

Numerical simulation of natural convection in square cavities with power function temperature wall by lattice Boltzmann method

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Abstract. In this study, a thermal lattice Boltzmann model is employed to investigate the natural convection in square cavities with power function temperature boundary. The horizontal wall of cavities is adiabatic, and the temperature at the west and east wall is set to constant or a power function. Four cases classified by temperature condition are investigated in detail at different Rayleigh number, 103, 104, 105, 106 and power index, 0, 0.5, 1, 2, 4 by Nusselt number and some hydrodynamic variables. Results show: though the average temperature different between west and east wall is same in the four cases on given power index, the thermal and hydrodynamic performance are of notable different, by which a useful suggestion about heat transfer rate in square cavities is put forward.

Introduction

Natural convection in closed enclosures has induced many researchers' interest due to significant industrial applications such as melting process, material processing, cooling systems for electronic devices etc. [1-4]. Natural convection in industry can be used in different conditions, such as different boundary condition, enclosure shape, or filled media, so a number of numerical and experimental studies have been carried for natural convection under different condition.

The studies of natural convection in different shape enclosures have been investigated widely, including rectangular, cubic triangular, cylinder and so on. Natural convection in rectangular is the most normal case, the research in which is thorough. A large cross section of fundamental research on this topic has been reviewed by Catton [5] and Bejan [6]. Davis [7] made a bench solution in square cavity with Rayleigh number up to 106. Hortmann and Peric [8] further the accuracy by finite volume and multi-grid technology for square cavity with heat left wall, cool right wall, and insulated top and bottom walls. Dillon, Emery, Mescher [9] made a deeply investigation for tall cavity with aspect ratios 8, 15, 20, and 33. Besides rectangular cavity, the triangular enclosure is usually studies under different boundary condition in recently years. Saha and Khan [10] studied natural convection within a tilted isosceles triangular enclosure with discrete bottom heating. And Many researchers paid their attention on inclined enclosures. Basak et al. [11] have studied the natural convection in isosceles triangular enclosures via heatline analysis for linear heating of inclined walls. Heo, Chung [12] made an experimental investigation of natural convection heat transfer on inclined cylinders for a range of Rayleigh numbers and inclined angle.

In recently years, natural convection in cavities filled with different media has been investigated, which include porous media, magnetic media, nanofluid. Vadasz [13] took an investigation to heat flux dispersion in natural convection in porous media. Amaguchi et al. [14] investigated the natural convection filled with magnetic fluid in a rectangular box. Oztop et al. [15] numerically analysis steady state natural convection in an enclosure filled with nanofluid.

A number of investigations to natural convection with different temperature boundary condition have been performed. Sathiyamoorthy et al. [16] made a numerical study to natural convection

flow in a closed square cavity when the bottom wall is uniformly heated and vertical wall(s) are linearly heated. V. Sivakumar et al. [17] analyzed the mixed convection in cavities with different lengths of the heating portion and different locations. Mahapatra et al. [18] investigated the effects of sine heating of wall on double-diffusive natural convection in square cavities.

Mathematic formulation and solution procedure

Problem statement

The geometry of the present problem is shown in Fig1. It displays a two-dimension enclosure with the height H . The temperatures of the west and east wall are set to Th , Tc or a power function of height y . In this article, Th and Tc are set to 1 and 0, respectively. The north and south walls are adiabatic. The enclosure is filled with fluid, which is ideal Newtonian, incompressible and laminar flow. Contrasted with normal natural convection, the main difference is the temperature of vertical walls is not a constant but a power function.

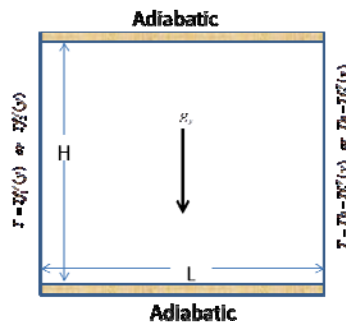


Fig1. Schematic diagram of the physical configuration and the coordinate system

In Fig. 1, g_y is gravity acceleration, and Tf_1^γ , Tf_2^γ are power functions with index γ , defined as

$$Tf_1^\gamma(y) = Th \cdot \left(\frac{y}{H}\right)^\gamma \quad (1) \quad Tf_2^\gamma(y) = Th \cdot \left(1 - \frac{y}{H}\right)^\gamma \quad (1)$$

where γ is power index. It is easy to be computed that let $\gamma \rightarrow 0$, then $Tf_1^\gamma, Tf_2^\gamma \rightarrow Th$, and let $\gamma \rightarrow \infty$, then $Tf_1^\gamma, Tf_2^\gamma \rightarrow 0$. Specially, let $\gamma = 1$, then Tf_1^γ, Tf_2^γ are linear function.

Lattice Boltzmann method

For the incompressible, the thermal lattice Boltzmann equation adopts a uniform lattice with BGK collision model. The two distribution functions model may be expressed as:

For the flow field:

$$f_i(x + e_i \Delta t, t + \Delta t) = f_i(x, t) - \frac{1}{\tau_f} [f_i(x, t) - f_i^{eq}(x, t)] + \Delta F_i \quad (2)$$

For the temperature field:

$$g_i(x + e_i \Delta t, t + \Delta t) = g_i(x, t) - \frac{1}{\tau_g} [g_i(x, t) - g_i^{eq}(x, t)] \quad (3)$$

where f_i and g_i are the particle density and energy distribution function along the particle velocity direction e_i , respectively, f_i^{eq} and g_i^{eq} are equilibrium distribution function, and τ_f and τ_g are the dimensionless relaxation times that control the rates approaching equilibrium. Also, F_i is an external force term along velocity direction e_i .

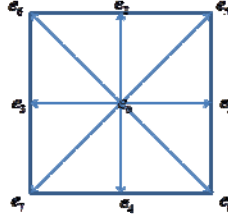


Fig2. Discrete velocity vectors for D2Q9 model

The equilibrium distribution functions, which depend on the local density and velocity, are given by the form

$$f_i^{eq} = w_i \rho \left[1 + \frac{e_i \cdot u}{c^2} + \frac{(e_i \cdot u)^2}{2c^4} - \frac{u \cdot u}{2c^2} \right] \quad (4)$$

$$g_i^{eq} = w_i \rho T \left[1 + \frac{e_i \cdot u}{c^2} + \frac{(e_i \cdot u)^2}{2c^4} - \frac{u \cdot u}{2c^2} \right] \quad (5)$$

where u and ρ are the macroscopic velocity and density, respectively, $c = \frac{\Delta x}{\Delta t}$ the lattice speed where Δx and Δt are set as unit, w_i is the weighting factor.

For D2Q9 model, the speed vector e_i and weight w_i are set as follows:

$$e_i = \begin{cases} 0 & i = 0 \\ c(\cos((i-1)\frac{\pi}{2}), \sin((i-1)\frac{\pi}{2})) & i = 1-4 \\ \frac{\sqrt{2}}{2} c(\cos((i-\frac{9}{2})\frac{\pi}{2}), \sin((i-\frac{9}{2})\frac{\pi}{2})) & i = 5-8 \end{cases} \quad (6)$$

$$w_i = \begin{cases} \frac{4}{9} & i = 0 \\ \frac{1}{9} & i = 1-4 \\ \frac{1}{36} & i = 5-8 \end{cases} \quad (7)$$

The corresponding kinematic viscosity and thermal diffusivity are then related to the relation times by:

$$\nu = [\tau_f - \frac{1}{2}] c_s^2 \Delta t \quad (8)$$

$$\alpha = [\tau_g - \frac{1}{2}] c_s^2 \Delta t. \quad (9)$$

where c_s is lattice sound speed in media, it is equals to $c_s = \frac{c}{\sqrt{3}}$.

As usual, in order to incorporate buoyancy force in the model, the Boussinesq approximation was applied, therefore the force term in the Eq [12] need to be calculated as below in vertical direction(y):

$$F_y = 3w_k g_y \beta (T - T_m) \quad (10)$$

Where g_y is the acceleration of gravity acting in the y direction of the lattice links; β is the thermal expansion coefficient and T_m is reference temperature.

Result and discussion

We use a uniform mesh of size $N_x \times N_y = N^2$ with N^2 equal to 32^2 , 64^2 , 128^2 , 256^2 to investigate the grid independence by Nusselt number Nu_{avg} and Nu_0 . In table [1] we tabulate the Nusselt number on different grids with power index $\gamma = 0$ in case I. From the table, we can clearly see that when N increases, the calculated average Nusselt quickly approaches the benchmark result. When N further increases from 128 to 256, there is not much improvement for the result. To short computing time, we set grids as 128^2 in following simulation.

To verify our codes validity, our result is compared with the benchmarks given by Hortmann[8], Davis[19], Mayne[20]. Table [2] shows the average Nusselt number Nu_{avg} , the maximum horizontal velocity on the vertical midplane of the cavity u_{max}^x , and the maximum vertical velocity on the horizontal midplane of the cavity u_{max}^y . It can be seen from table [1], that our result agrees well with the benchmarks.

Table 1 Grid dependence study

Ra	10^3	10^4	10^5	10^6
grids	Nu_{avg}			
32^2	1.1363	2.2636	4.5320	8.8445
64^2	1.1308	2.2624	4.5307	8.8389
128^2	1.1225	2.2541	4.5219	8.8031
256^2	1.1208	2.2446	4.5187	8.8190
grids	u_{max}^x			
32^2	3.6244	15.9956	34.7401	64.6905
64^2	3.6109	16.1123	34.7459	64.6771
128^2	3.6051	16.1561	34.7209	64.6537
256^2	3.5939	16.2274	34.7116	64.6468
grids	u_{max}^y			
32^2	3.7048	19.4691	68.9051	219.6021
64^2	3.6915	19.5567	68.7446	219.5949
128^2	3.6891	19.6296	68.6114	219.5016
256^2	3.6830	19.6540	68.6225	219.4843

Table 2 Comparison of Nusselt number and hydrodynamic variables with benchmark solutions.

Ra	10^3	10^4	10^5	10^6
	Nu_{avg}			
Present	1.1225	2.2541	4.5219	8.8031
ref [8]	1.118	2.243	4.519	8.800
ref [19]	-	2.245	4.52	8.825
ref[20]	-	-	-	-
	u_{max}^x			
Present	3.6051	16.1561	34.7209	64.6537
ref [9]	3.649	16.178	34.73	64.63
ref [19]	-	16.180	34.740	64.834
ref[20]	3.649	-	-	-
	u_{max}^y			
Present	3.6891	19.6296	68.6114	219.5016
ref [8]	3.697	19.617	68.59	219.36
ref [19]	-	19.630	68.640	220.461
ref[20]	3.696	-	-	-

Conclusion

In this work we employ the lattice Boltzmann method to investigate natural convection with power function temperature wall in square cavities. The coupled double distribution model is employed to simulation the flow and heat transfer: a D2Q9 scheme for mass and momentum conservation equation, and another D2Q9 scheme for the advection-diffusion equation for the temperature. This approach is valid under the Boussinesq approximation. The natural convection with power function type temperature wall is classified to four cases by different temperature setting on the vertical wall. Detail investigation is performed under different parameters value in cases.

We systematically study the validity and grid impendence of codes. The results show our codes are valid to realize the simulation by comparison with benchmark solutions.

By this investigation, a useful conclusion for natural convection in cavity can be make: if the average temperature difference between vertical walls is set as constant, the higher the temperature difference in the top is, the less the heat transfer effective is.

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