A Mechanical Fault Feature Extraction Method Based on Volterra Series Model for EEMD Decomposition

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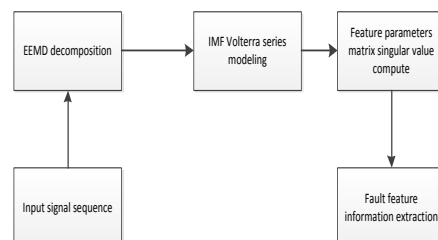
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Abstract: This paper, based on the existed EMD decomposition to extract the fault feature signal, will apply the analytical model named Volterra series model of chaotic time series to the fault diagnosis of rotating machinery. The method of combining EEMD decomposition and Volterra series model is proposed to extract the feature information of mechanical fault. Compared with the traditional fault feature extraction methods, this method has the advantages of adequate theoretical basis, novel method, obvious extraction features , better anti-noise interference ability，simple computation and so on. Simulation experiments show that the proposed method can effectively extract the feature parameters. By applying the method to the fault feature extraction of rotating machinery,the results are obtained satisfactorily.

Introduction

Actually, the actual system in engineering is almost always containing a wide variety of nonlinear factors,which contain a lot of fault feature information. To extract the relevant feature information from these vibration signals is the primary task of fault diagnosis and is also the key point. In the traditional sense,the fault feature extraction is based on the structural dynamic equations of the mechanical system. The theoretical basis of the mechanical structure is the inherent frequency, damping, stiffness and the excitation condition of the mechanical structure. So what are extracted from the traditional fault feature is the dynamic characteristic parameters, such as inherent frequency, vibratory mode and time-history response. However, in practical applications, when the system is affected by the vibration and noise, the parameters will lose the meaning of its faulty damage characteristics.There were some experiments showing that after processing the signals with HHT transform or wave transform,the results obtained by the two processing can be used to characterize fault damage feature of the mechanical structure , and the characteristic parameters are highly reliable, and are not affected easily by the outside conditions.

At present, because the Volterra series model has a clear physical meaning and can reflect the essential characteristics of the nonlinear system, it is becoming a new research direction to diagnose faulty based on the Volterra series model. In order to solve these problems, this paper proposes a method of combining EEMD with Volterra series model to extract mechanical fault feature (Fig.1).

Fig.1 The fault feature extraction by EEMD-Volterra method

Some experiments results conclude that it is not obvious to extract fault feature information with the single use of the Volterra model immediately, so this paper tries to decompose the original signal using EEMD method, so that the fault feature can be reasonably distributed in each IMF component. The next step is to build the Volterra model for the IMF component and to establish the parameter matrix using Volterra as the element,.Finally the singular value of matrix is used to characterize different mechanical fault characteristics.

In the practical application, considering that the decomposition of EMD[1] is more sensitive to the effects of noise and the endpoint, the effect of the EMD decomposition is affected by many factors[2,3], hence before decomposition ,something should be done with the signal. In this paper, the method of EEMD is proposed to take place of EMD to act as the tool to decompose the signal, which is based on the EMD decomposition method, and the effect ofthe decomposition of EEMD is expected to reduce the impact of noise to minimum.

EEMD-VolterraParameters Estimation

EEMD Decomposition Basis
In terms of the problem of the EMD decomposition method, EEMD is proposed,which is a method of auxiliary data analysis. EEMD decomposition principle is that when the additional white noise is distributed in the whole time frequency space, the time frequency space is composed of different components of the filter bank. When the signal is added up with distributed white noise background, the signal regions of different scales will be automatically mapped to the appropriate scale related to the background white noise. Since the noise is different , the noise will be eliminated when the total means of the adequate test are used. The mean of the whole will eventually be considered as the real result, the only persistent part is the signal itself, and many tests are added to eliminate the added noise. Based on the assumption that any signal is composed of a series of basic mode components, whose amplitude and phase are always changing with time, the basic mode component must satisfy two conditions, that is, the number of zeros and poles are equal   or at most differ by one, and the upper and lower envelope which is determined by its maximum value and minimum value are distributed symmetrically in locality by the time axis. Huang defined this mode component as the intrinsic mode function, that is IMF. The EEMD method can be used to screen the IMF component from the multi component signal. Its concrete steps are as follows：

Set the original signal $Y(t)$:

（1）To superimpose a set of Gauss white noise named $q(t)$ on the original signal $Y(t)$,then get a general signal,that is: $Y(t) = Y(t) + q(t)$;

（2）Calculate all the maximum and minimum values of $Y(t)$;

（3）According to the maximum value and minimum value,three-order spline interpolation was operated to construct the upper and lower envelope of $Y(t)$;

（4）According to the upper and lower envelope ,the local mean of $Y(t)$ named $m_{11}(t)$ was calculated and the difference value between $Y(t)$ andwas $m_{11}(t)$ defined as $h_{11}(t)$,that is: $h_{11}(t) = Y(t) - m_{11}(t)$;

（5）To replace the general signal,then circulate the step(1)(2)(3)，Until that the variance of $h_{1,k-1}$ and $h_{1,k}$ is less than a set value would $h_{11}(t)$ be considered   as a IMF component,

notes： $c_1 = h_{1,k-1}, r_1(t) = Y(t) - c_1(t), Y(t) = r_1(t)$;

Loop the last four step,untillrn(t) is less than a set value or it becomes a monotonic function. The EEMD decomposition of the original signal is over.thefinal decomposition type of $Y(t)$ is:

$$Y(t) = \sum_{i=1}^{N} c_i + r \qquad (1)$$

Volterra Series Model based on Chaotic Time Series
Volterra series is first put forward by the Italian mathematician Volterra, and Wiener[4],as the founder of control theory,is the first man to apply Volterra series to ananlyzenonlinear system. The method of delay coordinate state space reconstruction that put forward by Packad[5] is an important method to study the chaotic time series. Using the phase space reconstruction to study chaotic time series $\{U(n)\}$ was proposed by Packard and Takens.the theorem of Takens[6]proved, under the condition of determining the appropriate embedding dimension, if the delay coordinate dimension

$m \geq 2d + 1$ (d is the order of the original system), in the embedding dimension space , the regular trajectory (attractor and strange attractor) can be restored, that is ,the trajectory in the reconstruction of the space and the original dynamic system keep the homeomorphisms. Therefore, the next state of the system can be obtained by the current state of the system, so that the prediction value of the time series can be obtained. The essence of chaotic time series prediction is the inverse problem of a dynamical system, which is toreconsitute the dynamic model of the system by the state of the dynamical system $F(.) : U(n+T) = F(U(n))$ (2)

Of which, T is the forward prediction step (T>0). For the single input and single output nonlinear system with time-varying causality, the relations between input and output can be expressed as follows: Volterra functional series form:

$$y(n) = y0 + \sum_{n=1}^{N} \sum_{i_1=0}^{M} ... \sum_{i_k=0}^{M} [h_k(i_1, i_2, ..., i_k) \prod_{k=1}^{M} u(n-i_k)] + e(n) \qquad (3) N$$

is the Volterra model of the highest order,and M is the length of the memory, $u(t), y(t) \in R$

respectively is input and output of the system, $h_k(i_1, i_2, ..., i_k)$ is the k order Volterra time domain kernel or k order generalized impulse response function (GIRF), $e(n)$ is the truncation error.

In the system analysis, the dynamic characteristics of a large class of nonlinear systems can be described by the first three order Volterra series [7]. Based on this, this paper considers only the first three order Volterratime domain kernel function.The input observation matrix of the system is defined as: $U = [U(k), U(k+1), ..., U(k+N-1)]^T$ (4)

Of which， $U(k) = [u(k), ..., u(k-M+1), u^2(k), u(k)u(k-1), ..., u^N(k+N-1)]^T$ (5)

The output observation matrix of the system is defined as:

$Y = [y(k), y(k+1), .., y(k+N-1)]^T$ (6)

The Volterra kernel phasor of the system is defined as：

$H = [h_1(0), ..., h_1(M-1), h_2(0,0), h_2(0,1), ..., h_N(M-1, M-1)]^T$ (7)

The observation equation of Volterra system can be described as follow:

$Y(n) = U(n)H(n) + e(n)$ (8)

We can use the former equation to calculate Voterra series kernel,actually,the process of which is a standard least squares parameter estimation problem .That is：
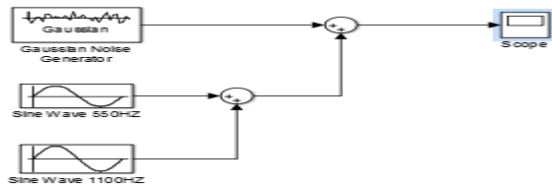
$\hat{H} = [U(n)^T U(n)]^{-1} U(n)^T Y(n)$ (9)

By using the normalized least square adaptive algorithm, the accurate value of the coefficient vector H can be obtained, and the current value of H contains the vast majority information of the system, which can be used as an important parameter of the system.
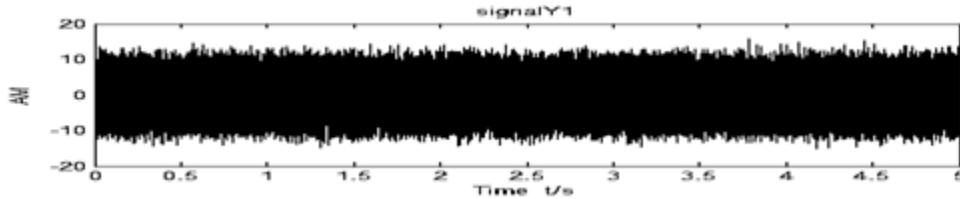
Simulation Experimental Analysis

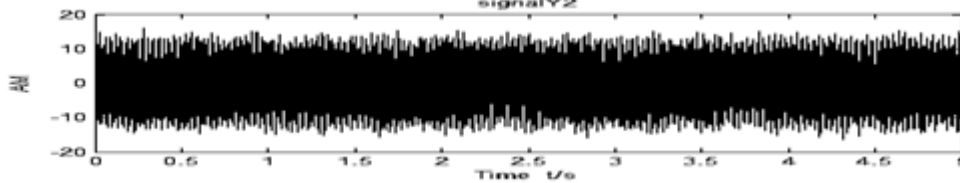### Structure of Simulation Signals with Noise.
In order to better simulate the real situation, a method of the based signal mixed with characteristic signal and noise is used to generate the simulation signal. of which the based signal is sinusoidal signal 550 Hz, and the characteristic signal is sinusoidal 1100 Hz, 1300 Hz,and 1500HZ. the noise added is white noise, the sampling frequency is 10 kHz, the sampling time is 5s,each group simulation signal of to 5 000 points as a sample generates 5 groups of sample signal. The following diagramsare the time domain diagram ofevery simulation signal.The simulation signals generator model shows as follow Fig.2.
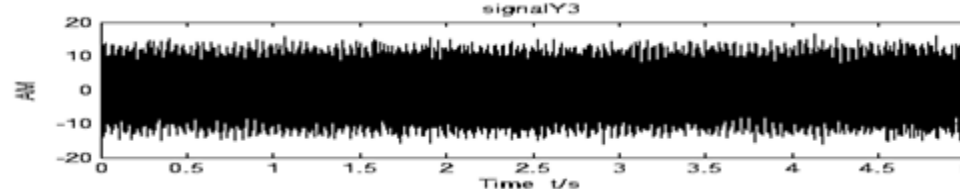
Fig.2 The simulation signals generator model

Fig.3 The first simulation signal

Fig.4 The seconds simulation signal

Fig.5 The third simulation signal

Experimental Results

5 samples of each simulation signal will all be undertaken EEMD decomposition. According to the actual situation, several or more IMF components are selected to establish the Volterra series model and predict the system parameters. It is pointed out that the embedding dimension (m=3)of chaotic time series Volterra prediction can be obtained by G-P algorithm, and the delay time t=10, which is obtained by C-C [8] method. Volterra expansion order is 3order.

Table 1 is a singular value calculated by the proposed method in the paper.

Table 1 EEMD-Volterramethodsingular value result

signal	Singular value				
	sample1	sample 2	sample 3	sample 4	sample 5
X1	1.0408	1.0411	1.0413	1.0439	1.0406
	0.959	0.9612	0.9614	0.9611	0.9658
	0.9468	0.9494	0.948	0.9503	0.9419
	0.6098	0.6072	0.6162	0.6039	0.604
X2	1.0554	1.0552	1.0543	1.0556	1.0543
	0.9559	0.9565	0.9566	0.9578	0.9576
	0.9556	0.9584	0.9577	0.9582	0.957
	0.6058	0.6043	0.6042	0.6052	0.6141
X3	1.0653	1.0669	1.0666	1.0646	1.0647
	0.9625	0.9616	0.9616	0.9613	0.9612
	0.9459	0.9404	0.9382	0.9398	0.9472
	0.6069	0.6087	0.608	0.6042	0.6082

It Can be seen from Table 1 that different samples of the same simulation signal,the singular value is nearly the same. Meanwhile, the singular value ofdifferent signal will differ fromdifferent signals.For example ,The singular values of signal X1 are all about 1. 04, and The singularvalues of signalX2and X3 are all respectively about 1.05 and 1.06. This difference also reflected on the

second number and the third number of the singular value, which shows that the method can effectively extract the different characteristics of the signal simulation steadily.
In order to verify furtherly the validity and accuracy of the proposed method.This paper will apply Wavelet-Volterra method to solve the system parameters, that is to compute singular values.On the same experimental condition, the Haar wavelet was used to have the above simulation signal 3-oder decomposed,then the signals that had composed are used to extract feature information by using Volterra prediction method .the results are shown in Table2.

Table 2 Wave-Volterra singular value results

signal	Singular value				
	sample1	sample2	sample3	sample4	sample5
X1	1.1313	1.0521	1.1293	1.1203	1.1114
	1.0737	1.0139	1.0539	1.0681	1.0686
	0.9497	0.9057	0.9496	0.9448	0.926
	0.6624	0.6224	0.6644	0.7069	0.6222
X2	1.0379	0.9861	1.0248	0.9894	0.9816
	1.0454	1.0096	1.0935	1.0235	0.9501
	0.9391	0.936	0.9527	0.9169	0.9799
	0.7116	0.6247	0.6238	0.6251	0.6225
X3	1.0166	0.9813	1.0386	1.0116	0.9814
	1.0551	0.9457	1.0718	1.0516	0.9469
	0.9507	0.8901	0.9491	0.949	0.887
	0.648	0.6229	0.6313	0.6406	0.6227

It can be seen from Table 2 that the decomposition of the singular values of each signal is disordered, and all the values aren't fixed in a certain range.The singular valuesare also different,even differ by a lot.The whole value are chaotic.

Conclusions

This paper undertakes the way of mechanical fault feature extraction.
1.The first one is to combine the decomposition of EEMD with and the Volterra series model prediction method. Compared with the wavelet-Volterra method, the method has the advantages of high stability，reliabilityand rapid calculation.
2.The simulation results are satisfactoryenough.This method can be an effective way to extract the fault information of   characteristic signals of rotating machinery.

Acknowledgements

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