

A novel image restoration technique based on anisotropic diffusion model

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Abstract. According to the principles of image restoration and non-linear constraint optimization, a novel image restoration algorithm based on the anisotropic diffusion model was proposed in this article. Based on the image textures and the gradient characteristics at the image edge, the adaptive anisotropic diffusion model was introduced and the cost function was minimized by the energy minimization scheme. Moreover, the nonlinear equations can be linearized using the fixed-point alternating iterative method. As the experimental results demonstrated, the proposed method can effectively restore the blurred images and enhance the restoration quality.

Introduction

During the acquisition process, the images are easily affected by the following factors such as the inaccurate focusing, the relative movement and the environmental influences in collecting and transmitting, all of which will lead to the reduction of imaging quality. The images will become blurred, and the noise will be added simultaneously^[1-11]. In this article, to reduce the step-shaped and fuzzy phenomena in images and acquire more favorable image restoration results, we focused on the texture and structural characteristics of the blurred images, and adopted the anisotropic diffusion method to process the images, with the aim of enhancing the quality of the original images.

Theoretical bases of image restoration

Aiming at solving the problem of image blurring, Y-K method uses the joint minimization approach and the constraint conditions for u and h can be written as:

$$\min_{u, h} J(u, h) = \min_{u, h} \frac{1}{2} \|h * u - u_0\|_{L^2(\Omega)}^2 + \alpha_1 \int_{\Omega} |\nabla u|^2 dx dy + \alpha_2 \int_{\Omega} |\nabla h|^2 dx dy. \quad (1)$$

Based on the Y-K method, C-W method put forward the following improvement model^[6]:

$$\min_{u, h} J(u, h) = \min_{u, h} \frac{1}{2} \|h * u - u_0\|_{L^2(\Omega)}^2 + \alpha_1 \int_{\Omega} |\nabla u|^2 dx dy + \alpha_2 \int_{\Omega} |\nabla h|^2 dx dy. \quad (2)$$

In this article, to keep the image smooth in the smooth regions and retain the texture details as much as possible in the texture regions, an image restoration method based on self-adaptive diffusion model was proposed, which combining the strengths of C-W method and Y-K method. In addition, the proposed method can reduce the noise in images as well.

Image restoration based on adaptive anisotropic diffusion model

In an image, the texture region mainly reflects its local gradient information. In this article, the joint minimization of the P-Laplace-based image restoration can be written as:

$$\min_{u,h} J(u, h) = \min_{u,h} \frac{1}{2} \|h * u - u_0\|_{L^2(\Omega)}^2 + \alpha_1 \int_{\Omega} |\nabla u|^{p(\nabla u)} dx dy + \alpha_2 \int_{\Omega} |\nabla h|^{p(\nabla h)} dx. \quad (3)$$

in which α_1 and α_2 are the positive parameters and $p(\nabla u)$ denotes the diffusion controlling factor, with the value range from 1 to 2, i.e., $1 \leq p(\nabla u) \leq 2$. Here:

$$p = \frac{2 + |\nabla u|}{1 + |\nabla u|}. \quad (4)$$

in which the gradient $|\nabla u|$ is determined by the neighboring eight pixels around the center pixel.

a. When the gradient approaches to the infinity, $p \rightarrow 1$ and the image restoration model can be converted to an anisotropic blind image restoration model by the C-W method;

b. When the gradient approaches to the zero, $p \rightarrow 2$ and the image restoration model can be converted to an anisotropic blind image restoration model by the Y-K method.

Then we conducted the processing on Eq. (3) and take the partial derivatives with respect to u and h . To simplify the calculations, the value of p is set and then the Euler-Lagrange Equation of Eq. (3) can be obtained. The partial derivative of Eq. (3) relative to u can be expressed as:

$$-p\alpha_1 \operatorname{div}(|\nabla u|^{p-2} \nabla u) + (h * u - u_0) * u(-x, -y) = 0. \quad (5)$$

To take the partial derivative on Eq. (5) relative to h , the obtained Euler-Lagrange can be written as:

$$(h * u - u_0) * h(-x, -y) - \alpha_2 \nabla \cdot \left(\frac{\nabla h}{|\nabla h|} \right) = 0. \quad (6)$$

Here we introduce a time variable t and adopt the gradient descent algorithm, the anisotropic diffusion model can be expressed as:

$$\begin{cases} u_t = \alpha_1 p \nabla \cdot (|\nabla u|^{p-2} \nabla u) - (h * u - u_0) * u(-x, -y) \\ h_t = \alpha_2 \nabla \cdot \left(\frac{\nabla h}{|\nabla h|} \right) - (h * u - u_0) * h(-x, -y) \end{cases} \quad (7)$$

To the most important diffusion term $\nabla \cdot (|\nabla u|^{p-2} \nabla u)$ in the model, we let:

$$\frac{\partial^2 u}{\partial x^2} = u_{xx}, \quad \frac{\partial^2 u}{\partial x \partial y} = u_{xy} \quad (8)$$

In a local coordinate system, we can obtain:

$$\frac{\partial u}{\partial t} = \nabla \cdot (|\nabla u|^{p-2} \nabla u) = |\nabla u|^{p-2} u_{\xi\xi} + (p-1) |\nabla u|^{p-2} u_{\eta\eta} \quad (9)$$

in which $u_{\xi\xi}$ and $u_{\eta\eta}$ denote the two-order partial derivatives in a local coordinate system. In the above Eq. (9), the diffusion coefficients are $|\nabla u|^{p-2}$ and $(p-1)|\nabla u|^{p-2}$.

Experimental results and analyses

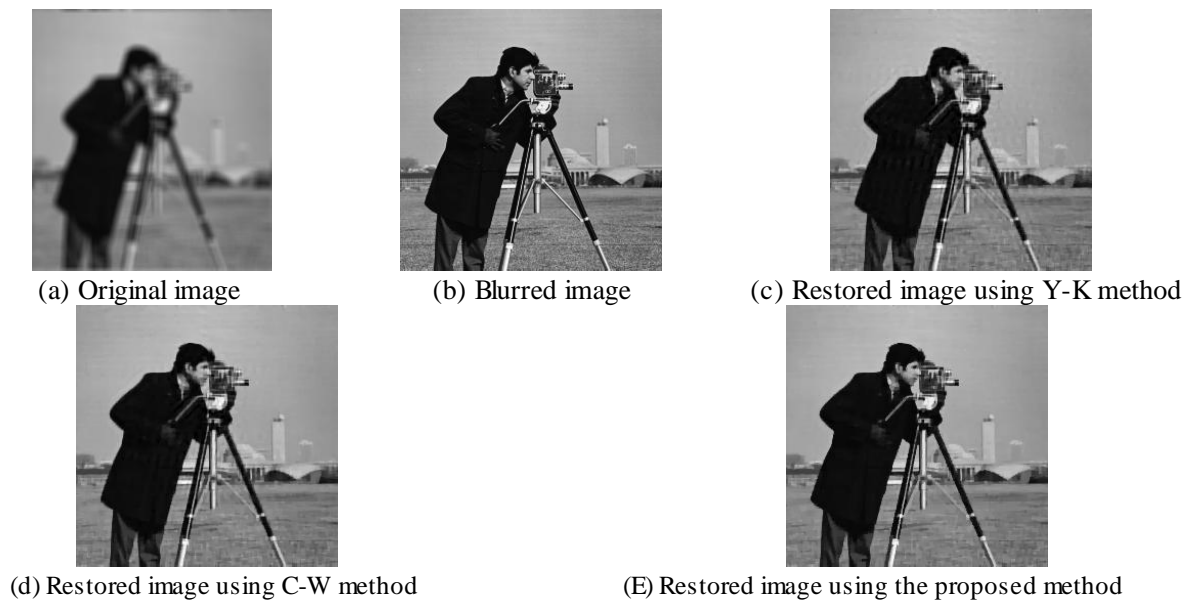


Fig. 1 Image restoration results using different methods

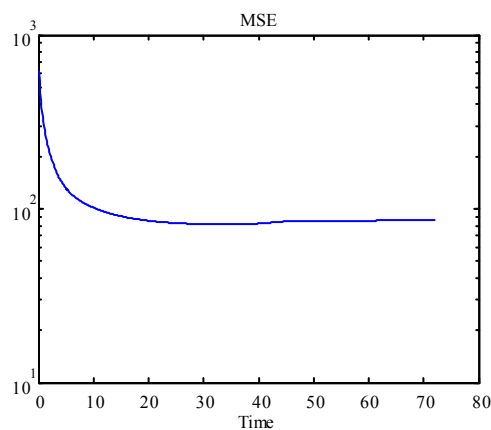


Fig. 2 Relationship between the processing velocity and the MSE of the results using the proposed method

Table 1 Comparison of PSNR values between the blurred images and the restored images using different methods

Image	The image after the addition of a 5*5 Gaussian kernel	The restored image using Y-K method	The restored image using C-W method	The restored image using the proposed method
Lena	24.16	29.23	29.76	30.78
Peppers	23.68	29.69	30.13	31.24
Barbara	22.27	30.14	30.49	31.58

Conclusions

The experimental results demonstrate that, using the proposed algorithm, the restored images can be significantly enhanced in quality, i.e., the effectiveness of the proposed method was proved.

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