

A Comparative Research on the Optimal Anchor Angle of Anchor Cable Frame Project

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Abstract. It often happens in anchor cable projects that the beams of anchor cable fail to exert the optimal effect or even the projects fail due to the irrational design of anchorage angle. In response to this, based on the principal theoretical formula, the expression of the optimal anchorage angle was deduced, taking the bending moment of beams of anchor cable. Relevant software was compiled. The validity of the results was verified. The calculation of the optimal anchorage angle under different theories was also researched to be compared with the results of this paper.

Introduction

The reinforcement technique for prestressed anchor cable frame refers to that the anchors of prestressed anchor cable is fastened to the frame structure, which frame, prestressed anchor cable and the foundation together take the load of the reinforcement structure^{[1]-[3]}. In the design, except the designing parameters of the frame itself, the anchorage angle of the prestressed anchor cable is a very significant parameter^{[4]-[5]}. Based on the Winkler foundation model, using backward beam method to calculate the internal force of anchor cable frame, the anchorage angle of the minimum bending moment is confirmed to be the optimal one.

The deduction of the theoretical formula

Let us suppose the anchor cable – frame-foundation system we are going to talk about is as shown in Fig.1.

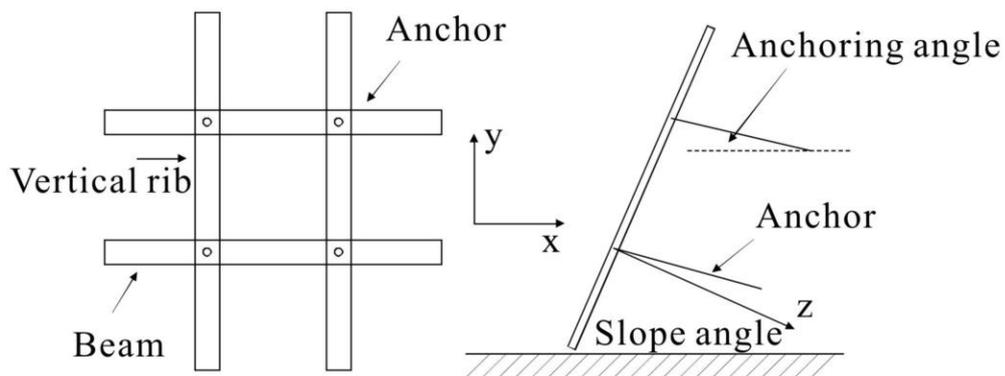


Fig. 1 Anchor cable frame diagram

For the distribution principle and calculation formula of forces on the beam and vertical rib, two important conditions are introduced in.

(1) Deformation compatibility condition: The beams are always in touch with the foundation soil. After deformation, the displacement generated by the intersections of the beams and vertical ribs must be equal. Namely,

$$Z_x = Z_y = Z \quad (1)$$

(2) Static equilibrium condition: The sum of forces distributed on the beam and the vertical rib should be equal to the component force in the direction vertical to the slope of the gross anchor cable force. Namely,

$$F \cos(90^\circ - \alpha - \beta) = F_x + F_y \quad (2)$$

In this paper, we take the bending moment between beams and vertical ribs as the objective function. We have deduced that when the function gets the minimum value, the correspondent anchorage angle is the optimal. The expression for bending moment of beams is similar to that of vertical ribs, so we choose the beam as a research example. Here is its bending moment expression:

$$M = (F_x / 4\lambda) e^{-\lambda x} (\cos \lambda x - \sin \lambda x) \quad (3)$$

In the expression: M stands for bending moment; F_x stands for the component force of anchor cable force on the beam.

$\lambda = \sqrt[4]{bk/(4EI)}$ stands for flexibility Eigen value; b stands for the width of beam; k stands for the modulus of sub grade reaction; E stands for the elasticity modulus of the beam; I stands for the flexural rigidity.

In expression(3), it can be seen that bending moment M changes according to the changes of x. M derivation of x, we get:

$$dM / dx = -(F \cos \lambda x) / (2e^{\lambda x}) \quad (4)$$

When $dM / dx = 0$:

We get: $\cos \lambda x = 0$

Namely: $x = \pi / (2\lambda) = 1.57 / \lambda$ after sorting we can get:

$$\lambda x = 1.57 \quad (5)$$

Expression (5) illustrates that the maximum bending moment happens when $x = 1.57 / \lambda$. Substitute expression (5) into expression (3):

$$M_{\max} = F_x / (4\lambda) \times 0.2022176 = F_x / (20\lambda) \quad (6)$$

In the expression: F_x stands for the component force of anchor cable force on the beam. Here is the expression.

$$F_x = C_a F \cos(90 - \alpha - \beta) \quad (7)$$

In the expression: $C_a = \lambda_y b_x / (\lambda_x b_y + \lambda_y b_x)$, since the material parameter and sectional dimension of beams and vertical ribs in general projects are the same, so $b_x = b_y$, $\lambda_x = \lambda_y$

Expression (7) can be simplified into:

$$F_x = 0.5 \times F \cos(90 - \alpha - \beta) \quad (8)$$

Substitute expression (8) into expression (6), and we get:

$$M_{\max} = \frac{1}{40} \frac{F \cos(90 - \alpha - \beta)}{\lambda} \quad (9)$$

In expression(9), we can see that the soil parameters of sliding mass is constant and the design parameters of beams and vertical ribs are fixed value. The design tension of the prestressed anchor cable is fixed. Then the maximum bending moment is related with the slope angle α of the side slope and anchorage angle β . Namely, $f(M_{\max}) = f(\alpha, \beta)$. We suppose the slope angle α of the side slope

is a constant, and anchorage angle β is a variable. Derivation of β :

$$M_{\max} / d\beta = (1/40) \times (F \sin(90 - \alpha - \beta) / \lambda) \quad (10)$$

When, $dM_{\max} / d\beta = 0$, $\sin(90 - \alpha - \beta) = 0$ then, $\alpha + \beta = 90^\circ$ namely

$$\beta_{opt} = 90^\circ - \alpha \quad (11)$$

According to the deduced formula(11), the included angle between the optimal anchorage angle β_{opt} and the slope angle α of the side slope is 90° .

The compilation of calculation software for the internal force of anchor cable frame

VB language is used for the software compilation in this paper. It offers a user-friendly interface and clear operation. The thinking of compiling this software is as shown in Fig. 2. Parameters input and results output are as shown in Table 1.

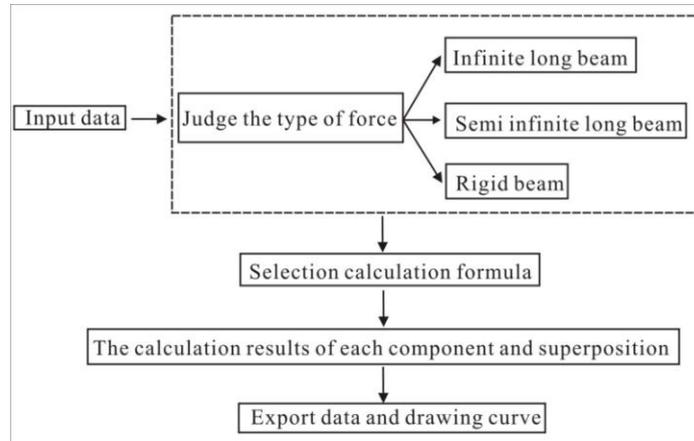


Fig.2 Software ideas flow chart

Table 1 The input and output table of the software

Parameters input	Design tension F	Slope angle of side slope	Anchorage angle	Sectional dimension b, h
		Frame geometric dimension	Sub grade reaction modulus	Material elasticity modulus E
Result output	Beam bending moment	Beam shearing force	Beam deflection	

There is a project using prestressed anchor cable frame to do reinforcement. Every piece of the frame is composed of two beams and two vertical ribs, with 4 prestressed anchor cables. Every anchor cable is made of $4\phi_{15}^J$ steel strand with design tension of 100 kN. The reinforced concrete frame adopts C25 concrete for spouting. The length of the vertical rib is 6 m while the length of the beam is 5m. The separation distance between vertical ribs is 2.5 m while that for beams is 2m. The slope angle of the side slope= 90° , and the anchorage angle= 30° . All frame dimensions are $b \times h = 0.3 \times 0.4 \text{ m}^2$

The reaction modulus of rock soil is $K_h = 2 \times 10^5 \text{ kN/m}^3$. The flexibility modulus of C25 concrete is $2.85 \times 10^7 \text{ kN/m}^2$. Compute the shearing force, bending moment and deflection of the frame. Input all the parameters into the software and press the computing button. Check the graphs of shearing force, bending moment and deflection through their results sections. The calculation results will be stored in .txt form, which is convenient for later procession. The calculation results are as shown in Fig.3- Fig.5. The calculation results can also be put into ORIGIN for plotting and analysis. As shown in Fig.6, it can be seen that the maximum bending moment, $-11.2 \text{ kN} \cdot \text{m}$, appears at the site of 1.25m and 3.75m.

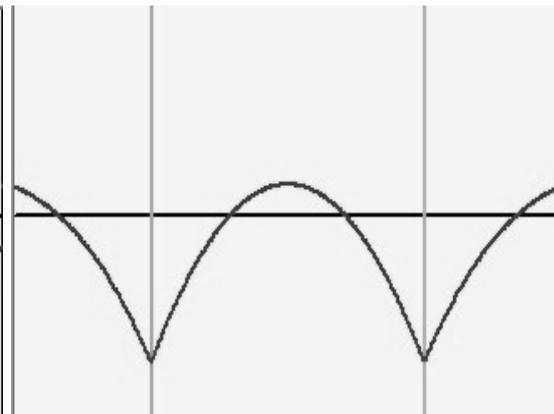
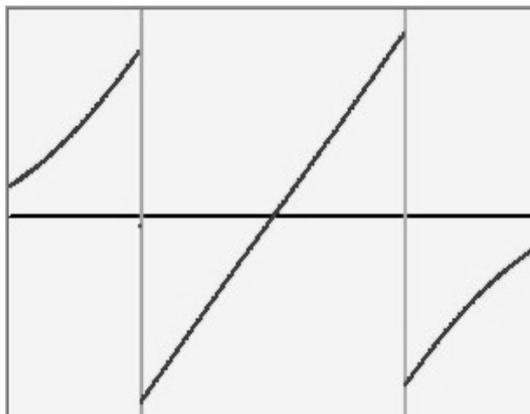


Fig.3 Beam shear force diagram (unit: kN)

Fig.4 Beam bending moment diagram (unit: kN·m)

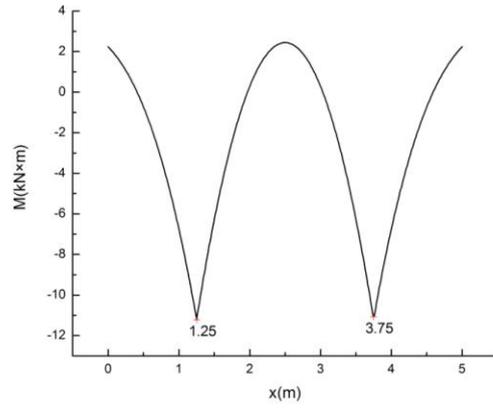
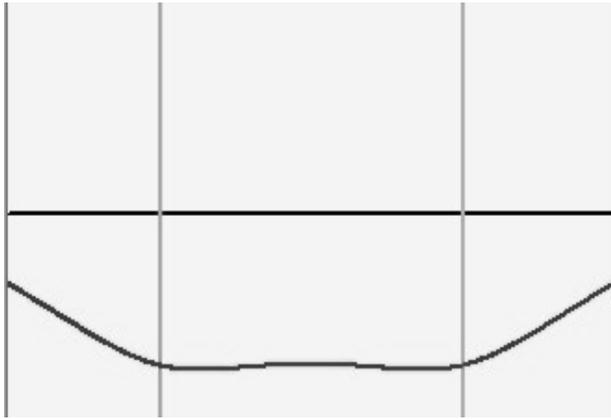


Fig.5 Beam deflection diagram (unit: m) Fig.6 Beam bending moment diagram drawn in ORIGIN

The verification of the calculation formula for the optimal anchorage angle

In order to verify the design formula $\beta_{opt} = 90^\circ - \alpha$ deduced in this paper for the optimal anchorage angle, the calculation software for internal force of anchor cable frame is used. During the verification, the design tension $F=100$ kN. The frame constitutes 2 beams and 2 vertical ribs. The length of the vertical rib is 6 m while the length of the beam is 5 m. The separation distance between vertical ribs is 2.5 m while that for beams is 2m. All the frame dimension is $b \times h=0.3 \times 0.4$ m². The reaction modulus of rock soil is $K_h=2 \times 10^5$ kN/m³. The flexibility modulus of C25 concrete is 2.85×10^7 kN/m². Take bending moment as the objective function, we can get results shown in Table 2 and Fig.7.

Table 2 The results of frame bending moment

Anchorage angle	$\alpha=10^\circ$	$\alpha=20^\circ$	$\alpha=30^\circ$	$\alpha=40^\circ$
$\beta=0^\circ$	2.27	4.43	6.65	8.47
$\beta=10^\circ$	4.38	6.71	8.30	9.82
$\beta=20^\circ$	6.51	8.60	9.82	11.57
$\beta=30^\circ$	8.36	9.95	11.18	12.51
$\beta=40^\circ$	10.03	11.29	12.52	13.02
$\beta=50^\circ$	11.45	12.51	13.27	13.32
$\beta=60^\circ$	12.15	12.71	14.45	12.77
$\beta=70^\circ$	12.77	13.10	14.24	12.15
$\beta=80^\circ$	13.52	12.93	13.59	11.51
$\beta=90^\circ$	12.98	12.77	11.25	10.24

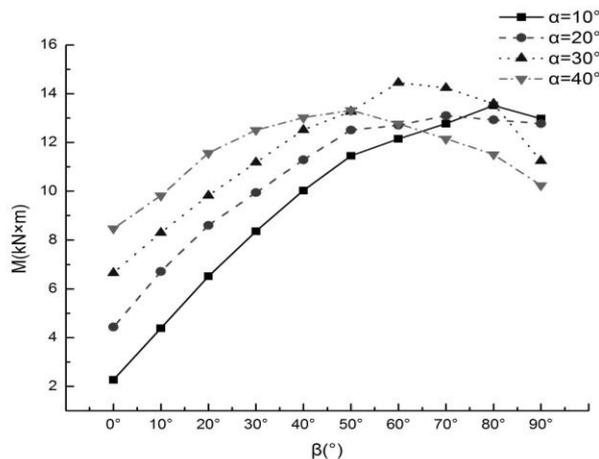


Fig.7 Different anchoring angle beam bending moment diagram

In the calculation chart, we can see when the slope angle of the side slope $\alpha=10^\circ$, the bending moment of the beams change according to the changes of the anchorage angle. When the anchorage angle $\beta=80^\circ$, the bending moment of the beams can reach the maximum. That is, under design tension 100 kN, the resistant function of beam on side slope reaches the maximum. Under this condition the effect of the anchor cable reaches the best. The results meet the calculation formula of the optimal anchorage angle deduced in this paper, $\beta_{opt} = 90^\circ - \alpha$, namely, $80^\circ=90^\circ-10^\circ$. From the verified results, we can see when the slope angle of side slope change into: $\alpha=20^\circ$, $\alpha=30^\circ$, $\alpha=40^\circ$, the corresponding optimal anchorage angle are: $\beta=70^\circ, \beta=60^\circ, \beta=50^\circ$, which conform with the results of the theoretical formula.

The comparison between methods for the optimal anchorage angle

Through relevant Chinese and foreign documents, other methods for the optimal anchorage angle are found.

The first method: According to document, the result can be concluded as follows:

$\beta_{opt} = 47.5^\circ - 0.7\alpha - 9(1 - \sin \phi) + 8FSR$ In the formula, α stands for slope angle of the side slope; $K=1-\sin\Phi$, $FSR = F \frac{\tan \alpha}{\tan \phi}$, Φ stands for the internal friction angle of the soil; F stands for assurance factor^[6].

The 2nd method: in loess strata: $\beta_{opt} = 60^\circ - \phi$. In the formula, Φ stands for the internal friction angle of the soil^[7].

The 3rd method: $\beta_{opt} = \frac{\pi}{4} + \frac{\phi}{2} - \alpha$. In the formula: Φ stands for the internal friction angle of the soil; α stands for slope angle of the side slope^[8].

The slope structure parameters and rock physical- mechanical indexes: sliding slope angle $\alpha=31^\circ$, the internal friction angle of the sliding slope, $\Phi=12^\circ$. The design tension of the anchor cable $T=100$ kN, and the cohesive force $c=10$ kPa. The design assurance factor $F=1.12$. It needs to be indicated that slope shearing and sliding often occur during rainy season. Therefore, the shearing resistant strength indexes of the rock mass, c 、 Φ are values in water-saturated state.

With different methods, the optimal anchorage angle can be acquired, as is shown in Chart 3. The calculation formula deduced in this paper, $\beta_{opt} = 90^\circ - 31^\circ = 59^\circ$ The first method:

$$\beta_{opt} = 47.5^\circ - 0.7 \times 31^\circ - 9(1 - \sin 9^\circ) + 8 \times 1.12 \frac{\tan 28^\circ}{\tan 20^\circ} = 43.9^\circ \quad \text{The second method:}$$

$$\beta_{opt} = 60^\circ - 12^\circ = 48^\circ \quad \text{The third method: } \beta_{opt} = \frac{\pi}{4} + \frac{31^\circ}{2} - 12^\circ = 41.5^\circ.$$

Table 3 The calculation results of the optimal anchorage angle

The optimal anchorage angle	Formula of this paper	Method1	Method2	Method3
β	59°	43.9°	48°	41.5°

From the Table 3, we can see there are differences among the results of these four methods. The formula deduced in this paper is a basic one with the fewest factors to consider. It only needs the slope angle of the side slope and meets the elastic foundation beam theory. Thus, this method can be used for the design of the optimal anchorage angle at the initial design stage.

Conclusions

(1)The formula deduced in this paper for the optimal anchorage angle, $\beta_{opt} = 90^\circ - \alpha$, is applicable for practical projects, which can help design the optimal anchorage angle easily and fast.

(2)The calculation software for the internal force of the anchor cable frame in this paper can help calculate the bending moment, shearing force and deflection under different conditions easily and fast. It also offers the function of curve plotting.

(3)In the comparison with other methods for the optimal anchorage angle, we can see there is no confirmed answer for designing the optimal anchorage angle, so general projects often adopt the experiential method.

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