Quantitative Analysis on Integrated Effect of Gears Mounting Errors to Point-Contact Tooth Surface Meshing Characteristics

Zhou Kaihong[1, a *], Wang Congyi[2,b *]

[1,2]College of Mechanical and Control Engineering,

Guilin University of Technology, Guilin, 541006, China

[a]681023@163.com, [b]wangcongyi126@126.com

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Abstract. A linear equations related to the position errors of the contact point on the tooth surface and the mounting errors was derived from calculating total differential of equations of meshing. The sensitivity of gears mounting errors to contact point position errors was studied by means of the singularity of the linear equations; the condition number of the resultant matrixes is used to evaluate the integrated effect of gears mounting errors to contact point position errors. The developed method was illustrated with numerical examples

Introduction

Tooth contact point location error is derived for the gear installation error of the system of linear equations. The contact point position error of the tooth surface of the relative position error sensitivity problem due to the singularity of the system of linear equations. The condition number of coefficient matrix in linear equations was proposed to quantitatively describe the relative location of two tooth surface error on the influence of contact point position error of the comprehensive method.

The Total Differential Of Meshing Equation To Installation Parameters Of Gear Pair And Each Gear Angle $\Sigma^{(1)}$

As is shown in Fig.1, $O^1-x^1y^1z^1$ and $O^2-x^2y^2z^2$ are two fixed coordinate systems, Among them, Axis z^1 and tooth surface $\Sigma^{(1)}$'s rotary axis of gear 1 coincidence. Axis z^2 and tooth surface $\Sigma^{(2)}$'s the rotary of gear 2 coincidence. The included angle of axis z^1 and axis z^2 of Angle gear pair (that is, the shaft Angle) is Σ, axis x^1 coincides with axis x^2,and pointing to the direction of the shortest distance line to the two gear axis, the shortest distance (namely the center distance, or offset distance) is E. $O^{(1)}-x^{(1)}y^{(1)}z^{(1)}$ is the moving coordinate system, which is fixed connected with the gear1. Axis z^1 coincides with axis $z^{(1)}$, H is the distance between O_1 and $O^{(1)}$, $O^{(1)}$ the root apex of the gear 1. $O^{(2)}-x^{(2)}y^{(2)}z^{(2)}$ the root apex of the gear 1. Axis z^2 coincides with axis $z^{(2)}$, J is the distance between O_2 and $O^{(2)}$, $O^{(2)}$ is the point of the gear 2`s root apex. φ_1 is the angle of rotation by moving coordinate system $O^{(1)}-x^{(1)}y^{(1)}z^{(1)}$ relative to fixed coordinate system $O^1-x^1y^1z^1$,and φ_2 is the angle of rotation by moving coordinate system $O^{(2)}-x^{(2)}y^{(2)}z^{(2)}$ relative to fixed coordinate $O^2-x^2y^2z^2$,Conform to the rules of right hand is positive direction, otherwise is negative direction. Is contact points. And this point position vector and the method of vector satisfy meshing equation:

$$H\overrightarrow{z^1}+\overrightarrow{r_1^{(1)}}=E\overrightarrow{x^1}+J\overrightarrow{z^2}+\overrightarrow{r_2^{(2)}} \tag{1}$$

$$\overrightarrow{n_1^{(1)}}=\overrightarrow{n_1^{(2)}} \tag{2}$$

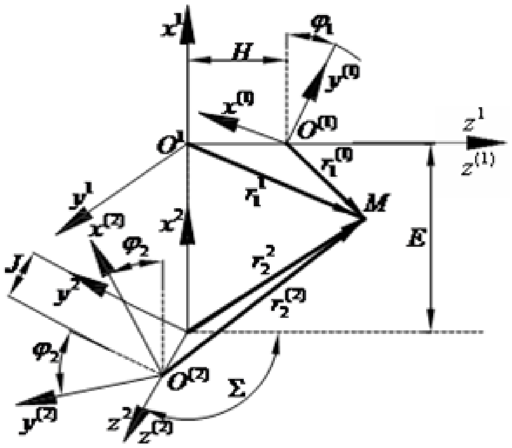

Fig. 1, Contact point M on the tooth surface of spatial gears in Cartesian coordinate systems.

The above two equations determine the location of each meshing point location in the process of the gear mesh, Equations contained in the installation position of gear set parameters E, H, J, Σ to determine the surface $\Sigma^{(1)}$ relative to tooth surface $\Sigma^{(2)}$ location tooth. When errors appear in these parameters, it will necessarily cause the mesh point location error. The relationship between the gear installation error and Meshing point location error can be caused by two tooth surface meshing equations(1),(2)for the parameters use the relative position E, H, J, Σ, φ_1 of total differential to determine:

$$d_1\overrightarrow{r_1^{(1)}} - d_2\overrightarrow{r_2^{(2)}} = dE\overrightarrow{x^1} + d\Sigma\overrightarrow{x^1} \times \overrightarrow{r_2^2} + dJ\overrightarrow{z^2} + \delta\varphi_2\overrightarrow{z^{(2)}} \times \overrightarrow{r_2^{(2)}} - dH\overrightarrow{z^1} \quad (3)$$

$$d_1\overrightarrow{n_1^{(1)}} - d_2\overrightarrow{n_2^{(2)}} = \left(d\Sigma\overrightarrow{x^1} + \delta\varphi_2\overrightarrow{z^2}\right) \times \overrightarrow{n_1^{(1)}} \quad (4)$$

Error $\delta\varphi_2$ is not an independent variable in the equation; it is caused by the gear installation error, so it can be represented by the tooth surface relative position errors. Equation (3) dot product $\overrightarrow{n_1^{(1)}}$, and noticed that $d_1\overrightarrow{r_1^{(1)}}$ and $d_2\overrightarrow{r_2^{(2)}}$ on common tangent plane in the two tooth surface, we can figure out that:

$$\delta\varphi_2 = \frac{\overrightarrow{n_1^{(1)}} \circ \left(dH\overrightarrow{z^1} - dE\overrightarrow{x^1} - d\Sigma\overrightarrow{x^1} \times \overrightarrow{r_2^{(2)}} - dJ\overrightarrow{z^2}\right)}{\overrightarrow{n_1^{(1)}} \circ \left(\overrightarrow{z^2} \times \overrightarrow{r_2^{(2)}}\right)} \quad (5)$$

Linearity And Sensibility Of Contact Points' Position Error And Gears' Installation Error

The vectors expressed by formula (3) and (4) are located in common tangent plane that through meshing point of tooth surface. As is shown in Fig.2, choose 2 orthogonal directions such as $\overrightarrow{e_{t1}^{(1)}}$ and $\overrightarrow{e_{t2}^{(2)}}$,from common tangent plane about contact point arbitrarily, in addition, $\overrightarrow{n_1^{(1)}} = \overrightarrow{e_{t1}^{(1)}} \times \overrightarrow{e_{t2}^{(1)}}$.Assuming $\kappa_{n1}^{(1)}$, $\kappa_{n2}^{(1)}$, $\tau_{g1}^{(1)}$ respectively for tooth surface $\Sigma^{(1)}$ of the normal curvature in the direction of $\overrightarrow{e_{t1}^{(1)}}$ and $\overrightarrow{e_{t2}^{(1)}}$ geodesic torsion in the direction of $\overrightarrow{e_{t1}^{(1)}}$; $\kappa_{n1}^{(2)}$, $\kappa_{n2}^{(2)}$, $\tau_{g1}^{(2)}$ respectively are tooth surface of $\Sigma^{(2)}$ the normal curvature in the direction of $\overrightarrow{e_{t1}^{(1)}}$ and $\overrightarrow{e_{t2}^{(1)}}$ geodesic torsion direction of $\overrightarrow{e_{t1}^{(1)}}$, Take the unit vector direction of $d\overrightarrow{r_1^{(1)}}$ as $\overrightarrow{\alpha^{(1)}}$,and the tooth surface $\Sigma^{(1)}$'s arc length in the direction is $s^{(1)}$,then $d_1\overrightarrow{r_1^{(1)}} = ds^{(1)}\overrightarrow{\alpha^{(1)}}$ take as the unit vector in the direction of $\overrightarrow{\alpha^{(2)}}$, the arc length in the direction of Tooth surface of $\Sigma^{(2)}$ is $s^{(2)}$, then $d_2\overrightarrow{r_2^{(2)}} = ds^{(2)}\overrightarrow{\alpha^{(2)}}$; and $\varphi_1 = 0$,multiplying $\overrightarrow{e_{t1}^{(1)}}$ and $\overrightarrow{e_{t2}^{(1)}}$ respectively by both ends of (3) and (4),we can figure out that:

$$\left.\begin{aligned}
&ds^{(2)}\cos\theta^{(2)} - ds^{(1)}\cos\theta^{(1)} = \overrightarrow{e_{t1}^{1}} \bullet \left(d_1\overrightarrow{r_1^{(1)}} - d_2\overrightarrow{r_2^{(2)}}\right)\\
&ds^{(2)}\sin\theta^{(2)} - ds^{(1)}\sin\theta^{(1)} = \overrightarrow{e_{t2}^{1}} \bullet \left(d_1\overrightarrow{r_1^{(1)}} - d_2\overrightarrow{r_2^{(2)}}\right)\\
&\kappa_{n1}^{(1)}ds^{(1)}\cos\theta^{(1)} + \tau_{g1}^{(1)}ds^{(1)}\sin\theta^{(1)} - \kappa_{n1}^{(2)}ds^{(2)}\cos\theta^{(2)}\\
&\quad - \tau_{g1}^{(2)}ds^{(2)}\sin\theta^{(2)} = \overrightarrow{e_{t1}^{1}} \bullet \left(d_1\overrightarrow{r_1^{(1)}} - d_2\overrightarrow{r_2^{(2)}}\right)\\
&\kappa_{n2}^{(1)}ds^{(1)}\sin\theta^{(1)} + \tau_{g1}^{(1)}ds^{(1)}\cos\theta^{(1)} + \kappa_{n2}^{(2)}ds^{(2)}\sin\theta^{(1)}\\
&\quad - \tau_{g1}^{(2)}ds^{(2)}\cos\theta^{(2)} = \overrightarrow{e_{t2}^{1}} \bullet \left(d_1\overrightarrow{r_1^{(1)}} - d_2\overrightarrow{r_2^{(2)}}\right)
\end{aligned}\right\} \tag{6}$$

Assuming:

$$\left.\begin{aligned}
ds_1^{(1)} &= ds^{(1)}\cos\theta^{(1)}\\
ds_2^{(1)} &= ds^{(1)}\sin\theta^{(1)}\\
ds_1^{(2)} &= ds^{(2)}\cos\theta^{(2)}\\
ds_2^{(2)} &= ds^{(2)}\sin\theta^{(2)}
\end{aligned}\right\} \tag{7}$$

In this: $ds_1^{(1)}, ds_2^{(1)}$ are respectively the variables of the tooth surface $\Sigma^{(1)}$'s meshing point along the direction $\overrightarrow{e_{t1}^{(1)}}, \overrightarrow{e_{t2}^{(1)}}$ respectively; $ds_1^{(2)}, ds_2^{(2)}$ are respectively the variables of the tooth surface $\Sigma^{(2)}$'s meshing point along the direction $\overrightarrow{e_{t1}^{(1)}}, \overrightarrow{e_{t2}^{(1)}}$ respectively. Substituting (3), (4), (5), (7) into (6) and get:

$$\begin{bmatrix} -1 & 0 & 1 & 0\\ 0 & -1 & 0 & 1\\ \kappa_{n_1}^{(1)} & \tau_{g_1}^{(1)} & -\kappa_{n_1}^{(2)} & -\tau_{g_1}^{(2)}\\ \tau_{g_1}^{(1)} & \kappa_{n_2}^{(1)} & -\tau_{g_1}^{(2)} & -\kappa_{n_2}^{(2)} \end{bmatrix} \begin{bmatrix} ds_1^{(1)}\\ ds_2^{(1)}\\ ds_1^{(2)}\\ ds_2^{(2)} \end{bmatrix} =$$
$$\left(\begin{bmatrix} -\overrightarrow{e_{t1}^{1}} \bullet \overrightarrow{z^1} & \overrightarrow{e_{t1}^{1}} \bullet \overrightarrow{z^2} & \overrightarrow{e_{t1}^{1}} \bullet \left(\overrightarrow{x^1} \times \overrightarrow{r_2^2}\right) & \overrightarrow{e_{t1}^{1}} \bullet \overrightarrow{x^1}\\ -\overrightarrow{e_{t2}^{1}} \bullet \overrightarrow{k^1} & \overrightarrow{e_{t2}^{1}} \bullet \overrightarrow{z^2} & \overrightarrow{e_{t2}^{1}} \bullet \left(\overrightarrow{x^1} \times \overrightarrow{r_2^2}\right) & \overrightarrow{e_{t2}^{1}} \bullet \overrightarrow{x^1}\\ 0 & 0 & \overrightarrow{e_{t1}^{1}} \bullet \overrightarrow{x^1} & 0\\ 0 & 0 & \overrightarrow{e_{t2}^{1}} \bullet \overrightarrow{x^1} & 0 \end{bmatrix} + \frac{1}{\overrightarrow{n_2^2} \bullet \left(\overrightarrow{z^2} \times \overrightarrow{r_2^2}\right)} \begin{bmatrix} \overrightarrow{e_{t1}^{1}} \bullet \left(\overrightarrow{z^2} \times \overrightarrow{r_2^2}\right)\\ \overrightarrow{e_{t2}^{1}} \bullet \left(\overrightarrow{z^2} \times \overrightarrow{r_2^2}\right)\\ \overrightarrow{e_{t1}^{1}} \bullet \left(\overrightarrow{z^2} \times \overrightarrow{n_2^2}\right)\\ \overrightarrow{e_{t2}^{1}} \bullet \left(\overrightarrow{z^2} \times \overrightarrow{n_2^2}\right) \end{bmatrix} \begin{bmatrix} \overrightarrow{n_1^1} \bullet \overrightarrow{z^1}\\ -\overrightarrow{n_1^2} \bullet \overrightarrow{z^2}\\ -\overrightarrow{n_1^1} \bullet \left(\overrightarrow{x^1} \times \overrightarrow{r_1^1}\right)\\ -\overrightarrow{n_1^1} \bullet \overrightarrow{x^1} \end{bmatrix}^T \right) \begin{bmatrix} dH\\ dJ\\ d\Sigma\\ dE \end{bmatrix} \tag{8}$$

Commanding as the square matrix of the equal sign on the left of the second term, is square, as the square matrix of the equal sign on the right, as array, so the (31) can be abbreviated:

$$KdS = WdB \tag{9}$$

Namely:

$$dS = K^{-1}WdB \tag{10}$$

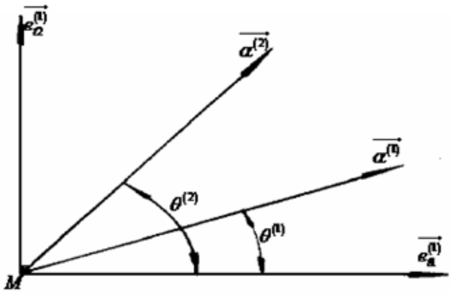

Fig. 2, The movement direction of contact point

K consists of second order parameters of the tooth surface, W consist of tooth surface location parameters and first order parameters(Fig.1,2). dS is the position error array of contact point, the element is error of two tooth surface meshing spots (superscript 1, 2) along the tangent direction of the contact trace (subscript 1) and the public on the tangent plane of vertical direction (subscript 2); dB is the position error to install array. If command

$$A = K^{-1}W \tag{11}$$

Because it is a linear relationship between these two kinds of error, so the mesh point position error of the sensitivity of the installation error is reflected by the condition number of matrix A Namely $Cond_2(A)$

$$Cond_2(A) = \|A\|_2 \cdot \|A^{-1}\|_2 \tag{11}$$

In which $\|A\|_2$ is 2 —induced norm of Matrix A .

By the theory of matrix[8]: The condition number $Cond_2(A)$ of coefficient matrix of a system of linear equation represents that relative error of the position error array dS of touch points is a multiple of dB to relative error of the position error array $Cond_2(A)$ of installation position. So $Cond_2(A)$ can be quantitatively express the contact point position error with the sensitivity of the installation position error.

Form the equation (11) we can know that: A is a square matrix that decided to mesh point location error square of the sensitivity of the installation error consists of two parts: The first part is zero order, first order of two gear tooth surface parameters; The other part is the second order two gear tooth surface parameter. It shows that the sensitivity of the mesh point location error to installation error associated with local geometrical structure of the tooth surface, so by optimizing the structure of tooth surface can reduce the sensitivity of the mesh point location error to installation error, which is one of the purposes of this study. If two tooth surface is a line contact. That is $\kappa_{n1}^{(1)} = \kappa_{n1}^{(2)}$, $\tau_{g1}^{(1)} = \tau_{g1}^{(2)}$ along the contact line direction, From the equation (11), we can draw a conclusion that if A is a singular matrix, the sensitivity of the mesh point location error to installation error $Cond_2(A)$ is infinite. In actual engineering, people can hardly meet the condition of meshing gear installation from the installation precision, which is consistent with the conclusion of practical engineering. Compared with the method of using each single error to measure the sensitivity of the installation error to the meshing point position error which introduced by the literature[4],this method from the singularity of the coefficient matrix of linear equations, considering the sensitivity of the mesh point location error to installation error, is more comprehensive and clear.

Examples

Using the above methods, when exposed to trace tangent tooth surfaces with different parameters of the second order, the use of mesh point of the arc helical gears and the point meshing of helical gear pair meshing point position error of the sensitivity of the installation error, the tooth face second-order parameters on meshing points to illustrate the influence of position error of the sensitivity of the installation error.

Table 1. Main geometric parameters of gear pair

The gear parameter names value	numerical
Teeth number of large gear z_1	42
Normal pressure angle of large gear positive turning surface α_{0t} /(∘)	20
Shaft angle Σ /(∘)	90
Reference cone angle of large gear δ_1 /(∘)	15.9454
Root angle of large gear δ_{f1} /(∘)	14.9517
Face cone of large gear $\delta_{\alpha 1}$ /(∘)	19.2041
Helix angle of midpoint on tooth width β /(∘)	34.3541
Cone distance of midpoint on tooth width R_m /(mm)	139.743
Normal modulus of midpoint on tooth width m_n /(mm)	5.282
Modification coefficient of large gear normal height x_{n1} /(mm)	-0.533
Modification coefficient of large gear tangential x_{t1} /(mm)	0.08
The normal addendum coefficient h_α^*	1.0
Normal tip clearance coefficient c^*	0.25
Cutter head radius r_0 /(mm)	139.743
Teeth number of pinion z_2	42
Modification coefficient of pinion normal height x_{n2} /(mm)	0.533
Modification coefficient of pinion tangential x_{t2} /(mm)	-0.08

Table.1 Main geometric parameters of gear pair Table.2 The second order point meshing gear tooth surface parameters,

$$\overrightarrow{z^1} = (0, \quad 0, \quad 1); \overrightarrow{x^1} = (1, \quad 0, \quad 0);$$

$$\overrightarrow{r_1^1} = \overrightarrow{r_2^2} = (-2.7285, \quad -38.3876, \quad 140.3921); \overrightarrow{n_1^1} = (0.3420, \quad -0.7753, \quad 0.5309);$$

$$\overrightarrow{e_{t1}^1} = (0.9822, \quad 0.1427, \quad 0.1221); \overrightarrow{e_{t2}^1} = (-0.0212, \quad -0.4268, \quad 0.6693);$$

and, $\overrightarrow{e_{t1}^1}$ tangent of the contact trace. The angle with tooth depth direction is 15° , $\overrightarrow{e_{t2}^1}$ and $\overrightarrow{e_{t1}^1}$ is the normal tooth surface and the tangent plane.

In table 3 ,there are sensitivity of the installation error of contact point position error for three kinds of design scheme. In this 3 design schemes, parameters and sensitivity of the installation error for scheme 1 and scheme 2 have little difference, But the main direction of the candidated of curvature is vary widely, the contact spot of the axis of the ellipse and tooth depth direction angle is different. It can be seen that ,in the design of tooth surface structure, the sensitivity of the installation error design is one side of the tooth surface second order parameter design.

Table 2 The second order point meshing gear tooth surface parameters

Project	1	2	3
$\kappa_{n1}^{(1)}$ / mm^{-1}	-0.0077	-0.0077	-0.0077
$\kappa_{n2}^{(1)}$ / mm^{-1}	-0.0011	-0.0011	-0.0011
$\tau_{g1}^{(1)}$ / mm^{-1}	-0.0029	-0.0029	-0.0029
$\kappa_{n1}^{(2)}$ / mm^{-1}	-0.0076	-0.0071	-0.0075
$\kappa_{n2}^{(2)}$ / mm^{-1}	0.0257	0.0254	0.0106
$\tau_{g1}^{(2)}$ / mm^{-1}	-0.0049	-0.0046	-0.0039

Table 3 The contact point position error of the sensitivity of the installation error

Plan	1	2	3
$Cond_2(A)$	2.13	1.89	23.4

The installation error sensitivity of Scheme 3 much larger than the other two schemes, this basically is the derivation of two gear tooth surface curvature and geodesic torsion induced difference is very small. However, tooth surface curvature and the contact stress of tooth surface is small, the contact strength is enhanced. But the tooth surface installation error cause much moving of the position of tooth surface contact, can be significantly reduce tooth bending strength. Therefore, when design tooth surface structure must be considered the size of the installation error sensitivity, in case of causing tooth bending strength weakened. At the same time considered the second order parameters effect on the contact strength.

Conclusion

By solving the total differential of the gear meshing condition equation to two tooth surface relative position parameters, the linear equations about position error of tooth surface contact and two tooth surface relative position errors is deduced. by the condition number of coefficient matrix quantitative evaluation to the system position error of gear tooth surface contact installation error sensitivity, provides a scientific and clear basis for the point meshing tooth surfaces second-order parameter design.

Acknowledgments

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