

# Course tracking and simulation of unmanned amphibious platform based on adaptive fuzzy sliding mode control

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**Abstract:** Unmanned amphibious platform (UAP in short) is a complex nonlinear system. It shows plenty of dynamic properties, such as parameter uncertainty and large time-lag, when controlled, which hinders the development of the approaches of exactly course tracking. Motivated by adaptive fuzzy sliding mode control method, a new course tracking method is proposed in this paper. Firstly, UAP's manipulating response function is established, and then the function is transformed to SISO system based on diffeomorphism. Secondly, course tracking method is established based on adaptive fuzzy sliding mode control method, and its global stability is testified theoretically under the favor of Lyapunov method. In the end, simulation is carried out based on a UAP, and the result illustrates that the approach is stable to disturbances and uncertainties.

## Introduction

With the development of modern technology, intelligent equipment such as unmanned ground vehicle, unmanned aerial vehicle, and unmanned surface vehicle, has made a big progress[1], which provides new power to the development of military equipment[2,3]. Under this background, unmanned amphibious platform (UAP in short) comes into being. It can carry out the beach landing assault task instead of traditional amphibious vehicles after equipped with remote weapon station, which reduces the casualties effectively[4]. However, UAP works in a complex environment, and it tends to be influenced by the outside natural environment such as wind, wave and flow. As a result, the movement of UAP shows strong nonlinearity, uncertainty and large time delay, etc. In order to improve the stability and maneuverability of UAP, it's necessary to establish an effective strategy to control UAP. Adaptive fuzzy sliding mode control (AFSMC in short) provides us an effective method for its insensitivity to system parameters' variations and the external disturbances[5]. Besides, the controller established based on AFSMC has a simple structure, which makes AFSMC become more and more popular in nonlinear systems[6]. Based on AFSMC, this paper is focused on the course tracking of UAP under the influence of uncertainties. UAP's manipulating response function is established firstly, and diffeomorphism is applied on it. Then the course tracking strategy is established based on AFSMC, which is validated by simulations in the end.

## Problem Statement

The unmanned amphibious platform studied in this paper is equipped with single steering and rudder. Being similar to a ship, we can get the maneuver response model of UAP with reference to Norbin model[7]:

$$Tj\ddot{\theta} + j\dot{\theta} + a_j\theta = Kd \quad (1)$$

Where  $d$  is the rudder angle,  $r = \dot{j}$  is the yaw rate,  $j$  is the heading angle,  $T$  is time constant,  $K$  is rudder gain, and  $a$  is Norbin coefficient.

The rudder system's mathematical model can be predigested as follows:

$$T_E \dot{d} + d = K_E d_E \quad (2)$$

Where  $d_E$  is the reference rudder angle,  $d$  is the actual rudder angle,  $K_E$  is the rudder system's control gain, and  $T_E$  is the rudder system's time constant.

Choose  $x_1 = j$ ,  $x_2 = \dot{j} = r$ ,  $x_3 = d$ ,  $u = d_E$  as state variable, and UAP's maneuvering mathematical model can be obtained according to Eq. (1) and Eq. (2):

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{1}{T}x_2 - \frac{a}{T}x_2^3 + \frac{K}{T}x_3 \\ \dot{x}_3 = -\frac{1}{T_E}x_3 + \frac{K_E}{T_E}u \\ y = x_1 \end{cases} \quad (3)$$

Apply diffeomorphism to Eq. (3), and consider that

$$\begin{cases} z_1 = x_1 \\ z_2 = x_2 \\ z_3 = -\frac{1}{T}x_2 - \frac{a}{T}x_2^3 + \frac{K}{T}x_3 \end{cases}$$

Then Eq. (4) can be obtained:

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = z_3 \\ \dot{z}_3 = f(z) + g(z)u \\ y = z_1 \end{cases} \quad (4)$$

Where  $g(z) = \frac{KK_E}{TT_E}$  is an unknown parameter, and  $f(z) = -\frac{z_2 + az_2^3}{TT_E} - \frac{z_3 + 3az_2^2z_3}{T} - \frac{z_3}{T_E}$  is an unknown nonlinear function. Therefore, thanks to diffeomorphism, the course tracking problem of UAP is equivalent to stabilize Eq.(4), which is a nonlinear unmatched SISO system. And the control objective is to design a feedback control law to ensure system (4) is stable.

For convenience of the following analysis, an assumption is proposed:

*Assumption 1:* The sign of function  $g(z)$  is known, and without loss of generality, assume  $g(z) > 0$  and that there are given functions  $M_{g1}(z)$  and  $M_{g2}(z)$  that make  $f(z)$  and  $g(z)$  satisfy the following inequalities:

$$\begin{aligned} |f(z)| &\leq M_f(z) \\ 0 < M_{g1}(z) &\leq g(z) \leq M_{g2}(z) \end{aligned}$$

*Definition 1:* The operator Proj[8] has the following form:

$$\text{Proj}_{\hat{q}_i}(y) = \begin{cases} 0 & \hat{q}_i = q_{\min}, y < 0 \\ 0 & \hat{q}_i = q_{\max}, y > 0 \\ 0 & \text{others} \end{cases}$$

Where  $\theta(t)$  is a unknown vector with parameters time-varying, and  $\hat{q}(t)$  is the estimate of  $\theta(t)$ .

The properties of operator Proj is shown as follows:

- (1)  $\hat{q} \in \Omega_{\hat{q}} = \{\hat{q} \mid q_{\min} \leq \hat{q} \leq q_{\max}\}$
- (2)  $(\hat{q} - q)(\text{Proj}_{\hat{q}}(y) - y) \leq 0, \forall y$

In order to deal with the unknown nonlinear system, a control law based on fuzzy logic control with input singleton fuzzification, product inference and center average defuzzification, is established. Suppose that the  $i$ -th fuzzy rule is shown as following:

$R_i$ : if  $x_1$  is  $F_{i1}$ ,  $x_2$  is  $F_{i2}$ , ..., and  $x_n$  is  $F_{in}$ , then  $y=w_i$ .

Where  $x=[x_1, x_2, \dots, x_n]^T$  and  $y$  denote fuzzy logic system's input and output respectively;  $n$  denotes the number of variables, and  $m$  denotes the number of fuzzy rules;  $w_i$  denotes the singleton value of the  $i$ -th fuzzy rule, and  $F_{ij}$ , whose membership function is as following, denotes the domain  $X_i$ 's fuzzy set:

$$m_{F_{ij}}(x_j) = \exp\left(-\left(\frac{x_j - \bar{x}_{ij}}{s_{ij}}\right)^2\right) \quad (5)$$

Hence, combining all the single fuzzy rule sets, we can obtain the fuzzy system's model:

$$y(x) = \frac{\sum_{i=1}^m w_i \left( \prod_{j=1}^n m_{F_{ij}}(x_j) \right)}{\sum_{i=1}^m \left( \prod_{j=1}^n m_{F_{ij}}(x_j) \right)} \quad (6)$$

Consider that the membership's parameters such as  $\bar{x}_{ij}$  and  $s_{ij}$  are fixed, and the fuzzy singleton  $w_i$  are tunable, then Eq. (6) can be transferred as follows:

$$\begin{cases} y(x|q) = \sum_{i=1}^m q_i h_i(x) = q^T h(x) \\ h_i(x) = \frac{\prod_{j=1}^n m_{F_{ij}}(x_j)}{\sum_{i=1}^m \left( \prod_{j=1}^n m_{F_{ij}}(x_j) \right)} \end{cases} \quad (7)$$

Where  $h(x)=[h_1(x), h_2(x), \dots, h_m(x)]^T$  is called fuzzy function vector, and  $\theta=[\theta_1, \theta_2, \dots, \theta_m]^T \in R^m$  is called parameter vector.

**Lemma 1:** For any given real positive number real  $\varepsilon$  and any real continuous function  $y$  which is defined on a compact set  $X \subset R^n$ , there is a fuzzy logic system  $y^*$  with Eq. (8) form that satisfies the equation  $\sup_{x \in X} |y^*(x|q) - y(x)| < \varepsilon$ .

According to Lemma 1, function  $f(x)$  can be described as

$$f(x) = \mathbf{q}^T h(x) + \Delta f(x), \quad \forall x \in X \subset \mathbb{R}^n$$

Where the approximate error satisfies the equation:  $\sup_{x \in X} |\Delta f(x)| < \epsilon$

## Design of Controller

(1) Design of equivalent controller

Applying multiple sliding mode control to system (4), three sliding mode surfaces are defined as follows:

$$s_i = z_i - z_{id}, \quad i = 1, 2, 3$$

Where  $z_{id}$  denotes the desired value of state variable, and  $z_{1d} = \psi_d$ . The design procedure is as follows:

Step 1: In sight of Eq. (4), the derivative of the first sliding surface  $s_1 = z_1 - \psi_d$  is

$$\dot{s}_1 = \dot{z}_1 - \dot{\psi}_d = \dot{z}_1 - \dot{z}_{1d} \quad (8)$$

In order to stabilize Eq. (8), the virtual control  $z_{2d}$  should be designed as

$$z_{2d} = \dot{z}_{1d} - c_1 s_1 \quad (9)$$

Where  $c_1$  is a positive design parameter.

Step 2: In sight of Eq. (4), the derivative of the second sliding surface is

$$\dot{s}_2 = \dot{z}_2 + \dot{z}_{3d} - \dot{z}_{2d} \quad (10)$$

In order to stabilize Eq. (10), the virtual control  $z_{3d}$  should be designed as

$$z_{3d} = \dot{z}_{2d} - c_2 s_2 \quad (11)$$

Where  $c_2$  is a positive design parameter.

Step 3: In sight of Eq. (4), the derivative of the third sliding surface is

$$\dot{s}_3 = \dot{z}_3 - \dot{z}_{3d} = f(z) + g(z)u - \dot{z}_{3d} \quad (12)$$

In order to stabilize Eq. (12), the control  $u$  should be designed as

$$u = \frac{1}{g(z)} [-f(z) + \dot{z}_{3d} - c_3 s_3] \quad (13)$$

However, it is impossible to design controller  $u$  because  $f(z)$  and  $g(z)$  are unknown nonlinear function. Thanks to fuzzy logic control,  $f(z)$  and  $g(z)$  can be approximately replaced by the fuzzy logic function  $\hat{f}(z)$  and  $\hat{g}(z)$ :

$$\hat{f}(z) = \mathbf{q}_1^T h_1(z) \quad (14)$$

$$\hat{g}(z) = \mathbf{q}_2^T h_2(z) \quad (15)$$

Therefore, the equivalent controller  $u_{CE}$  can be derived by replacing  $f(z)$  and  $g(z)$  in Eq. (13) with  $\hat{f}(z|\mathbf{q}_1)$  and  $\hat{g}(z|\mathbf{q}_2)$ :

$$u_{ce} = \frac{1}{\hat{g}(z|q_2)} \left[ -\hat{f}(z|q_1) + \mathfrak{z}_3 - c_3 s_3 \right] \quad (16)$$

Substituting  $u_{CE}$  into system Eq. (4) yields:

$$\dot{\mathfrak{z}}_3 = \Delta f(z) + \Delta g(z)u_{ce} - c_3 s_3 \quad (17)$$

Define Lyapunov function as

$$V_1 = \frac{1}{2} s_3^2 \quad (18)$$

In sight of Eq. (18), the derivative of  $V_1$  is

$$\dot{V}_1 = -c_3 s_3^2 + [\Delta f(z) + \Delta g(z)u_{ce}] s_3 \quad (19)$$

It is obvious that system Eq. (4) can't be stabilized by  $u_{CE}$  alone because of the existence of approximate errors  $\Delta f(z)$  and  $\Delta g(z)$ . And compensate controller  $u_s$  (as is shown below) should be designed either.

$$u = u_{ce} + u_s \quad (20)$$

(2) Design of compensate controller

Substituting Eq. (20) into system (4) yields:

$$\dot{\mathfrak{z}}_3 = \Delta f(x) + \Delta g(x)u_{ce} - c_3 s_3 + g(x)u_s \quad (21)$$

Taking the Lyapunov function Eq. (18) into account gives

$$\begin{aligned} \dot{V}_1 &= -c_3 s_3^2 + s_3 [\Delta f(x) + \Delta g(x)u_{ce} + g(x)u_s] \\ &\leq -c_3 s_3^2 + |s_3| [|\Delta f(x)| + |\Delta g(x)||u_{ce}|] + s_3 g(x)u_s \\ &\leq -c_3 s_3^2 + |s_3| \{ |f(x)| + |\hat{f}(x|q_1)| + [g(x) + |\hat{g}(x|q_2)|] |u_{ce}| \} + s_3 g(x)u_s \end{aligned} \quad (22)$$

And according to *assumption 1*, we can get

$$|f(x)| \leq M_f(x)$$

$$0 < M_{g1}(x) \leq g(x) \leq M_{g2}(x)$$

In order to satisfy the inequality  $\dot{V}_1 \leq -c_3 s_3^2$ , the compensate controller should be designed as

$$u_s = -\frac{1}{M_{g1}(x)} \{ M_f(x) + |\hat{f}(x|q_1)| + [M_{g2}(x) + |\hat{g}(x|q_2)|] |u_{ce}| \} \text{sat}(s_3 / l) \quad (23)$$

Where  $l$  is the thickness of boundary layer.

In the end, choose the parameter vector  $q$ 's self adaptive law:

$$\begin{aligned} \dot{q}_1 &= r_1 \text{Proj}_{q_1}(s_3 h_1(x)) \\ \dot{q}_2 &= r_2 \text{Proj}_{q_2}(s_3 h_2(x) u_{ce}) \end{aligned} \quad (24)$$

Where  $r_1$  and  $r_2$  are the learning rates.

## Stability Analysis

Theorem can be established on summary of all the above analyses.

*Theorem 1:* As to the closed-loop system Eq. (4) consisted of controller Eq. (23) and parameter adaptive law Eq.(24), under the precondition of *assumption 1*, for the given compact set  $\Omega_n \subset R^n$ , the closed-loop system Eq. (4) is bounded while its initial state satisfies  $x(0) \in \Omega_n$ . Moreover, the tracking error can converge to any given sliding mode saturated layer.

Proof: Define Lyapunov function as

$$V = \frac{1}{2} s_3^2 + \frac{1}{2r_1} \hat{q}_1^* \hat{q}_1^* + \frac{1}{2r_2} \hat{q}_2^* \hat{q}_2^* \quad (25)$$

The derivative of  $V$  is

$$\begin{aligned} \dot{V} &= -c_3 s_3^2 + s_3 \{ f(x) - \hat{f}(x|q_1) + [g(x) - \hat{g}(x|q_2)] u_{ce} \} + g(x) u_s + \frac{1}{r_1} \hat{q}_1^* \dot{\hat{q}}_1^* + \frac{1}{r_2} \hat{q}_2^* \dot{\hat{q}}_2^* \\ &= -c_3 s_3^2 + s_3 \{ f(x) - \hat{f}(x|q_1^*) + [g(x) - \hat{g}(x|q_2^*)] u_{ce} \} + g(x) u_s + s_3 \{ \hat{f}(x|q_1^*) - \hat{f}(x|q_1) + [\hat{g}(x|q_2^*) - \hat{g}(x|q_2)] u_{ce} \} \\ &\quad + \frac{1}{r_1} \hat{q}_1^* \dot{\hat{q}}_1^* + \frac{1}{r_2} \hat{q}_2^* \dot{\hat{q}}_2^* \\ &= -c_3 s_3^2 + s_3 \{ f(x) - \hat{f}(x|q_1^*) + [g(x) - \hat{g}(x|q_2^*)] u_{ce} \} + g(x) u_s + \hat{q}_1^* \left[ -\frac{1}{r_1} \dot{\hat{q}}_1^* - s_3 h_1(x) \right] + \hat{q}_2^* \left[ -\frac{1}{r_2} \dot{\hat{q}}_2^* - s_3 h_2(x) u_{ce} \right] \quad (26) \end{aligned}$$

According to the parameter self adaptive law, compensate controller and the 2<sup>nd</sup> property of discontinuous projection algorithm, we can obtain

$$\dot{V} \leq -c_3 s_3^2 \quad (27)$$

## Simulation study

This section illustrates the performance of the designed trajectory tracking control system via numerical simulation. The control system was simulated in MATLAB/Simulink. The model parameters of the UAP are given as follows:  $K=0.49$ ,  $T=208.91$ ,  $K_E=1$ ,  $T_E=2.5$ ,  $a=30$ .

The design parameter of the control strategy was set to be  $r_1=10$ ,  $r_2=5$ ,  $c_1=0.5$ ,  $c_2=10$ ,  $c_3=1$ ,  $l=50$ . Besides, the desired course was set as a constant value:  $\psi_d=20$ .

Simulation results were obtained for two methods: traditional PID control method, and control strategy proposed in this paper based on AFS method. In all of the following simulations we used the same initial state of UAP:  $x_1(0)=x_2(0)=x_3(0)=0$ .

Simulation results are shown from Fig. 1 to Fig. 4. As is shown in Fig. 1 and Fig. 2, it is obvious that the convergency speed of AFSMC is faster than that of traditional PID, which validated the effectiveness of the control strategy proposed based on adaptive fuzzy sliding mode control method.

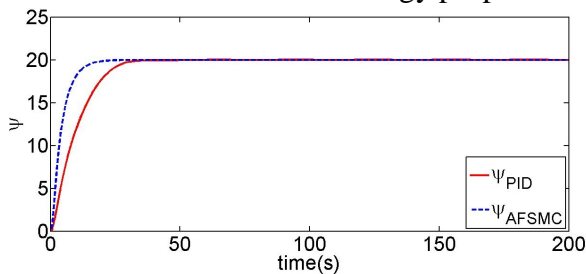


Fig. 1 Time response of heading angle

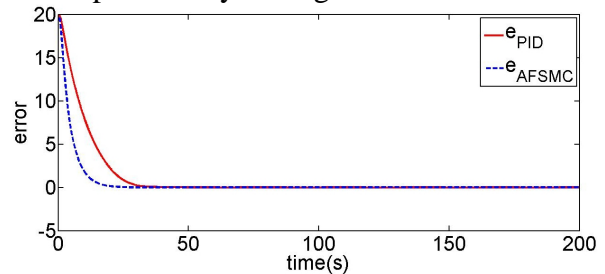


Fig. 2 Time response of heading angle error

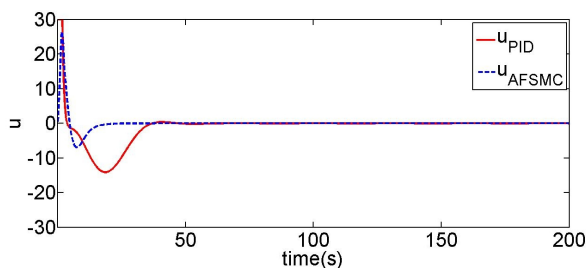


Fig. 3 Time response of rudder

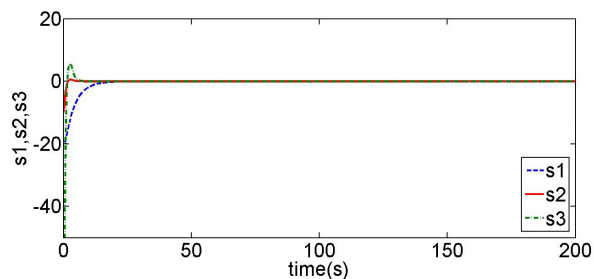


Fig. 4 Time response of system state

## Conclusions

In this paper, UAP's course tracking strategy was established based on adaptive fuzzy sliding mode control, and simulation was conducted which validated the effectiveness of the established strategy. However, only simple model without disturbances was taken into account, which is the next work to do.

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