Fault Tolerant Control of Quadrotor based on Parameter Estimation Techniques and Reconfigurable PID Controller

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Abstract. This paper focuses on the fault tolerant control of quadrotor. In this paper, the dynamic model of quadrotor and its measurements model will be given, and a PID controller will be designed based on the models. Then we propose an approach to detect and diagnose the fault of rotor with the help of parameter estimation techniques and adjust the controller to achieve better control results under fault. The proposed fault detection and diagnosis approach is designed for being insensitive to noise of measurements for reliability and robustness.

Introduction

Quadrotor is a new popular UAV, and its rapid development attracts attention all over the world. Such as any vehicle flying in airspace, quadrotor is required to meet rules on safety and mitigates any faults that can lead to a feared event. Therefore fault tolerant control becomes an important issue in research. Fault-Tolerant Control System (FTCS) is able to accommodate the fault of component automatically. Generally, the FTCS can be classified into two types: Passive Fault-Tolerant Control Systems (PFTCS) and Active Fault-Tolerant Control Systems (AFTCS) [1]. In this paper, we will focus on AFTCS, which include the Faults Detection and Diagnosis (FDD) and controller reconfiguration. A typical AFTCS structure is shown in Fig. 1.

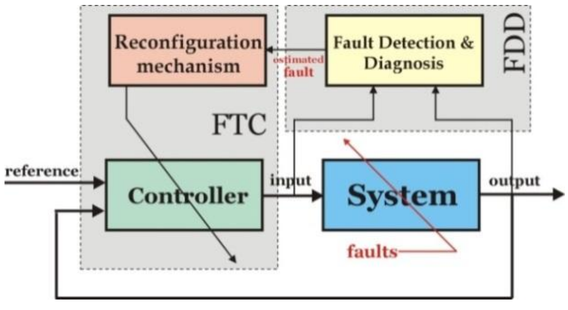

Fig. 1 Fault-Tolerant Control System

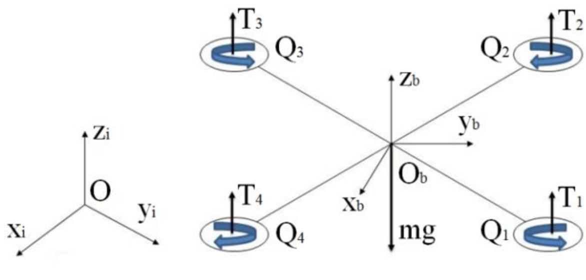

Fig. 2 Mechanical Analysis of Quadrotor

The parameters of quadrotor used in this paper are shown in table 1. In this paper, dynamics model and measurement model will be given first, which corresponds to the system part in the figure. Then PID controller will be introduced briefly. At last faults in rotor will be considered, which will influence the thrust coefficient k (unit $N/(rad/s)^2$) and moment coefficient b (unit $N \cdot m/(rad/s)^2$). FDD and controller reconfiguration mechanism will be designed for the fault.

Dynamics & Measurement

Before giving the dynamics model, we need to define the coordinate system. As shown in Fig. 2, body coordinate system and inertial coordinate system are introduced, and vectors in one system can be transformed to ones in another system by multiplying them with transformation matrix [2]. The rotation sequence from the inertial system to the body system is z, y and x. The forces concerned are the gravity of quadrotor and the lifts of 4 rotors, and the moments concerned are the torques of 4 rotors, while other forces and moments are too small to be considered.

Table 1 Quadrotor Parameters

Parameters	Value
Thrust Coefficient k	9.1×10^{-6}
Moment Coefficient b	1.365×10^{-6}
Rotational inertia I_x	0.0040 $kg\ m^2$
Rotational inertia I_y	0.0040 $kg\ m^2$
Rotational inertia I_z	0.0079 $kg\ m^2$
Arm of force L	0.18 m
Mass m	0.5 kg

Table 2 Measurement Parameters

Accelerometer	Noise	$2.0\times10^{-4}m^2/s^4$
	Bias	$5.0\times10^{-4}m/s^3$
	Sampling Frequency	105.5Hz
Gyroscope	Noise	$2.0\times10^{-7}m^2/s^4$
	Bias	Neglected
	Sampling Frequency	105.5Hz

Dynamics model are given from (1) to (3). $\tau_{bx},\tau_{by},\tau_{bz}$ are the moments of quadrotor in the body coordinate system, and Φ,θ,ψ are roll angle, pitch angle and yaw angle respectively.

$$\begin{pmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{pmatrix}=\begin{pmatrix}0\\0\\-g\end{pmatrix}+\frac{T_1+T_2+T_3+T_4}{m}\begin{pmatrix}cos\Phi sin\theta cos\psi+sin\Phi sin\psi\\ cos\Phi sin\theta sin\psi-sin\Phi cos\psi\\ cos\Phi cos\theta\end{pmatrix}\tag{1}$$

$$\begin{pmatrix}\tau_{bx}\\ \tau_{by}\\ \tau_{bz}\end{pmatrix}=\begin{pmatrix}0\\0\\Q_1+Q_2+Q_3+Q_4\end{pmatrix}+\begin{pmatrix}T_1+T_2-T_3-T_4\\T_2+T_3-T_1-T_4\\0\end{pmatrix}\frac{\sqrt{2}}{2}L\tag{2}$$

$$\begin{pmatrix}\ddot{\Phi}\\ \ddot{\theta}\\ \ddot{\psi}\end{pmatrix}=\begin{pmatrix}1 & sin\Phi tan\theta & cos\Phi tan\theta\\ 0 & cos\Phi & -sin\Phi\\ 0 & sin\Phi sec\theta & cos\Phi sec\theta\end{pmatrix}\begin{pmatrix}I_x & 0 & 0\\0 & I_y & 0\\0 & 0 & I_z\end{pmatrix}^{-1}\begin{pmatrix}\tau_{bx}\\ \tau_{by}\\ \tau_{bz}\end{pmatrix}\tag{3}$$

In addition, the motor response model should be included, as shown in equation (4).

$$\omega(s)=\ \omega_{con}(s)\frac{1}{0.05s+1}\tag{4}$$

Measurement model is also constructed to take the noise into account. Measurement equipment includes accelerometer, gyroscope and GPS. Accelerometer and gyroscope parameters are shown in Table 2, while the bias of gyroscope is so small that it is neglected. To simplify the problem, GPS is considered to be accurate which is used for calibration of positon and velocity every 0.25s (4Hz).

As measurements of accelerometer and gyroscope are in the body coordinate system, navigation system is in need to get the position and attitude in the earth coordinate system. We could use equation (5) and (6) to estimate the angular velocity and acceleration. Attitude, velocity and positon can be obtained by integration. As the bias of measurement exists in accelerometer, calibration of velocity and position will be applied with the help of GPS.

$$\begin{pmatrix}\dot{\Phi}_e\\ \dot{\theta}_e\\ \dot{\psi}_e\end{pmatrix}=\begin{pmatrix}1 & sin\Phi_e tan\theta_e & cos\Phi_e tan\theta_e\\ 0 & cos\Phi_e & -sin\Phi_e\\ 0 & sin\Phi_e sec\theta_e & cos\Phi_e sec\theta_e\end{pmatrix}\begin{pmatrix}p_m\\ q_m\\ r_m\end{pmatrix}=\begin{pmatrix}1 & sin\Phi_e tan\theta_e & cos\Phi_e tan\theta_e\\ 0 & cos\Phi_e & -sin\Phi_e\\ 0 & sin\Phi_e sec\theta_e & cos\Phi_e sec\theta_e\end{pmatrix}\omega\tag{5}$$

$$\begin{pmatrix}\ddot{x}_e\\ \ddot{y}_e\\ \ddot{z}_e\end{pmatrix}=\begin{pmatrix}0\\0\\-g\end{pmatrix}+\begin{pmatrix}\cos\theta_e\cos\psi_e & \sin\Phi_e\sin\theta_e\cos\psi_e-\sin\psi_e\cos\Phi_e & \cos\Phi_e\sin\theta_e\cos\psi_e+\sin\Phi_e\sin\psi_e\\ \sin\psi\cos\theta_e & \sin\psi\sin\theta_e\sin\Phi_e+\cos\Phi_e\cos\psi_e & \cos\Phi_e\sin\theta_e\sin\psi_e-\sin\Phi_e\cos\psi_e\\ -\sin\theta_e & \cos\theta_e\sin\Phi_e & \cos\Phi_e\cos\theta_e\end{pmatrix}\begin{pmatrix}\ddot{x}_m\\ \ddot{y}_m\\ \ddot{z}_m\end{pmatrix}\tag{6}$$

To estimate the quadrotor's moment, angular acceleration is estimated by equation (7). Here ω_n signifies the measurement of gyroscope in time nT and T is the sampling time of gyroscope.

$$\dot{\omega}_{n-1}=\frac{1}{2T}(\omega_n-\omega_{n-2})\tag{7}$$

PID Controller

PID controller for position control is designed for quadrotor and it is achieved through two steps. The first step is the attitude control, and the second step is the position control, as shown in Fig.3 and Fig. 4. We need to restructure the dynamics model through equation (8). Combined with equation (1), U_1, desired roll angle Φ^* and desired pitch angle θ^* can be expressed by desired yaw angle ψ^* and desired acceleration in three axes, which corresponds to the block parameter calculation in Fig. 3.

$$\begin{cases} U_1 = T_1 + T_2 + T_3 + T_4 \\ U_2 = (T_1 + T_2 - T_3 - T_4)\dfrac{\sqrt{2}}{2}L \\ U_3 = (T_2 + T_3 - T_1 - T_4)\dfrac{\sqrt{2}}{2}L \\ U_4 = Q_1 - Q_2 + Q_3 - Q_4 \end{cases} \tag{8}$$

The thrusts and moments of rotors can be obtained in equation (9). Combined with equation (8), the desired rotation speed of motor can be determined, which corresponds to the block rotation speed allocation in Fig. 4.

$$\begin{cases} T_i = k\omega_i^2 \\ Q_i = b\omega_i^2 \end{cases} \qquad i = 1,2,3,4 \tag{9}$$

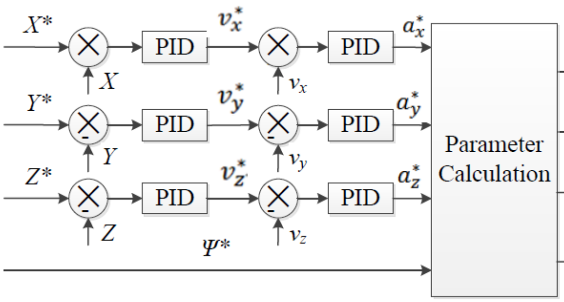

Fig. 3 Position Control

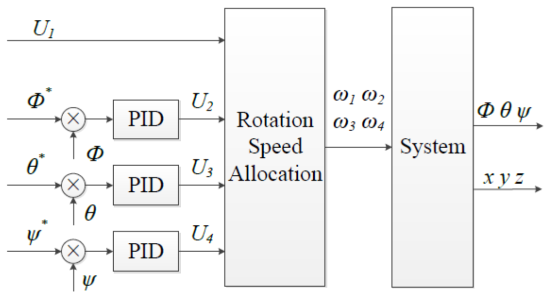

Fig. 4 Attitude Control

Parameters of the PID controller are fixed and tests are made. This controller show short response time, satisfying precision and enough robustness. However, when faults in rotor happen, the flight fluctuates a lot although it becomes stable at last. Such result can be optimized by FTCS, which will be shown after. The simulation result will be compared to prove the advantage of FTCS.

Fault Detection and Diagnosis

Fault Detection and Diagnosis refers to recognizing a fault that has occurred and pinpointing one or more root causes of the fault. As it is well-known, an FDD scheme has three tasks: (1) fault detection indicates that something is wrong in the system; (2) fault isolation determines the location and the type of the fault; (3) fault identification determines the magnitude of the fault. Fault isolation and identification are usually referred as fault diagnosis in the literature [3].

In this paper, only rotor fault is considered. This fault corresponds to the deformation, damage, wreckage of blade, which changes the thrust coefficient k and moment coefficient b. As the model structure is already known and this fault changes model parameters, we choose parameter estimation techniques to detect and diagnose the fault [4]. To simplify the problem, two hypotheses are made: (1) the fault is static, which means the parameter only changes from one constant to another constant after the fault appears; (2) it is impossible to appear two faults simultaneously.

We will use equation (10) to detect and diagnose the fault. From the equation, we can see that we need the real motor rotation speed, the total thrust and the moment in three axes. Real motor rotation speed ω can be calculated from control signal ω_{con} if there is no fault in motor, as shown in equation (4). The total thrust can be obtained from the estimated acceleration in the earth coordinate system, and the moment in three axes can be obtained from the estimated angular acceleration in the body coordinate system.

$$\begin{cases} T = k_1\omega_1^2 + k_2\omega_2^2 + k_3\omega_3^2 + k_4\omega_4^2 \\ \tau_{bx} = (k_1\omega_1^2 + k_2\omega_2^2 - k_3\omega_3^2 - k_4\omega_4^2)\dfrac{\sqrt{2}}{2}L \\ \tau_{by} = (k_2\omega_2^2 + k_3\omega_3^2 - k_1\omega_1^2 - k_4\omega_4^2)\dfrac{\sqrt{2}}{2}L \\ \tau_{bz} = b_1\omega_1^2 - b_2\omega_2^2 + b_3\omega_3^2 - b_4\omega_4^2 \end{cases} \tag{10}$$

There are 8 unknowns and only 4 equations, so it is necessary to use recursive least square estimators to estimate the unknowns [5]. Traditionally, the thrust coefficients can be estimated by the first three equations and the moment coefficients can be estimated by the last one. The four thrust coefficients (or moment coefficients) are estimated all at once, which make its estimation inaccurate with the existence of noise. To eliminate the influence of noise, a new parameter estimation method is proposed which combines the recursive least square estimators and logic operations.

The first step is to detect the fault. The detection is achieved through estimation of thrust coefficient. For a certain rotor, for example, rotor 1, the first three equations of (10) can be written as equation (11).

$$\begin{cases} k_1^a = \dfrac{T - k_2\omega_2^2 - k_3\omega_3^2 - k_4\omega_4^2}{\omega_1^2} \\ k_1^b = \dfrac{\dfrac{\sqrt{2}\tau_{bx}}{L} - k_2\omega_2^2 + k_3\omega_3^2 + k_4\omega_4^2}{\omega_1^2} \\ k_1^c = \dfrac{-\dfrac{\sqrt{2}\tau_{by}}{L} + k_2\omega_2^2 + k_3\omega_3^2 - k_4\omega_4^2}{\omega_1^2} \end{cases} \tag{11}$$

If there is no fault or the fault is only in rotor 1, the thrust coefficient of other rotors is determined and already known, and the thrust coefficient of rotor 1 calculated by the three equations will always be the same (or fluctuate in reasonable interval under noise). Otherwise, the three results will be very different. (Note: Such conclusion is made based on the assumption of single fault) To reduce the fluctuation of estimation results, we introduce a recursive least square estimator with forgetting factor 0.9 for each equation. Until now, the detection logic is clear: if the thrust coefficients calculated by the three equations respectively are close, the estimation is reliable and we can check the estimation to diagnosis the fault; otherwise, the fault must be in other rotor. If we apply such process to each rotor, we can locate the fault and estimate the thrust coefficient. After we locate the fault, we can estimate the moment coefficient of the rotor with fault by the last equation in (10). This process is also achieved by the recursive least square estimator.

Simulation is made to test the performance. We suppose that the fault happens in rotor 1 in the 5$^{\text{th}}$ second, which makes k_1 becomes 4.55×10^{-6} from 9.1×10^{-6}, and b_1 becomes 0.6825×10^{-6} from 1.365×10^{-6}. Simulation results are shown in Fig. 5. We can see that the fault is precisely detected and diagnosed.

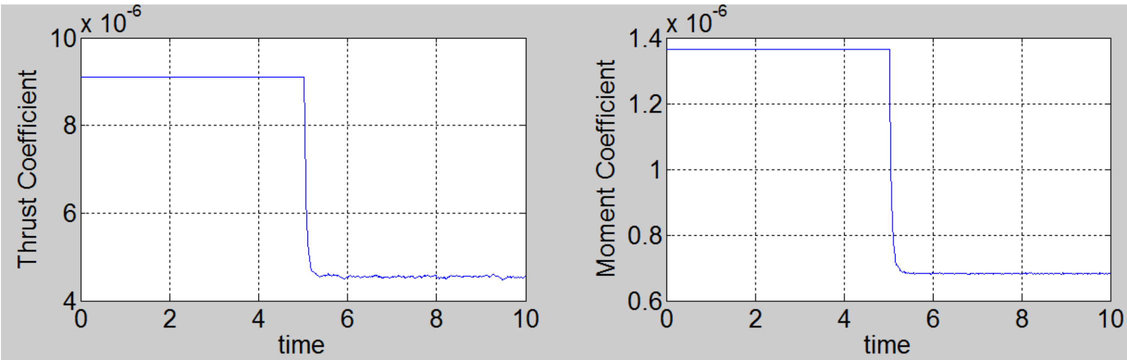

Fig. 5 FDD Results of Rotor 1

Controller Reconfiguration

Controller will be reconfigured automatically after the fault appears. As introduced before, motors' rotation speeds are allocated based on equation (8) and equation (9). After fault appears, thrust coefficient k and moment coefficient b in equation (9) should be adjusted to the diagnosis results and the allocated rotation speed should be recalculated to adapt to the new situation. So the new approach of rotation speed allocation is shown in the following equation.

$$\begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} = \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \\ \frac{\sqrt{2}}{2}k_1L & \frac{\sqrt{2}}{2}k_2L & -\frac{\sqrt{2}}{2}k_3L & -\frac{\sqrt{2}}{2}k_4L \\ -\frac{\sqrt{2}}{2}k_1L & \frac{\sqrt{2}}{2}k_2L & \frac{\sqrt{2}}{2}k_3L & -\frac{\sqrt{2}}{2}k_4L \\ b_1 & -b_2 & b_3 & -b_4 \end{bmatrix}^{-1} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} \tag{12}$$

Simulation is made to test the performance. We suppose that the waypoints of quadrotor are (0,0,0), (0,0,1), (1,0,1), (1,1,1), and the fault happens in rotor 1 in the 5th second, which makes k_1 becomes 4.55×10^{-6} from 9.1×10^{-6}, and b_1 becomes 0.6825×10^{-6} from 1.365×10^{-6}. Comparison is made between traditional PID controller and fault tolerant controller. We can see the fault tolerant controller works better than traditional one.

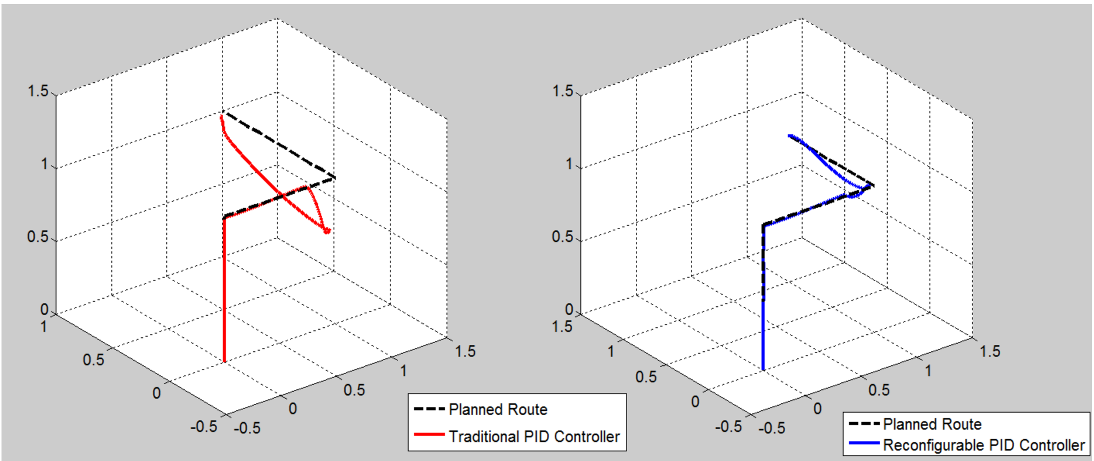

Fig. 6 Fault Tolerant Control Results

Conclusions

This paper has proposed a fault-tolerant control method which includes a fault detection and diagnosis approach for the rotor fault of quadrotor and develops a reconfigurable PID controller. This method successfully improves the performance of quadrotor under fault, as shown in our simulation. Future work will be done to develop new fault-tolerant control method which can cope with more faults. There are always new things for us to explore.

References

[1] Mahmoud, Magdi S., and Yuanqing Xia. Analysis and Synthesis of Fault-tolerant Control Systems. John Wiley & Sons, 2013.

[2] Beard, Randal W. "Quadrotor dynamics and control." Brigham Young University (2008).

[3] Youmin Zhang, and Jin Jiang. "Bibliographical review on reconfigurable fault-tolerant control systems." Annual reviews in control 32.2 (2008): 229-252.

[4] Isermann, Rolf. Fault-diagnosis systems: an introduction from fault detection to fault tolerance. Springer Science & Business Media, 2006.

[5] Toscano, Rosario. "Commande et diagnostic des systèmes dynamiques." Elipses, Paris (2005).