neq i}^{n+1} \text{prob} \left[ \left( H - b_i < |H - b_j| \right) \middle| \left( H > b_i \right) \right] &= \frac{\text{prob} \left( H - b_i < |H - b_j|, b_i < H \right)}{\text{prob}(b_i < H)} \\ &= \frac{\text{prob}\left(v_j < b^{*-1}(2H - b_i), v_i < b^{*-1}(H)\right)}{\text{prob}(v_i < b^{*-1}(H))} \end{aligned} \quad (5)$$

(5)

It can be easily derived that

$$\text{prob} \left[ \left( H - b_i < |H - b_j| \right) \middle| \left( H > b_i \right) \right] = \text{prob}\left(v_j < b^{*-1}(2H - b_i)\right) \quad (6)$$

Owing to the rational person supposition, to obtain profit maximization means to achieve utility maximization, so

$$MaxEu_i = (b_i - v_i) \prod_{j=1, j \neq i}^{n+1} prob\left(v_j < b^{*-1}(2H - b_i)\right) = (b_i - v_i) \left[ \int_L^{\phi(2H - b_i)} f(x) dx \right]^n \quad (7)$$

Then we can obtain,

$$\left[ \int_X^{\phi(2H - b_i)} f(x) dx \right]^n = n(b_i - v_i) \bullet \left\{ \left[ \int_X^{\phi(2H - b_i)} f(x) dx \right]^{n-1} \phi'(2H - b_i) f[\phi(2H - b_i)] \right\} \quad (8)$$

$$\int_X^{\phi(2H - b_i)} f(x) dx = n(b_i - v_i) \phi'(2H - b_i) f[\phi(2H - b_i)] \quad (9)$$

Since  $\phi(2H - b_i) = v_i$ , we have  $\int_X^{v_i} f(x) dx = n(b_i - v_i) v_i' f(v_i)$

Let  $g(c) = \int_X^c f(x) dx$ , so

$$g(v_i) = n(b_i - v_i) f(v_i) \frac{dv_i}{db_i} \quad (10)$$

It can be easily derive that

$$\frac{db_i}{dv_i} - \frac{nb_i f(v_i)}{g(v_i)} = \frac{-nv_i f(v_i)}{g(v_i)} \quad (11)$$

$$b_i = e^{-\int \frac{-nf(v_i)}{g(v_i)} dv_i} \left\{ \int \left[ \frac{-nv_i f(v_i)}{g(v_i)} \right] \cdot e^{\int \frac{-nf(v_i)}{g(v_i)} dv_i} dv_i + L \right\} \quad (12)$$

Especially,  $f(x) \sim U[0,1]$ , so  $f(x) = 1$ ,  $g(v_i) = v_i$ ,

Based on that, we can rewrite the function above as follows:

$$b_i = e^{-\int \frac{-n}{v_i} dv_i} \int \left( \frac{-nv_i}{v_i} \right) e^{\int \frac{-n}{v_i} dv_i} dv_i = v_i^n \left( \int -nv_i^{-n} dv_i \right) = v_i^n \left( -n \frac{v_i^{-n+1}}{n-1} \right) = \frac{n-1}{n} v_i \quad (13)$$

To summarize, bidding rivals' average tender offer under the condition of the stochastic compound base price is subject to  $B = (n-1)v/n$ . We call  $v$  as the average of tendering valuation. Therefore, it's key to estimate bidding rivals' tendering valuation when determining their average tender offer. While in practice,  $v$  is the social average cost of the engineering project and can be compiled by budge quota and adjusted by

coefficient from earlier project experiences. The relationship of variables mentioned above can be seen from Figure 1.

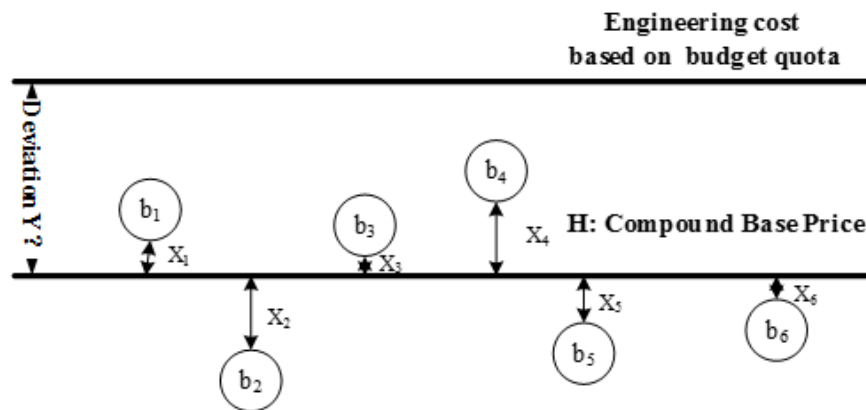


Figure 1. Relationships between bidding rivals' average tender offer other variables

In conclusion bidding rivals' average tender offer is approximate to engineering cost multiplying coefficient  $(n-1)/n$  under the condition of the stochastic compound base price, where budget quota act as basis of the compound base price.

### Numerical Example Analysis

In this section, we assume a virtual tender and use the theoretical model mentioned above to simulate the process of tendering. This project applies compound base price mode in the tender documents. The base bid price is not stated in the documents but announced in public when opening bids, the weight of which in bid evaluation is 0.4~0.6. Correspondingly, the weight of average tender offer is 0.6~0.4. Thus, the weighted average number of those figures multiplying the flotation efficiency 10% can be regarded as base bid price. Therefore, the theoretical stochastic model can be applied to this situation to predict average tender offer.

First, we need the number of tenders when using the model based on the field reconnaissance and tendering Q & A organized by the bidder. The estimated number of bidders participating in the project bidding are six.

Second, the base bid price is another important figure to be predicted according to the bidders' previous tendering offer of the similar project. Generally, it is determined by prevailing profession quota and comprehensive information combined with the project tender documents, construction drawings, site survey, etc., as well as market price information released by engineering cost management institution. The engineering cost of this project is RMB 56,121,750 and the base bid price RMB 50,509,575.

Third, the weight of base bid price is proposed by the evaluation committee in an independent, anonymous manner and the extent of normal reference value is 0.48~0.51, which obeys normal distribution. According to the characteristics of normal distribution of uncertain variables may be located above the average may be located below the average, but are more likely to be close to the average and not far from the mean, therefore, predict the most likely to take the value of 0.5.

Finally, the average tender offer is approximate to engineering cost multiplying coefficient  $(n-1)/n$  under the condition of the stochastic compound base price. In this particular case, the average tender offer is RMB  $(7-1)/7 \times 56,121,750 = 48,104,357$

### Conclusions and Future Research

This paper apply the method of probability theory to solve the model and bidding rivals' average tender offer and provide a little enlightenment about estimating bidding rivals' average tender offer. Different from

other estimating models, we consider the standard form of the stochastic compound base price bid evaluation as well as the maximizing probability of winning the bid.

The chance of acceptance of the bid and bid winner's profit margins is to some extent determined by tender offer under the condition of the stochastic compound base price. Thus estimating bidding rivals' average tender offer becomes a key issue of the project bidders. Taking maximum probability of winning bid as the main object of studying, this article considers the standard form of the stochastic compound base price bid evaluation, and constructs an estimating model of bidding rivals' average tender offer through improving and integrating the existing model. Then method of probability theory is applied to solve the model and bidding rivals' average tender offer is obtained. The results of this study solve to a certain extent stochastic problem in theoretical model of optimal average tender offer, as well as provided a little enlightenment about estimating bidding rivals' average tender offer under the condition of the stochastic compound base price.

Although this model contribute to a certain degree to estimate bidding rivals' average tender offer, we assume that all players are interest-oriented and risk-neutral. However, in practice, the hypothesis may be too strict. Therefore, our model can be extended to consider the players' risk preference to present fruitful opportunities for future research.

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