Theory and Experiments of a Rolling-ball Mechanism for Horizontal Vibration Isolation

Gaofeng Guan[a*] Qiang Yu[b] Fei Shen[c]

School of Mechanical and Electronic Engineering, University of Electronic Science and Technology of China, Chengdu, SiChuan, China

[a]gyjj2008@aliyun.com, [b]yuqiang199104@163.com, [c]shenflying@hotmail.com

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Abstract: It's significant to isolate micro-vibration interference from ground to precision instruments, so lots of scholars have paid attention to study vibration isolation mechanisms and got a fruitful outcome. The most important characteristics of a passive vertical vibration isolator are natural frequency and payload capacity while horizontal vibration isolator only concerns the former. So the performance of a passive vibration isolator in low input frequencies is considerably improved by reducing its natural frequency. This paper studied a rolling-ball mechanism that the ball rolls relative to two ball sockets with unequal radii. When the motion was assumed as slight movement, according to relational geometric relationship, the Lagrange dynamics equation of rolling angle β as generalized coordinate was established. The horizontal natural frequency has been also obtained. Then the simulation analysis of influences of various parameters on natural frequency was studied. Finally, low natural frequency has been realized in the experiments which confirmed its validity.

1. Introduction

In order to achieve a better performance during the use in the field of precision instruments, high-end biological and chemical experiments or optical apparatus, the devices should be installed in foundation platform that can isolate micro-vibration disturbance [1].Passive isolators is most widely used in vibration elimination as they are low cost, easy to implement and so on. The vibration isolation performance of a passive vibration isolator in low input frequencies is considerably improved by reducing its natural frequency, and lower frequency means better performance [2, 3].

Non-independent isolators are often composed of several regularly arranged vibration isolation mechanisms with the same basis, where rolling types of horizontal vibration isolation mechanisms have attracted the attention of many scholars. Lin [4] presented a free rolling mechanism to isolate seismic damage to buildings of which a sphere was placed between two smooth surfaces, relying on rolling friction to consume energy to the superstructure, but it could not restore to the initial position and limit horizontal displacement. Zhou [5] proposed an improved way that placed the ball between two ball sockets, and earthquake simulation tests confirmed that it had a better seismic isolation performance. Jiangid [6, 7] proposed two improved methods about free rolling mechanism, one is to replace the ball with ellipsoid, and the other is to use parallel with cantilever beam or spring. YasudaA [8] also proposed another free rolling mechanism, he replaced the upper and lower areas of the ball with a part of sphere of unequal radii, respectively. Butterworth [9] carried out a detailed theoretical modeling and parameters analysis, and equivalent linearization of stiffness of natural frequency of

YasudaA's improved mechanism, finally, simulation analysis confirmed its effectiveness. Harvey [10, 11] has investigated numerically the chaotic response of a rolling mechanism with curved slots of varied radius and rich chaotic behavior has exhibited in the case where the response includes impacts.

Based on the General, a rolling ball mechanism with an upper and lower ball sockets with non-equal radii is studied in this paper. The natural frequency has been obtained by theoretical analysis and the simulation shows how each parameter to influence on the natural frequency. Then the mechanism was applied to precision vibration isolation to suppress the slight vibrations of the ground. The experimental results verifies that the institution could achieve a lower frequency, which confirmed that the vibration isolation system is efficient.

2. Theoretical analysis

2.1 The basic configuration

The basic configuration of the vibration isolation mechanism is showed in Fig. 1(a). It is composed of an upper plate, a ball and a lower plate, and the upper plate and the lower plate both have a ball socket called upper ball socket and lower ball socket.
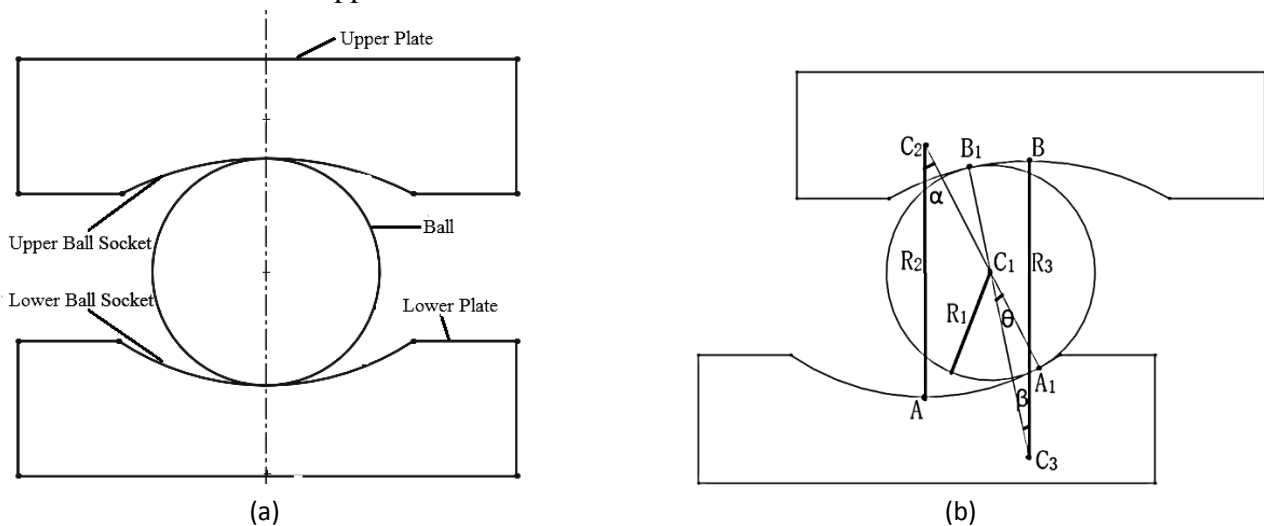

Fig.1 (a) Diagram of the mechanism in balanced state; (b) Diagram of the mechanism in motion state

Table 1

The parameters of the mechanism in Fig.1 and Fig. 2.

Symbol	Description
C_1	The center of the ball
C_2	The center of the lower ball socket
C_3	The center of the upper ball socket
R_1	The radius of the ball
R_2	The radius of the lower ball socket
R_3	The radius of the upper ball socket
A	The contact point of the ball and the lower ball socket in balanced state
B	The contact point of the ball and the upper ball socket in balanced state
A_1	The contact point of the ball and the lower ball socket in motion state
B_1	The contact point of the ball and the upper ball socket in

	motion state
α	The rolling angel of the ball and the lower ball socket in motion state
β	The rolling angel of the ball and the upper ball socket in motion state
γ	The rolling angel of C_2 and C_3 relative to C_1

2.2 Geometric constraints and relationships

To be general, we assume that:

(1) The radii of the ball sockets in the upper and lower plates are unequal, such as $R_3 > R_2$;

(2) The upper and lower ball sockets and the ball are rigid, without distortion;

(3) The motion type of the ball relative to the ball sockets in the lower and upper plates is rolling without sliding.

As shown in Fig. 1(a), when the mechanism is in balance state, the centers of the ball sockets in the upper plate and lower plate and the center of the ball are in the same vertical line.

As shown in Fig. 1(b), whenever the mechanism is in the motion state and the ball rolls relative to the lower and the upper ball sockets, the center of the upper ball socket C_3, the center of ball C_1 and the contact point of the ball and the upper ball socket B_1 are in the same line, so as the center of the lower ball socket C_2, the center of the ball C_1 and the contact point of the ball and the lower ball socket C_2. A is the contact point of the ball and the lower ball socket and B is the contact point of the ball and the upper ball socket in balanced state, and A_1 is the contact point of the ball and the lower ball socket and B_1 is the contact point of the ball and the upper ball socket in motion state.

The arc concave AA_1 is unequal to BB_1:

$$AA_1 = R_2 * \alpha \tag{1}$$
$$BB_1 = R_3 * \beta \tag{2}$$

Besides, from the geometric relationships, we gets equations (3) and (4):

$$\alpha = \theta + \beta \tag{3}$$
$$R_2 * \beta + R_1 * \theta = R_2 * \alpha \tag{4}$$

Transform Eq. (3) and Eq. (4), relationship of Eq. (5) is utilized.

$$\alpha = \frac{R_3 - R_1}{R_2 - R_1} * \beta \tag{5}$$

2.3 Vibration equation

As the disturbance from the ground to the lower plate is very slight, we assume the lower plate is static and the ball roll relative to the lower and upper plates. Be aware of that the mechanism will rotate when used lonely, so it is better to use in parallel with three or above.

When the mechanism is in motion state, the horizontal and vertical displaces x and y of the upper plate are

$$x = (R_3 - R_1)\sin\beta + (R_2 - R_1)\sin\alpha \tag{6}$$

$$y = (R_2 + R_3 - 2R_1) - [(R_2 - R_1)\cos\alpha + (R_3 - R_1)\cos\beta \tag{7}$$

Taking Eq. (5) into (6) and (7), the horizontal and vertical speeds $\dot{x}$ and $\dot{y}$ are obtained

$$\dot{x} = (R_3 - R_1)(\cos\beta + \cos\frac{R_3 - R_1}{R_2 - R_1}\beta)\dot{\beta} \tag{8}$$

$$\dot{y} = (R_3 - R_1)(\sin\beta + \sin\frac{R_3 - R_1}{R_2 - R_1}\beta)\dot{\beta} \tag{9}$$

The synthetic speed v can be expressed as follow

$$v = \sqrt{\dot{x}^2 + \dot{y}^2} = (R_3 - R_1)\dot{\beta}\sqrt{2 + 2cos\frac{R_2-R_3}{R_2-R_1}\beta}$$ (10)

The kinetic energy of the upper plate is

$$T = \frac{1}{2}Mv^2$$ （11）

Where M is the quality of the upper plate. Assuming the potential energy of the upper plate is zero in balance state, then now we get potential energy at present

$$V = Mgy$$ （12）

Equations (13) to (18) are achieved by Lagrange equation

$$L = T - V$$ （13）

$$\frac{\partial L}{\partial \dot{\beta}} = M(R_3 - R_1)^2(2 + 2cos\frac{R_2-R_3}{R_2-R_1}\beta)\dot{\beta}$$ (14)

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\beta}}\right) = M(R_3-R_1)^2\left(2 + 2cos\frac{R_2-R_3}{R_2-R_1}\beta\right)\ddot{\beta} + M(R_3-R_1)^2\left(-2\frac{R_2-R_3}{R_2-R_1}sin\frac{R_2-R_3}{R_2-R_1}\beta\right)\dot{\beta}^2$$ (15)

Besides

$$\frac{\partial L}{\partial \beta} = -Mg(R_3-R_1)\left(sin\beta + sin\frac{R_3-R_1}{R_2-R_1}\beta\right) + M(R_3-R_1)^2\left(-\frac{R_2-R_3}{R_2-R_1}sin\frac{R_2-R_3}{R_2-R_1}\beta\right)\dot{\beta}^2$$ (16)

From Lagrange's equations

$$M(R_3-R_1)^2\left(2 + 2cos\frac{R_2-R_3}{R_2-R_1}\beta\right)\ddot{\beta} + Mg(R_3-R_1)\left(sin\beta + sin\frac{R_3-R_1}{R_2-R_1}\beta\right) - M(R_3-R_1)^2\left(-\frac{R_2-R_3}{R_2-R_1}sin\frac{R_2-R_3}{R_2-R_1}\beta\right)\dot{\beta}^2 = 0$$

(17)

The third part of Eq. (17) is a higher order term and it can be omitted. From basic mathematical theory

$$\left(sin\beta + sin\frac{R_3-R_1}{R_2-R_1}\beta\right) = 2sin\frac{R_2+R_3-2R_1}{2(R_2-R_1)}\beta cos\frac{R_2-R_3}{2(R_2-R_1)}\beta$$ (18)

As β is very small, $cos\frac{R_2-R_3}{2(R_2-R_1)}\beta \approx 1$, Eq. (18) turns into Eq. (19)

$$sin\beta + sin\frac{R_3-R_1}{R_2-R_1}\beta = 2sin\frac{R_2+R_3-2R_1}{2(R_2-R_1)}\beta$$ （19）

Taking Eq. (19) into (17)

$$M(R_3-R_1)^2\left(2 + 2cos\frac{R_2-R_3}{R_2-R_1}\beta\right)\ddot{\beta} + 2Mg(R_3-R_1)sin\frac{R_2+R_3-2R_1}{2(R_2-R_1)}\beta = 0$$ (20)

As above, $cos\frac{R_2-R_3}{R_2-R_1}\beta = 1, sin\frac{R_2+R_3-2R_1}{2(R_2-R_1)}\beta \approx \frac{R_2+R_3-2R_1}{2(R_2-R_1)}\beta$, Eq. (20) is simplified as follow

$$4(R_3-R_1)\ddot{\beta} + g\frac{R_2+R_3-2R_1}{R_2-R_1}\beta = 0$$ (21)

The horizontal natural frequency is obtained

$$f = \frac{1}{4\pi}\sqrt{\frac{g(R_2+R_3-2R_1)}{(R_2-R_1)(R_3-R_1)}}$$ (22)

3. Simulations and experiments

3.1 Simulation

Set the range of R_1 is from 0 to 0.05m, and the radii of the upper and the lower ball sockets changes in the range of R_1 to 0.5m. The Influence of R_1 and R_2 ($R_2=R_3$) on natural frequency is shown in Fig. 2, and the Influence of R_2 and R_3 on natural frequency when R_1 is 0.005m, 0.01m and 0.02m, respectively, are shown in Fig. 3 to fig.5.
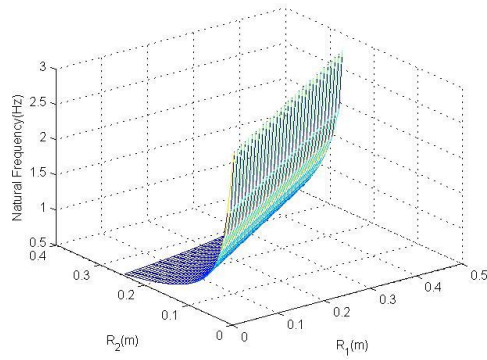
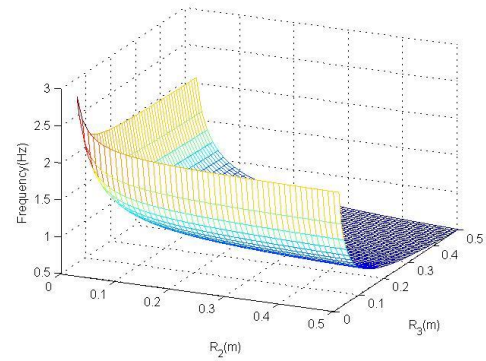Fig.2 Influence of R_1 and R_2 ($R_2=R_3$) on natural frequency

Fig.3 Influence of R_2 and R_3 on natural frequency under the condition: $R_1=0.005m$
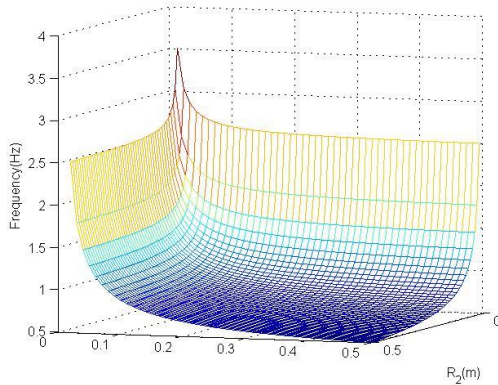

Fig.4 Influence of R_2 and R_3 on natural frequency

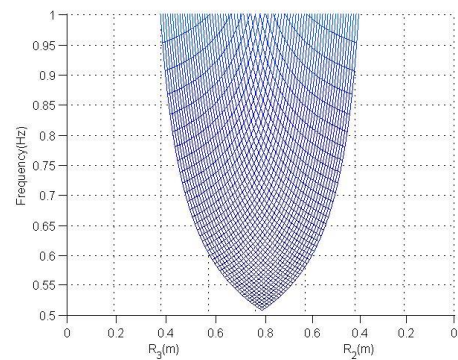under the condition: $R_1=0.01m$

Fig.5 Influence of R_2 and R_3 on natural frequency

under the condition: $R_1=0.02m$

Through the simulation analysis, it can be seen that the vibration natural frequency of the mechanism f decreases with R_2 and R_3 increase, but the speed decreases; f decreases with R_1 decreases, and the speed decreases. In addition, within the scope the lowest natural frequency can be close to 0.5Hz and with the increase of R_1, the lowest natural frequency increases slightly, but the variation is very small. If the natural frequency achieves sub-Hz only when R_2 and R_3 are minimum close to 0.4m.

3.2 Experiments design

For stability considerations, the design is shown in Fig.6. According to the actual situation, the material of the ball is 45 steel and R_1 is 0.02m, meanwhile the material of the upper and lower ball sockets is aluminum alloy 6061T and R_3 and R_2 are 0.3m and 0.4m. The upper and lower sheet metal plates of the vibration isolation system are aluminum alloy 6061T, between the two plates are four parallel mechanisms with symmetrical distribution in a rectangular form, and the lower sheet metal

places on the ground, and the upper sheet metal supports the load. The experiment use BK (Bruel＆

Kjær) as the vibration measurement system, and the vibration of the ground is the input signal and the vibration of the upper plate is the output signal, then the experimental setup is shown in Fig. 7. The experiments were divided into four groups and the corresponding load are 10Kg, 20Kg, 40Kg and 80Kg, respectively. Each group test 3 times and then make date fitting by Matlab after the experimental results are averaged. The x-axis represents the ground vibration frequency and the y-axis represents the acceleration transfer ratio.
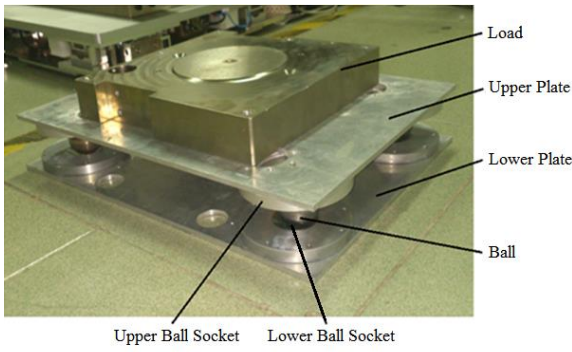
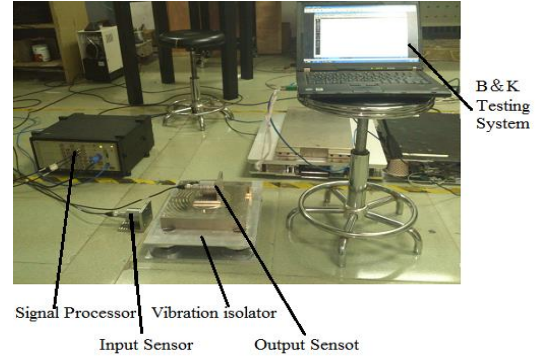Fig.6 Photograph of the rolling ball-ball vibration isolation system   Fig.7 Photograph of the experimental system
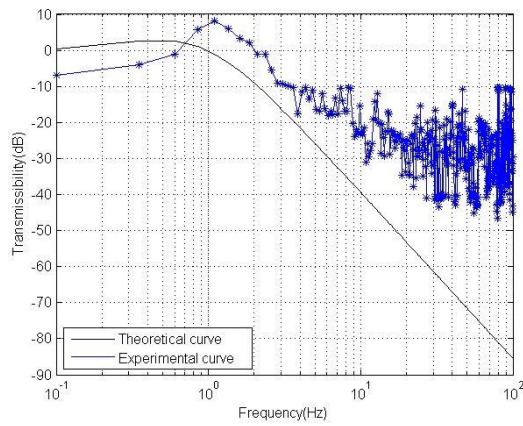
3.3 Experimental consequences

Fig.8 The theoretical and measured consequences
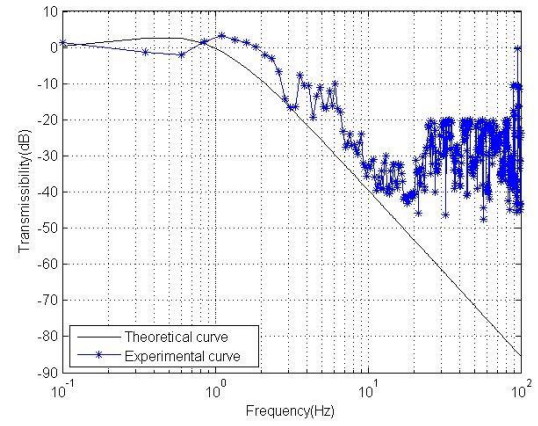
when the payload M is 10Kg

Fig.9 The theoretical and measured consequences

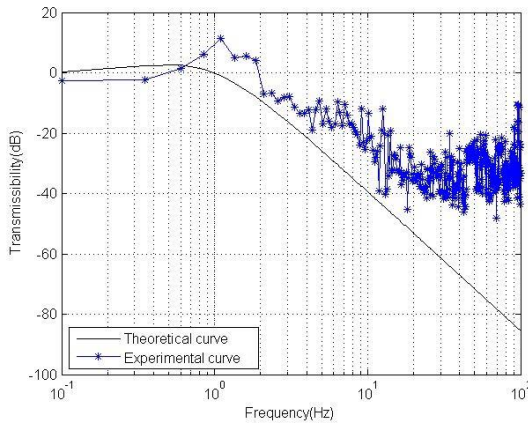when the payload M is 20Kg

Fig.10 The theoretical and measured consequences
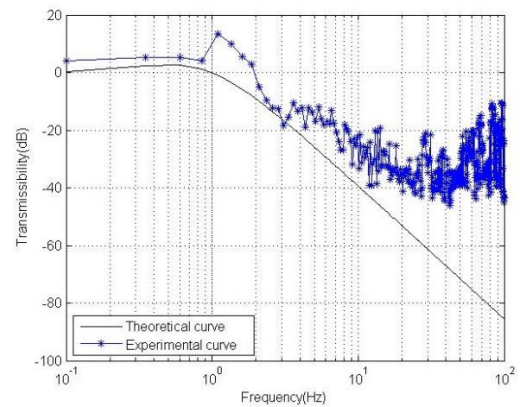
when the payload M is 40Kg

Fig.11 The theoretical and measured consequences

when the payload M is 80Kg

It can be seen from Fig. 8 to Fig. 11 that the natural frequency of experimental results are about to 1Hz, which are close to theoretical results 0.62Hz, and it may be due to the deviation between the hypothesis and the practice. Besides, it also can be seen the natural frequency of the system will not change with variable payload. The vibration transmissibility reaches to -20dB before the excitation frequency reaches to 10Hz, which confirms that the vibration isolation system is efficient.

4. Conclusion

This paper studied a rolling ball mechanism that the ball rolls relative to two ball sockets with unequal radii theoretically. Then we explore further the performance of vibration isolation system

which composed of a plurality of the mechanisms. Experiments showed that the vibration transmissibility has reached to -20dB before the excitation frequency of the ground reaches to 10Hz, and the author is trying to employ active control technology to optimize further the performance of vibration isolation. In addition, the damping effect caused by the contact problem is also discussing. The vibration isolation system can be applied to the field of ultra-precision instrument, high-end physical and chemical experiments, which has a guiding significance for theory analysis and engineering design of the rolling ball vibration isolation mechanism.

References

[1]   D.B.Debra. Vibration isolation of precision machine tools and instruments. CIRP Annals, 1992, 41(2), 711-718.

[2]   Zhang Liguo, Zhang Jiazhong, Jia Liping, et al. Future and development of air springs [J].Journal of Vibration and Shock,2007,26(2):146-154.

[3]   Erin C and Wilson B. An improved model of a pneumatic vibration isolator theory and experiment [J].Journal of Sound and Vibration, 1998, 218 (1):81-101.

[4]   Lin, T. and W. and Hone, C. C. (1993). Base isolation by free rolling rods under basement [J].Earthquake Engineering and Structural Dynamics, Vol. 22, No. 3, pp. 261–273.

[5]   Qiang Zhou. Experimental study and numerical analysis on structure base-isolated by ball system with restoring property [D].Thesis of the Master's Degree. Xi'an University of Architecture & Technology, Xi'an, China, 1997.

[6]   R.S.Jangid and Y.B.Londhe. Effectiveness of elliptical rolling rods for base isolation [J].J.Struct.Eng, 1998, 124:469-472.

[7]   R.S.Jangid. Stochastic seismic response of structures isolated by rolling rods [J].Engineering Structures, 2000, 22:937-946.

[8]   Masashi. Yasuda. Vibration control unit and vibration control body [P].US 20030052247A1, 2003.

[9]   J.W.Butterworth. Seismic response of a non-concentric rolling isolator system [J].Advances in Structural Engineering, 2006, 9(1).

[10]   P.S.Harvey Jr., R. Wiebe,H.P.Gavin. On the chaotic response of a nonlinear rolling isolation system [J]. Physica D, 256–257 (2013): 36–42.

[11]   P.Scott Harvey Jr. and Henri P.Gavin. The nonholonomic and chaotic nature of a rolling isolation system [J].Journal of Sound and Vibration, 332 (2013):3535–3551.