# Design of 3-Dimensional Discrete Sliding Mode Guidance Law Based on ESO

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**Abstract.** In the paper ,a new approach to design discrete sliding mode guidance laws is presented based on the target-missile relative motion equation in three-dimensional space. The paper also proposes a new method to estimate the real-time target acceleration through the Extended State Observer. Finally, by mathematical simulation, the design significantly reduces system chattering caused by the variable structure control and has a strong robustness for hitting the maneuvering target.

## Introduction

The guidance law based on 3-dimensional kinematics is more authentic and feasible in project, because the chasing problems of missile-target happen in 3-dimensional space during the process of actual interception. At the same time，the design of discrete-time guidance law is of universal significance, as the computer real-time control parts of guidance structure are discrete-time control systems in the actual engineering application[1]。Therefore, this paper, based on the model of target-missile relative motion in three-dimensional space, directly develops a method for discrete sliding mode guidance laws.

## Modeling

The fundamental formula for the chasing of missile-target in three-dimension can be concluded as Eq. 1.

$$\begin{aligned}
&\ddot{r} - r\dot{\varphi}_L^2 \cos^2\theta_L - r\dot{\theta}_L^2 = a_{T_{XL}} - a_{M_{XL}} \\
&r\ddot{\varphi}_L \cos\theta_L + 2\dot{r}\dot{\varphi}_L \cos\theta_L - 2r\dot{\theta}_L\dot{\varphi}_L \sin\theta_L = a_{T_{YL}} - a_{M_{YL}} \\
&r\ddot{\theta}_L + 2\dot{r}\dot{\theta}_L + r\dot{\varphi}_L^2 \sin\theta_L \cos\theta_L = a_{T_{ZL}} - a_{M_{ZL}}
\end{aligned} \tag{1}$$

As same as the planar guidance laws, $a_{M_{XL}}$ has to make the relative velocity of missile-target less than zero on the premise of stabilization of the line of sight in the process of terminal guidance. How to calculate $a_{M_{YL}}$ and $a_{M_{ZL}}$ by the feasible methods ,which make angular rate of Line of Sight (LOS) $\dot{\theta}_L$ and $\dot{\varphi}_L$ close to zero, is the key to designing guidance laws[2]. It's obviously that there have a serious of crossover and coupling between elevation angle and azimuth angle of LOS, which makes the state of the system as nonlinear equations.

## Estimating the real-time target acceleration through ESO

The Extended State Observer (ESO), which is the element of the Active Disturbance Rejection Control (ADRC), not only can get the state of the uncertain target, but also can obtain the real-time control quantity of the model internal and external disturbances [3].

On the basis of Eq.1,set $x_1 = \dot{\theta}_L$, $x_2 = \dot{\varphi}_L$ and $u_1 = a_{M_{ZL}}$, $u_2 = a_{M_{YL}}$, $w_1 = a_{T_{ZL}}$, $w_2 = a_{M_{YL}}$ then first-order and time-varying differential equation can be written as Eq.2.

 

$$\dot{x}_1(t) = -\frac{2\dot{r}(t)}{r(t)}x_1(t) - x_2{}^2(t)\sin\theta_L(t)\cos\theta_L(t) - \frac{u_1(t)}{r(t)} + \frac{w_1(t)}{r(t)}$$
$$\dot{x}_2(t) = -\frac{2\dot{r}(t)}{r(t)}x_2(t) + 2x_1(t)x_2(t)\tan\theta_L(t) - \frac{u_2(t)}{r(t)\cos\theta_L(t)} + \frac{w_2(t)}{r(t)\cos\theta_L(t)} \quad (2)$$

$w_1$ and $w_2$ are considered as disturbance variables, which are required to estimate in real time. Then the second-order ESO can be established on account of Eq.2.

$$\begin{cases} e_2 = z_2 - y_2 \\ \dot{z}_2 = z_{22} - \beta_{21} fal(e_2, \alpha_{21}, \delta_{21}) + f_2(z_1, z_2) + b_2 u_2 \\ \dot{z}_{22} = -\beta_{22} fal(e_2, \alpha_{22}, \delta_{22}) \end{cases}$$
$$\begin{cases} e_1 = z_1 - y_1 \\ \dot{z}_1 = z_{11} - \beta_{11} fal(e_1, \alpha_{11}, \delta_{11}) + f_1(z_1, z_2) + b_1 u_1 \\ \dot{z}_{11} = -\beta_{12} fal(e_1, \alpha_{12}, \delta_{12}) \end{cases} \quad (3)$$

Therefore, by regulating the parameters of $\beta$ ,the observer of Eq.3 can estimate the state variables $x$ and the system disturbance $w$ .

**Design for discrete sliding mode guidance law**

The Power Function $fal(s,\alpha,\delta)$ ,a nonlinear function, has plenty of intelligent performances. The function $fal$ can be described as Eq.4.

$$fal(s,\alpha,\delta) = \begin{cases} |s|^{\alpha} sign(\text{s}) \,, & |s| \geq \delta \\ \dfrac{s}{\delta^{1-\alpha}} \,, & |s| < \delta \end{cases} \quad (4)$$

On the basis of the Power Function, the new form reaching law of sliding mode guidance law can be obtained as Eq.5.

$$s(k+1) = (1 - qT)s(k) - \varepsilon T fal(s,\alpha,\delta) \quad (5)$$

The reaching law in Eq.5 can satisfy the stability of discrete sliding mode guidance law [4].Meanwhile, it makes $s(k)$ reduce to zero monotonously without the alternation of the positive and negative, which weakens the chattering problem of variable structure control system effectively[5].Based on Eq.1,the new discrete domain state equations are described as Eq.6.

$$x(k+1) = f(x(k)) + g_1(x(k))u + g_2(x(k))w \quad (6)$$

By combining Eq.5with Eq.6,the control inputs are designed as Eq.7：

$$u_L = \begin{bmatrix} (\lambda_1+2)|\dot{r}(k)|x_1(k) - x_2{}^2(k)\sin x_3(k)\cos x_3(k) r(k) + w_1 \\ \cos x_3(k)(\lambda_2+2)|\dot{r}(k)|x_2(k) + 2x_1(k)x_2(k)r(k)\sin(x_3(k)) + w_2 \end{bmatrix}$$
$$u_N = \begin{bmatrix} \gamma_1 fal(x_1(k),\alpha_1,\delta_1) \\ \cos x_3(k)\gamma_2 fal(x_2(k),\alpha_2,\delta_2) \end{bmatrix} \quad (7)$$

In Eq.7, $w_1$ and $w_2$ have been already obtained from ESO estimation, which can compensate the control input directly.

## Simulation and Results

By taking the simulation of the missile intercepting maneuvering target in 3-D space as an example, the hitting effect of the designed guidance laws will be illustrated. The initial position of missile is original point, the missile's initial velocity is 1000m/s, and both the trajectory inclination angle and deflection angle are 0 °.As for the maneuvering target, the initial position is (10000,10000, 0), the initial velocity is 1000 m/s, the trajectory inclination angle is 0 °, and the deflection angle is 225 °. The target maneuvers with sinusoidal acceleration in 3-Dimensional space. The normal accelerations of maneuvering target are set as $aty = 100(sin((t-1)\pi))$ and $aty = 50sin(5t)$ .The parameters of ESO are set as $\beta_{11}$=10, $\beta_{12}$=100, $\beta_{21}$=20, $\beta_{22}$=200, $\alpha_{11}$=0.5, $\alpha_{12}$=0.25, $\alpha_{21}$=0.5, $\alpha_{22}$=0.25, $\delta_{11}$=0.001, $\delta_{12}$=0.001, $\delta_{21}$=0.001, $\delta_{22}$=0.001. The parameters of discrete sliding mode controller are $\lambda_1$=2, $\lambda_2$=2, $\varepsilon_1$=0.5, $\varepsilon_2$=0.5, $\alpha_1$=0.5, $\alpha_2$=0.5, $\gamma_1$=0.01, $\gamma_2$ =0.01. Simulation result is showed as Fig.1.

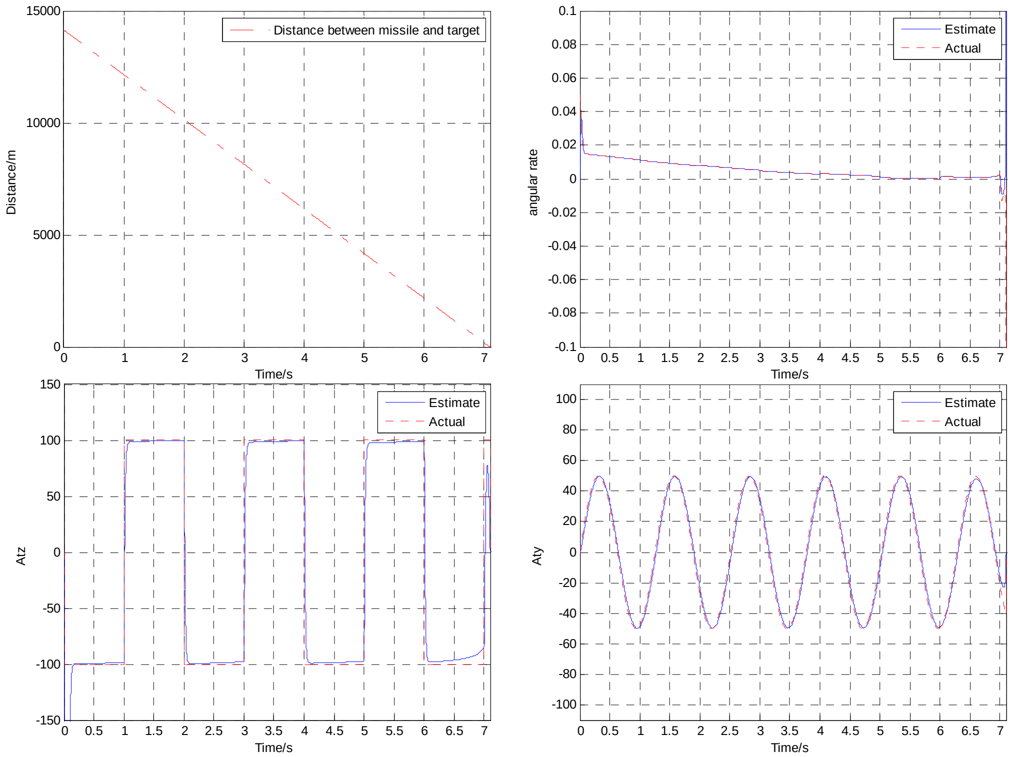


Fig.1.Simulation Results

The simulation results can be concluded that intercepting time of discrete sliding mode guidance law is 7.101 s, miss distance is 0.255 m. The change rates of elevation angle and azimuth of LOS converge monotonously to the boundary layer of zero and the chattering of LOS angular rate changed little in magnitude when reaching the seeming sliding mode. The designed guidance laws always try to make the relative velocity deflection angle of missile and target approaching to zero , making the missile always attacking aimed at the target in direct. In this way, the results can achieve shorter interception time and faster attacking speed. By ESO, both the sinusoidal acceleration and the switch acceleration can be estimated exactly and rapidly.

## Conclusions

The paper presents a new form discrete sliding mode guidance law, which has a strong anti-interference ability and robustness,and the algorithm is simple and easy to implement in guidance system. To some extent, it can inhibit the guidance system chattering which brought from sliding mode variable structure control. By ESO , the discrete variable structure guidance law can overcome the impact of uncertainty of target acceleration on the guidance system effectively. Finally, the sliding mode control theory based on the Power Function and the characteristics of ESO meet the needs of missile-target interceptor and have good prospects of projects.

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