Heat Conduction Analysis of the Composite Wall by the Extended Finite Element method

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Abstract. Recent years, the “green” building is booming. As an important component of this environmentally friendly building, the composite wall insulation system has attracted a lot of attentions. In this paper, the widely used extended finite element method (XFEM) is adopted to analyze heat conduction problem in the composite wall. Numerical study on practical case demonstrates the validity of the XFEM for thermal diffusion in heterogeneous walls.

Introduction

As we all know, buildings in construction or in use need to consume a large quantities of natural resources, and will also emit large amount of harmful gases. In the past years, human beings have gradually realized the serve negative effect of building on the climate and the environment. Under this background, the green building, aiming to the harmony among people, architecture and nature, comes into our sights.

In order to better meet the various needs of “green”, e.g., good performance in thermal insulation, sound isolation, natural lighting and ventilation, new building materials are being developed and adopted in practical engineering. The traditional walls, such as the concrete walls and masonry walls, are improved with some green materials (e.g., corn straw and cotton stalk) [1, 2]. Such multi-layered (composite) walls have excellent properties in demanded aspects.

Recent years, the study on thermal performance of the composite wall has become a hot topic. As a key tool, the numerical methods are very attractive attributing to their higher efficiency and lower cost. Among the frequently used approaches, the finite element method (FEM) [3] is a representative. The FEM uses continuous shape functions, which require continuous material properties interior an element. Accordingly, when analyzing the composite walls, material interfaces must be consistent with element edges, which may increase the meshing cost to some extent.

To conquer the disadvantage of the FEM, the extended finite element method (XFEM) [4-6] is developed to solve discontinuous problems efficiently and accurately. In the XFEM, the mesh can be independent of internal physical boundaries (e.g., the physical interfaces) through the use of enrichment functions in the approximation. Yu and his coauthors [7, 8] applied the XFEM to solve heat transfer problem in non-homogeneous materials. In the present paper, the XFEM is applied to investigate heat diffusion problem in composite walls.

Basic equations of the XFEM for thermal analysis

For steady heat conduction problem without heat source, the heat flux distribution is [9]

$$-\nabla \mathbf{q} = 0 \tag{1}$$

where ∇ is the gradient operator and $\mathbf{q}$ is the heat flux vector. The associated boundary conditions are

$$T = \bar{T} \qquad \text{on } \Gamma_1 \tag{2}$$

$$-k\frac{\partial T}{\partial \mathbf{n}} = \bar{q} \qquad \text{on } \Gamma_2 \tag{3}$$

$$-k\frac{\partial T}{\partial \mathbf{n}} = h\left(T - T_f\right) \qquad \text{on } \Gamma_3 \tag{4}$$

where Γ_1, Γ_2 and Γ_3 are, respectively, the first, second and third boundaries. T denotes the temperature and $\mathbf{n}$ is the outward unit normal to the domain. $\bar{T}$ and $\bar{q}$ are, respectively, the prescribed temperature and heat flux on the corresponding boundary. k is the thermal conductivity and h is the convection heat-transfer coefficient. T_f is the environmental temperature.

To solve the above problem by the XFEM, the temperature approximation is written as [10]

$$T^h(\mathbf{x}) = \sum_{i \in I} N_i(\mathbf{x})T_i + \sum_{j \in J} N_j(\mathbf{x})y(\mathbf{x})a_j \tag{5}$$

where $N_i(\mathbf{x})$ is the nodal shape functions and $\mathbf{x} = (x, y)$. I and J are, respectively, the set of all nodes and enriched nodes. T_i and a_j are, respectively, the conventional and additional thermal unknowns. $y(\mathbf{x})$ is the enrichment function. For multi-layered structure, $y(\mathbf{x})$ is chosen as [9]

$$y(\mathbf{x}) = \sum_m \left| j_m \right| N_m(\mathbf{x}) - \left| \sum_m j_m N_m(\mathbf{x}) \right| \tag{6}$$

where $j_m(\mathbf{x}) = \pm \min \left\| \mathbf{x} - \mathbf{x}_\Gamma \right\|$ with $\mathbf{x}_\Gamma$ the layer interface as illustrated in Fig. 1.

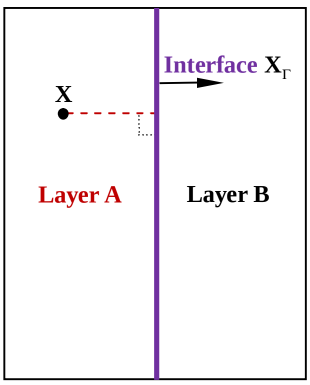

Fig. 1. An illustration for the computation of enrichment function

With the variation principle, the XFEM discrete equations for the thermal analysis of multi-layered structure are derived as

$$\mathbf{KT} = \mathbf{F} \tag{7}$$

where $\mathbf{T}$ is the vector of thermal unknowns. $\mathbf{K}$ and $\mathbf{F}$ are, respectively, the global thermal conductivity matrix and equivalent thermal load vector. For more details, please see reference [1].

Numerical examples

In this part, the thermal conductivity of a typical composite wall in northern China using different insulation materials is investigated with the XFEM. The main objectives of this test are to validate the proposed method and to numerically compare the insulation properties of different materials as well.

The corresponding physical model is plotted in Fig. 2, in which the inner and outer wythe are, respectively, made of the reinforced concrete and small hollow concrete block. The thickness of the inner layer, insulation layer, air layer and outer layer is, respectively, 190mm, 90mm, 10mm and 90mm. Three kinds of insulation materials, that is, the XPS (the polystyrene resin mixed with other raw materials and polymers), the EPS (the molded expanded polystyrene foam) and the corn straw are, respectively, considered. For computational purpose, the thermal conductivity of the associated materials is listed is Table 1.

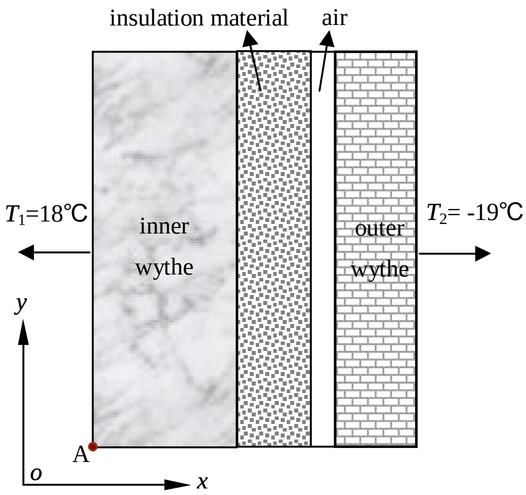

Fig. 2. Physical model of the composite wall

Table 1 Thermal conductivity of different materials

material	thermal conductivity (W/m/K)
reinforced concrete	1.74
XPS	0.028
EPS	0.042
corn straw	0.056
air	0.071
concrete block	0.86

In view that the thermal fields are independent of the height of the wall (that is, the dimension along *y*-axis in Fig. 2), regular mesh with 125*2=250 square elements (125 divisions along *x*-axis) and 378 nodes is used for XFEM modeling. The choice of such kind of mesh configuration is to specially produce the inconsistence of the element edges with the material interface, which is a major advantage of the XFEM compared with the FEM, as already described in the Introduction section.

To compare the thermal performance of different insulation materials, the temperature at a fixed point, i.e., point A in Fig. 2 is calculated and the associated results are summarized in Table 2. As we can see from this table, the thermal insulation performance of the composite wall using the XPS is the best, while that using the EPS is better than that using the corn straw, which can be inferred from the thermal conductivity presented in Table 1.

Conclusions

In this paper, the XFEM is applied to analyze the thermal property of composite wall containing special insulation materials. To represent the existence of material interfaces in the wall, the enrichment function is incorporated in the temperature approximation. The XFEM discrete equations are briefly presented and the proposed method is verified through a typical engineering problem. The powerful capability of the XFEM for thermal analysis of heterogeneous structure is well demonstrated.

Table 2 Temperature at point A under different insulation material

insulation material	Temperature of point A (℃)
XPS	16.85
EPS	16.39
corn straw	15.97

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