# The Improvement of Transitional Butterworth-Chebyshev Filters

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**Abstract.** According to a simple design method of transitional filters which was proposed by C.S.GARGOUR, the transfer function of transitional lowpass filter is computed, and the transitional lowpass filter is improved from the poles of the transfer function with better linear phase response. And the filter was simulated by the Matlab software. The results of simulation verify that the method is feasible. On one hand, the step responses of the filters possess shorter raising time and litter overshoot. On the other hand, the phase character's curve of ITBC filter is smoother than Butterworth filters. Consequently the linearity and stability of the filter is better.

## Introduction

A filter is a device that eliminates interference signals of the transmitting end and receiving end of the equipment. When designing a filter, usually traditional filters,such as Butterworth, Chebyshev and so on,would be chosen. But there is a contradiction between above filters' phase-frequency characteristics and amplitude-frequency characteristics. In order to solve this question, some scholars begin to study transitional filters, attempting to find a balance between the phase-frequency characteristic and amplitude-frequency characteristic.

C.S.GARGOUR proposed a simple design method of transitional Butterworth-Chebyshev filters(TBC) in literature [1]. The designed TBC filter's amplitude-frequency characteristics was the same as the corresponding Chebyshev filter's, and the linear phase response got improved. Scholars improved directly the Chebyshev filter's transfer function, optimizing pole distribution appropriately to make poles far away from the imaginary axis, thus improving the system stability in literature [2].

On predecessors' research foundations, an Improved Transitional Butterworth-Chebyshev (ITBC) filter was designed by combining above two design methods. Compared with the Chebyshev filter and Butterworth filter, the new filter shows better linear response and system stability.

## The ITBC lowpass filter's design model

Traditional Chebyshev lowpass filter's amplitude square is:

$$\left|H(j\omega)\right|^2 = \frac{1}{1+\varepsilon^2 C_n^2(\omega)} \qquad (1)$$

In the formula, $\omega$ is the Kuibyshev lowpass filter's angular frequency variable, $C_n(\omega)$ is a n-order Chebyshev polynomial, and $\varepsilon$ is the coefficient of fluctuation.

At the same time, the Chebyshev lowpass filter's transfer function can be indicated by the form of poles, and its expression is following:

$$\left|H(s)\right| = \frac{\alpha}{\prod_{k=1}^{n}(s-s_k^{'})} \qquad (2)$$

In the formula, $\alpha$ is amplification factor whose specific values can be found in Table 1 of literature [2], $s$ is the the Chebyshev lowpass filter's frequency variable, and $s_k^{'}$ is the improved filter's left plane pole, the calculation is following[2]:

$$s_k^{'} = \sigma_k^{'} + j\omega_k^{'} \qquad (3)$$

In the formula:

$$\sigma_k^{'} = -\sinh(\frac{1}{n}\sinh^{-1}\frac{1}{\varepsilon})\sin\theta_k \,;\, \omega_k^{'} = \cosh(\frac{1}{n}\sinh^{-1}\frac{1}{\varepsilon})\cos\theta_k \qquad (4)$$

a) When n is an even number,

$$\theta_k = \begin{cases} \frac{2k-1}{2n}\pi + \frac{1}{2n}\pi & k = 1,2,\cdots,\frac{n}{2} \\ \frac{2k-1}{2n}\pi + \frac{1}{2n} & k = \frac{n}{2}+1,\cdots n \end{cases} \qquad (5)$$

b） When n is an odd number,

$$\theta_k = \begin{cases} \frac{2k-1}{2n}\pi + \frac{1}{2n}\pi & k = 1,2,\cdots,\frac{n+1}{2}-1 \\ \frac{2k-1}{2n}\pi & k = \frac{n+1}{2} \\ \frac{2k-1}{2n}\pi - \frac{1}{2n}\pi & k = \frac{n+1}{2}+1,\cdots n \end{cases} \qquad (6)$$

After squaring the formula(2), its result is equal with the formula(1), among it, $s = j\omega$, the equation gotten is following:

$$|H(s)|^2 = [\frac{\alpha}{\prod_{k=1}^{n}(s-s_k^{'})}]^2 = \frac{1}{1+\varepsilon^2 C_n^2(\frac{s}{j})} \qquad (7)$$

With the deformation of the formula(7), the improved second-order Chebyshev polynomial is :

$$C_n^2(\frac{s}{j}) = \frac{[\prod_{k=1}^{n}(s-s_k^{'})]^2 - \alpha^2}{\alpha^2\varepsilon^2} \qquad (8)$$

After merging terms that contain frequency variable $\omega$ of the amplitude square function of the traditional Chebyshev filter and Butterworth filter, the amplitude square function of the TBC filter's transfer function gotten is following:

$$|H(\mathrm{j}\omega)|^2 = \frac{1}{1+\varepsilon^2[\omega^{n_1}C_n^{n_2}(\omega)]^2} \qquad (9)$$

After substituting the improved Chebyshev polynomial of the formula(8) into the denominator of the formula(9), the amplitude square function of the ITBC lowpass filter that only contains the variable $s$ gotten finally is:

$$|H(s)|^2 = \frac{(\alpha\varepsilon)^{2n_2}}{(\alpha\varepsilon)^{2n_2} + \varepsilon^2(\frac{s}{j})^{2n_1}[\prod_{k=1}^{n}(s-s_k^{'})^2 - \alpha^2]^{n_2}} \quad n = 1,2\cdots n'-1 \qquad (10)$$

In above formula, $n$ is the order of the Chebyshev lowpass filter, and $n'$ is the order of the ITBC lowpass filter which is one order higher than $n$.

**The simulation analysis before and after the improvement**

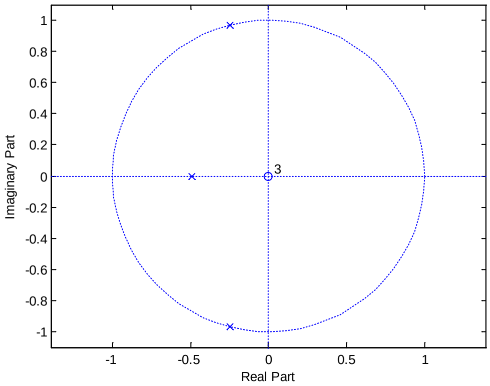
The ITBC filter was improved on the base of the TBC filter, in order to reduce the ITBC filter's complexity, we established $n'=3$, the minimum order that could achieve the TBC filter, as the order of the ITBC filter. At this point, in the amplitude square function of the ITBC filter transfer function. $n_1 = 1$ , $n_2 = 1$ , $n = 2$ . Choosing the passband maximum attenuation ripple as 1, namely $\alpha_{\max} = 1dB$ , $\varepsilon = 0.5088$ could be gotten. After substituting it into the formula(10), the amplitude square function of the ITBC lowpass filter could be calculated:

$$|H(\omega)|^2 = \frac{1}{-0.2589s^6 - 0.8039s^5 - 0.9465s^4 - 0.5005s^3 - 0.0054s^2 + 1} \qquad (11)$$

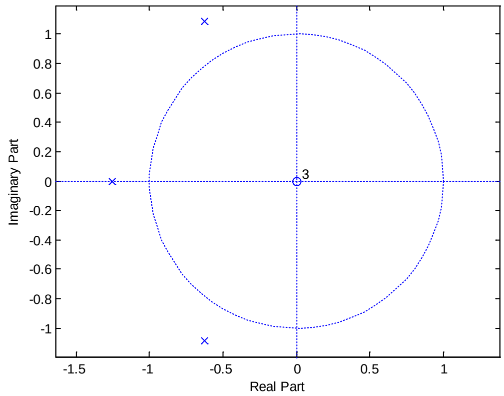
The corresponding transfer function could be calculated by the amplitude square function of the ITBC lowpass filter of the formula(11), and the amplitude square function of the ITBC lowpass filter finally calculated was following:

$$H(s) = \frac{4.5829}{s^3 + 4.1362s^2 + 6.6896s + 4.5829} \qquad (12)$$

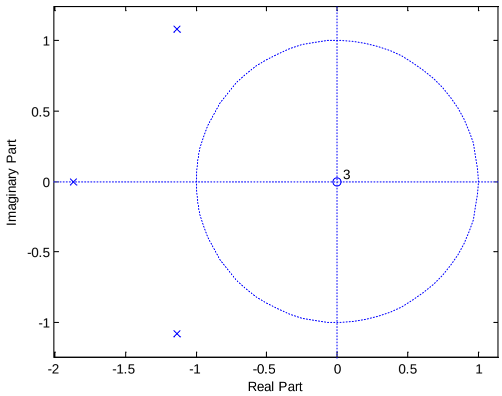
In order to verify the ITBC filter's frequency characteristic, the ITBC lowpass filter could be compared with the Chebyshev lowpass filter and Butterworth lowpass filter which were third-order isoflux with ripple.




(a) Chebyshev lowpass filter's zero-pole graph




(b) Butterworth lowpass filter's zero-pole graph




(c) ITBC lowpass filters' zero-pole graph

Fig.1 The zero-pole graph of three lowpass filters

We used Matlab[3] to draw associated parametric graph according to the three kinds of filters. We analyzed the transfer function, then observed the zero-pole graph of the three kinds of lowpass filters of Figure 1, finding that the three kinds of filters didn't have zeros, only had three poles all distributed on the left hand of the S plane, which showed that the system was stable. The condition of the distance from the poles of the three kinds of filters to the imaginary axis was that Chebyshev was the nearest, Butterworth was the next, and ITBC was the farthest. The nearer the pole away from the imaginary axis was, the bigger the damping ratio was , the faster the attenuation was, and the faster the system response was. This point also could be seen in the step response graph of Figure 2, compared with other two kinds of filter, the ITBC lowpass filter's rise time was shortened, and its overshoot was decreased.

Through observing and comparing the frequency response graph of the three kinds of lowpass filters in Figure 3, it could be found that compared the amplitude-frequency characteristic of the ITBC lowpass filter with the Butterworth lowpass filter, its passband was smooth, but the bandwidth was narrower, the transition band's attenuation was slow. In the phase-frequency response graph, the ITBC lowpass filter was the gentlest, and its linear phase response was the best of the three kinds of filters. The phase linear was often used to evaluate signal's phase distortion degree after filtering, and the better the linear phase response was, the smaller the signal's distortion degree was , then the more stable the system was[4].

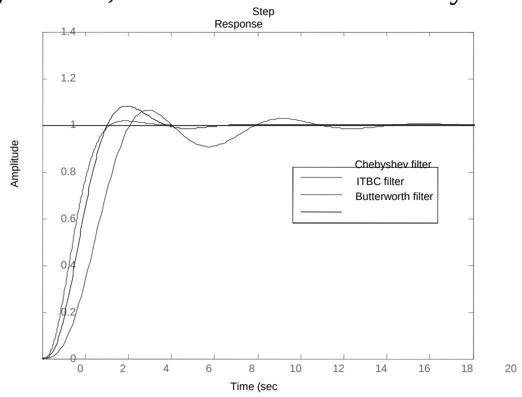



Fig.2 The step response graph of three lowpass filters

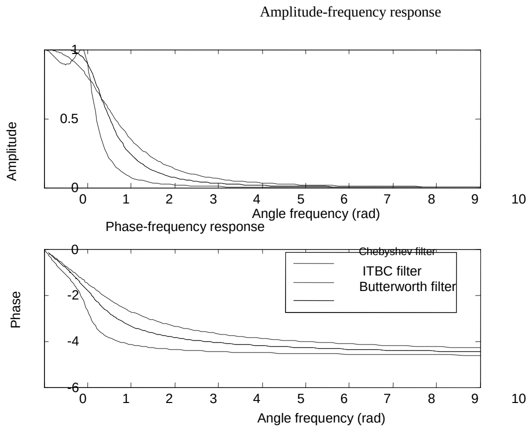



Fig.3 The frequency response graph of three lowpass filters

## The ITBC band-stop filter's design

This design used a Rogers5880 dielectric slab whose relative dielectric constant was 2.2 and whose thickness was 1mm, then replaced the capacitance and inductance with $\lambda/4$ series-parallel microstrip lines after the ITBC third-order lowpass prototype's frequency transformation, finally designed a 1.880GHz-2.145GHz band-stop filter whose center frequency was 2GHz and whose band-stop range is set to the frequency of 3G communication signals. It could be used in test facilities or front-end facilities of the experimental system, to eliminate the influence of unnecessary 3G signals on the measurement environment and facilities. Its structure was shown in Figure 4:

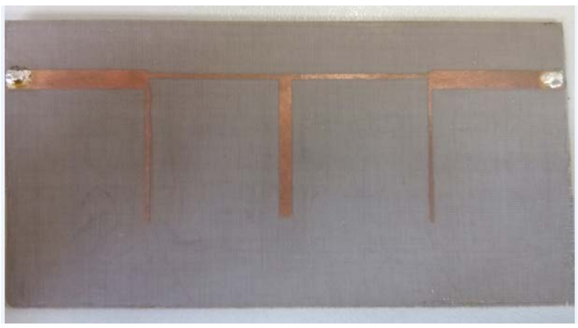
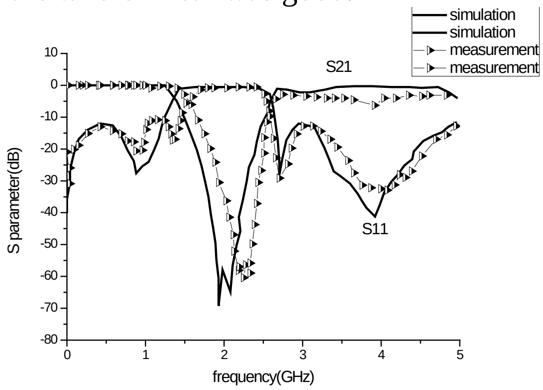


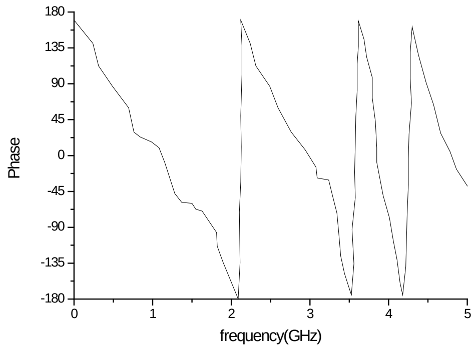
Fig.4 Physical Picture

After making the ITBC band-stop filter's physical, we used a Agilent N5230A vector network analyzer to measure it, and measured S parameter and frequency phase response, as was shown in Figure 5.

Observing the result of Figure 5(a), the simulation result's center resonance frequency was 2GHz, and when it was between 1.880GHz and 2.145GHz, its inhibition was greater than 40dB. In the measured result, the resonance frequency was 2.3GHz, deviating 0.3GHz, which possibly due to the the inexactitude of the design of the lowpass filter made up of lumped elements. The insertion loss was near the 4GHz pass-band, we preliminarily conjectured that it was influenced by Skin Effect at high frequencies, leading to microstrip lines loss' relatively large value. When the return loss in two pass-band ranges that was below 1.36GHz and above 2.3GHz was under 10dB, all indicators satisfied design requirements basically. Though the phase measured result of Figure5(b) fluctuated slightly, the whole linear was good.




The ITBC band-stop filter's phase response surveyed drawing




The ITBC band-stop filter's frequency response surveyed drawing

Fig.5 The ITBC filter's test and simulation result

## Conclusion

The paper presented another kind of lowpass filter. Through using Matlab for the program and simulation, we drew the ITBC lowpass filter's zore-pole, phase characteristic, step response and other parametric graphs for the comparative analysis with the Chebyshev lowpass filter and Butterworth lowpass filter. The results showed that the ITBC lowpass filter represented excellent linear phase response in the phase-frequency curve. In addition, it had a shorter rise time and a smaller overshoot. This showed that the design idea that the TBC filter modified into the ITBC filter was viable, which provided a new method for the design reconciling amplitude and phase. This time the paper designed the ITBC lowpass prototype as a 3G communication signal band-stop filter with a $\lambda/4$ open circuit microstrip line structure, its simulation results reached theory requirements basically, and achieved good phase characteristics as well. In the future, the ITBC filter will be applied to microstrip filters with more complicated structure [5].

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