

Application of MTSP in the transport of Ebola vaccine

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Abstract. West Africa is hit by the most unprecedented outbreak of Ebola virus and the medication that can resist the virus has been discovered. This paper establishes a mathematical model to discuss determination of the transportation organization scheme. In the model, we choose several major Ebola treatment centres as the locations of delivery and use the MTSP model to establish a reasonable transport organization plan. Then, we take Sierra Leone for example in our model, after determination of the locations, we transfer the MTSP to TSP by Gorenstein's method. Next, we can get the result by using the traditional TSP method. In our result, the total distance is just almost 1448km if we use three planes in Sierra Leone.

The central idea of MTSP model

Before explaining MTSP model, we will state the idea of a classical model called TSP. TSP (Travelling salesman problem) model is used to find the shortest possible route that visits each city exactly once and returns to the origin city after given a list of cities and the distances between each pair of cities. TSP can be modeled as an undirected weighted graph, such that cities are the graph's vertices, paths are the graph's edges, and a path's distance is the edge's length. It is a minimization problem starting and finishing at a specified vertex after having visited each other vertex exactly once. If no path exists between two cities, adding an arbitrarily long edge will complete the graph without affecting the optimal tour. The arithmetic can find the shortest circle by modifying the edge when it is still a circle. The problem can be formulated as an integer linear program. Label the cities with the numbers $0, 1, 2, \dots, n$ and define:

$$x_{ij} = \begin{cases} 1 & \text{the path from city } i \text{ to city } j \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

For $i=0, 1, 2, \dots, n$, let u_i be an artificial variable, and finally take c_{ij} to be the distance from city i to city j . Then TSP can be written as the following integer linear programming problem:

$$\min_{0 \leq x_{ij} \leq 1} \sum_{i=0}^n \sum_{j \neq i, j=0}^n c_{ij} x_{ij} \quad i, j = 0, \dots, n \quad (2)$$

$$u_i \in \mathbf{Z} \quad i = 0, \dots, n \quad (3)$$

$$\sum_{i=0, i \neq j}^n x_{ij} = 1 \quad j = 0, \dots, n \quad (4)$$

$$\sum_{j=0, j \neq i}^n x_{ij} = 1 \quad i = 0, \dots, n \quad (5)$$

$$u_i + u_j + n x_{ij} \leq n - 1 \quad 1 \leq i \neq j \leq n \quad (6)$$

The result found by this arithmetic is not proved to be optimal, but it is accurate and can be improved by changing the origin circle.

Now, the question turns to be MTSP (multiple Travelling Salesman Problem). In this problem, we should take multiple salesman into consideration. For example, m piece of salesman go to n cities. Each city must be visited exactly once. Then, we should found a model to figure out the method which has the lowest cost, considering some feasible factors like time, money or distance. However, Scientists and Scholars have not found an exact method to solve this problem directly. So we should analyze the problem at another angle.

The idea is to transform MTSP to TSP. The basic strategy is to add $(m-1)$ virtual cities.

We set these cities in order to separate the cities visited by each people. Then, we can get a Matrix $[(n+m-1) \times (n+m-1)]$ of distances between $(n+m-1)$ cities. The distance between virtual cities is infinity. The current problem is getting back to the origin problem-----TSP. Then, the

problem can be formulated as an integer linear program. And in the same way as before, we can finally find the method.

The Foundation of Model

At the beginning, we select six ETCs (Ebola treatment centers) in Sierra Leone. Here, we only consider about Sierra Leone. Because Sierra Leone has the highest prevalence and only one country can simplify computation. As is shown in the Fig 5, we select 6 ETCs. Because each one can supply its surrounding area. Also, when we find some adjacent ETCs, we only choose the most representative one. We regard these ETCs as six vertexes.



Fig 1: The distribution of Ebola treatment center

Then, we measure the distances between each vertex by Photoshop 7.0. The distances are shown in the table 1. We make the value of m equal to 3. Vertex 7 and vertex 8 represent virtual ETCs.

Table 8: the distance between vertexes

	1	2	3	4	5	6	7	8
1	0	138.75	177.5	239.167	259.167	302.5	∞	∞
2	138.75	0	103.75	134.167	188.333	190.833	138.75	138.75
3	177.5	103.75	0	76.25	93.3333	157.083	177.5	177.5
4	239.167	134.167	76.25	0	55.4167	105.417	239.167	239.167
5	259.167	188.333	93.3333	55.4167	0	120.833	259.167	259.167
6	302.5	190.833	157.083	105.417	120.833	0	302.5	302.5
7	∞	138.75	177.5	239.167	259.167	302.5	0	∞
8	∞	138.75	177.5	239.167	259.167	302.5	∞	0

Then, we get a symmetrical matrix (8×8) of distance:

$$\begin{pmatrix}
 0 & 138.75 & 177.5 & 239.167 & 259.167 & 302.5 & \infty & \infty \\
 138.75 & 0 & 103.75 & 134.167 & 188.333 & 190.833 & 138.75 & 138.75 \\
 177.5 & 103.75 & 0 & 76.25 & 93.3333 & 157.083 & 177.5 & 177.5 \\
 239.167 & 134.167 & 76.25 & 0 & 55.4167 & 105.417 & 239.167 & 239.167 \\
 259.167 & 188.333 & 93.3333 & 55.4167 & 0 & 120.833 & 259.167 & 259.167 \\
 302.5 & 190.833 & 157.083 & 105.417 & 120.833 & 0 & 302.5 & 302.5 \\
 \infty & 138.75 & 177.5 & 239.167 & 259.167 & 302.5 & 0 & \infty \\
 \infty & 138.75 & 177.5 & 239.167 & 259.167 & 302.5 & \infty & 0
 \end{pmatrix}$$

MTSP has been transformed into TSP. So we can use the amendment-circle algorithm to compute the shortest route when m equals to 3. In this route, the total time depends on the longest circle. This is very similar to cask principle.

Solution and Result

- When m equals to 3:
 - 1) The solution of the amendment-circle algorithm:

The route is 1 to 2 to 7 to 4 to 5 to 3 to 8 to 6 to 1. The total shortest distance is 1447.917 km. The distance of the longest circle is 565.417 km.

2) Results:

The route is 1 to 2 to 1 and 1 to 4 to 5 to 3 to 1 and 1 to 6 to 1.

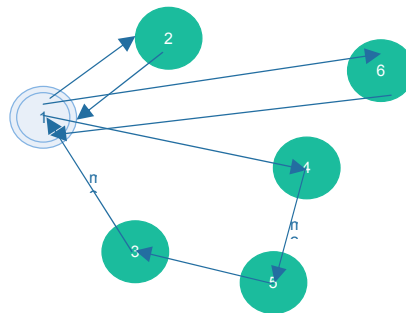


Fig 2: Optimal transportation route

- When m equals to 2

1) The solution of the amendment-circle algorithm:

The route is 1 to 2 to 7 to 3 to 4 to 5 to 6 to 1. The total shortest distance is 1009.999 km. The distance of the longest circle is 732.499 km.

2) Results:

The route is 1 to 2 to 1 and 1 to 3 to 4 to 5 to 6 to 1.

- When m equals to 1

1) The solution of the amendment-circle algorithm:

The route is 1 to 2 to 3 to 4 to 5 to 6 to 1. The total shortest distance is 797.499 km. The distance of the longest circle is 732.499 km

2) Results:

The route is 1 to 2 to 3 to 4 to 5 to 6 to 1.

Analysis of the Result

According to the result, corresponding to different value of m , there is always a shortest route. The result shows that there is a positive correlation between the value of m and the total distance. We think that it is due to the increasing number of times returned to the starting point. That does not mean more transportations more cost. Because the result also shows that there is a negative correlation between the value of m and the distance of the longest circle. That means more transportations less cost. Because we just consider about the total time. But we think that the transportations must have a upper limit. Because too much transportations must be too much cost.

So we could give m more value, we can get more distance of the longest circle. Then, we can get a relation between m and the distance of the longest circle. We can get a better result.

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