The Travelling Wave Solutions of KdV-Burger Equations

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Abstract. This paper is concerned with the existence of exact traveling wave solutions of some nonlinear evolution equations by the tanh-function method. The validity and reliability of this method is demonstrated by applying it to the KdV-Burger equations.

Keywords: Traveling wave solutions; Tanh-function method; KdV-Burger equations.

1. Introduction

Nonlinear phenomena play a fundamental role in applied mathematics, physics, power electronics, oscillatory systems, earthquakes and tremor recently. The study of nonlinear partial differential equations in modelling physical phenomena has become an important tool. The investigation of the travelling wave solutions plays an important role in nonlinear sciences. There are usually no analytical solutions for these problems, especially when the nonlinear terms are complicated. Therefore, it is practical importance to find travelling solutions.

The methods of looking for exact traveling wave solutions of nonlinear evolution equations, has been tremendous development in recent decades, such as Hirota's bilinear technique [5], MSE method [9],F-expansion method [13,19], the Painlve expansion method [18]. In 1990s, Huibin and Kelin [7] proposed a new method. The main idea of this method is taking hyperbolic tangent function of the power series as possi-ble traveling wave solutions of the nonlinear evolution equations. Then they substituted the power series directly to KdV equation, and obtained the coefficients of the power series. However this method involved very complicated algebra computation. In order to reduce the complex algebra computation, Malfiety [10–12] proposed the tanh-function method. Fan et al [3] proposed the extended hyperbolic tangent method, which replace the tanh-function by the solutions of Riccati equation. In [1, 4, 14–16], using the tanh function method, they got the exact form of traveling wave solutions of various types of evolution equations.

2. The Tanh-function Method

Let's consider the nonlinear partial differential equations

$$N\left(u,u_t,u_x,u_{xx},u_{xxx}\cdots\right)=0, \tag{2.1}$$

Where $u\left(x,t\right)$ is the real function on R^2. At first, we assume the traveling wave solutions of (2.1) are the form of

$$u\left(x,t\right)=U\left(\omega\right)=U\left(c\left(x-vt\right)\right), \tag{2.2}$$

With the velocity v, and the constant c. Submitted (2.2) into (2.1), we can get the ODEs about ω

$$N\left(U,U',U'',U''',\cdots\right)=0. \tag{2.3}$$

Second, we assume the possibly traveling wave solutions can be written

$$u\left(x,t\right)=U\left(\omega\right)=H\left(Y\right)=\sum_{i=0}^{K}a_iY^i, \tag{2.4}$$

Where $Y=\tanh\left(\omega\right)=\dfrac{e^{\omega}-e^{-\omega}}{e^{\omega}+e^{-\omega}}$, the highest order K will be determined late.

Then we can get

$$\frac{dY}{d\omega}=1-Y^2,\frac{dU}{d\omega}=\left(1-Y^2\right)H',\frac{d^2U}{d\omega^2}=\left(1-Y^2\right)\left(-2Y\cdot H'+\left(1-Y^2\right)H''\right).$$

$$\frac{d^3U}{d\omega^3}=\left(1-Y^2\right)\left(6Y^2-2\right)H'-6Y\left(1-Y^2\right)H''+\left(1-Y^2\right)^2H''',$$

Submitted above equations into (2.3), we can get the ODEs with Y

$$N\left(Y,H,H',H'',H''',\cdots\right)=0, \tag{2.5}$$

Where $H'=\frac{dH}{dY}$. To determine the parameter K, we usually balance the nonlinear term and the highest order derivative term in equation (2.5). Then, we submitted (2.4) (with the determined K) into (2.5), and get the polynomial equation with Y, Collecting all the coefficients of power of Y; and letting the coefficients of each power of Y to be vanished,we can determined all the coefficient $a_1,a_2,\cdots,a_k$. According to (2.4), we can get the traveling wave solutions of (2.1).

3. Application

We consider the KdV-Burger equations

$$u_t+\alpha uu_x+\beta u_{xxx}+\gamma u_{xx}=0, \tag{3.1}$$

Where α,β are non-zero constants. Submitting $u(x,t)=U(\omega)=U\left(c\left(x-vt\right)\right)$ into (3.1), we get

$$-cvU'+\alpha cU\cdot U'+\beta c^3U'''+\gamma c^2U''=0.$$

Integrating the above equation, we have

$$-vU+\frac{1}{2}\alpha U^2+\beta c^2U''+\gamma cU'=0. \tag{3.2}$$

In view of (2.4), proceeding as before, we obtain

$$-vH+\frac{1}{2}\alpha H^2+\beta c^2\left(1-Y^2\right)\left[-2YH'+\left(1-Y^2\right)H''\right]+\gamma c\left(1-Y^2\right)H'=0.$$

Balancing the terms Y^4H'' with H^2, we get $K=2$. So, we can obtain

$$H\left(Y\right)=a_0+a_1Y+a_2Y^2,H'=a_1+2a_2Y,H''=2a_2.$$

Submitting it into (3.2), we obtain the following algebraic equations:

$$\begin{cases}-va_0+\frac{1}{2}\alpha a_0^2+2\beta c^2a_2+c\gamma a_1=0,\\ -va_0+\alpha a_0a_1-2a_1\beta c^2+2c\gamma a_2=0,\\ -va_2+\frac{1}{2}\alpha a_1^2+a_0a_2-8\beta c^2a_2-c\gamma a_1=0,\\ \alpha a_1a_2+2\beta c^2a_1-2c\gamma a_2=0,\\ \frac{1}{2}\alpha a_2^2+6\beta c^2a_2=0.\end{cases}$$

Solving the algebraic equations, we get two sets of solutions:

$$v=-\frac{6\gamma^2}{25\beta},c=-\frac{\gamma}{10\beta},a_0=-\frac{3\gamma^2}{25\alpha\beta},a_1=-\frac{6\gamma^2}{25\alpha\beta},a_2=-\frac{3\gamma^2}{25\alpha\beta}$$

$$v=\frac{6\gamma^2}{25\beta},c=-\frac{\gamma}{10\beta},a_0=\frac{9\gamma^2}{25\alpha\beta},a_1=-\frac{6\gamma^2}{25\alpha\beta},a_2=-\frac{3\gamma^2}{25\alpha\beta}.$$

From (2.2)-(2.4) we obtain two traveling wave solutions of (3.1)

$$u(x,t)=-\frac{3\gamma^2}{25\alpha\beta}+\frac{6\gamma^2}{25\alpha\beta}\tanh\left(\frac{\gamma}{10\beta}\left(x+\frac{6\gamma^2}{25\beta}t\right)\right)-\frac{3\gamma^2}{25\alpha\beta}\tanh^2\left(\frac{\gamma}{10\beta}\left(x+\frac{6\gamma^2}{25\beta}t\right)\right) \tag{3.3$_1$}$$

$$u(x,t)=\frac{9\gamma^2}{25\alpha\beta}+\frac{6\gamma^2}{25\alpha\beta}\tanh\left(\frac{\gamma}{10\beta}\left(x-\frac{6\gamma^2}{25\beta}t\right)\right)-\frac{3\gamma^2}{25\alpha\beta}\tanh^2\left(\frac{\gamma}{10\beta}\left(x-\frac{6\gamma^2}{25\beta}t\right)\right) \quad (3.3)_2$$

In a similar way, we can get the following solutions

$$u(x,t)=-\frac{3\gamma^2}{25\alpha\beta}+\frac{6\gamma^2}{25\alpha\beta}\coth\left(\frac{\gamma}{10\beta}\left(x+\frac{6\gamma^2}{25\beta}t\right)\right)-\frac{3\gamma^2}{25\alpha\beta}\coth^2\left(\frac{\gamma}{10\beta}\left(x+\frac{6\gamma^2}{25\beta}t\right)\right) \quad (3.4)_1$$

$$u(x,t)=\frac{9\gamma^2}{25\alpha\beta}+\frac{6\gamma^2}{25\alpha\beta}\coth\left(\frac{\gamma}{10\beta}\left(x-\frac{6\gamma^2}{25\beta}t\right)\right)-\frac{3\gamma^2}{25\alpha\beta}\coth^2\left(\frac{\gamma}{10\beta}\left(x+\frac{6\gamma^2}{25\beta}t\right)\right) \quad (3.4)_2$$

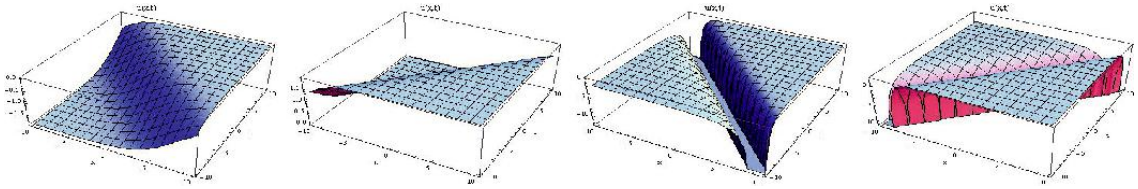

Fig. 1. the travelling wave solutions of (3.3-3.4), when we take $\alpha=\beta=1,\gamma=2$ and $(x,t)\in[-10,10]\times[-10,10]$.

4. Conclusion

In this paper, we have applied the tanh-function method to construct a series of traveling wave solutions for some special types of equations: nonlinear beam equations, Fisher-KKP equations and KdV-Burger equations. These traveling waves solutions are expressed in terms of hyperbolic tangent or hyperbolic cotangent functions depending on different pa-rameters. The tanh-function method is direct, concise and effective, which can be applied to many other nonlinear evolution equations.

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