The Triple-shear Unified Elasto-plastic Constitutive Model and the slope stability analysis

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Abstract： The traditional method in solving geotechnical engineering problems can only come to the theoretical solutions. The triple-shear unified elasto-plastic constitutive model is proposed based on the triple-shear unified yield criterion, and the secondary development to ANSYS with the complicated practical engineering problems. The coincidence between the theoretical resolutions and the numerical resolutions for circular tunnels under the static pressure indicates that the proposed constitutive model and the subroutine codes are convincible. As an example, the slope stability is analyzed with the developed ANSYS software and come to the numerical resolutions. The results show the stability of landslide increases with the increase of the intermediate principal stress.

Introduction

The triple-shear unified yield criterion can reflect the basic yield characteristics of the rock and soil, and can correct the shortcomings of the twin-shear unified yield criterion in which the double slip angles under some specific stress states exist[1]. The triple-shear unified yield criterion, through changing its intermediate principal stress effect parameter b, can also degenerate to the other criteria and increase the range of its applications. Nowadays, the criterion has been applied to some simple geotechanical engineering problems[2-5]. However, for the more complicated problems, the elasto-plastic constitutive model based on the triple-shear unified yield criterion should be researched and combined it with the numerical methods.

The triple-shear unified elasto-plastic constitutive model and the elasto-plastic stiffness matrix suitable for the FEM calculations are proposed. The secondary development to the ANSYS with this model is coded to analyze the complicated practical engineering problems. Comparison of resolutions between the theoretical and numerical methods for a circular tunnel under the static pressures indicates the feasibility of the proposed constitutive model and the subroutine codes. In addition, the case study of the slope stability is cited here.

The triple-shear unified elasto-plastic constitutive model

The triple-shear unified yield criterion based on the shear failure mechanism is expressed as

$$f=(a s_1-s_3)(s_1-s_3)+b(a s_1-s_2)(s_1-s_2)+b(a s_2-s_3)(s_2-s_3)-(1+b)(s_1-s_3)f_t=0 \quad (1)$$

where f_t is uniaxial compressive strength of material, a is the ratio of the uniaxial tensile to the compressive strength of material. Its stress tensor invariant expressions is

$$f\left(I_1,\sqrt{J_2},q\right)=\frac{I_1}{3}(a-1)\left\{\sin(q+\frac{p}{3})+b\left[\sin q-\sin(q-\frac{p}{3})\right]\right\}+$$

$$\frac{\sqrt{J_2}}{\sqrt{3}}\left\{\begin{array}{l} a\left[\sin(2q+\frac{p}{3})+\frac{\sqrt{3}}{2}\right]+b\left[\sqrt{3}-\sin(2q+\frac{2p}{3})-\sin 2q\right]+ \\ ab\left[\sin(2q-\frac{2p}{3})-\sin(2q-\frac{p}{3})+\sqrt{3}\right]+\left[\sin 2q+\frac{\sqrt{3}}{2}\right] \end{array}\right\}-(1+b)\sin(q+\frac{p}{3})f_t=0 \quad (2)$$

According to the incremental theory of elasto-plasticity and the associated flow rule，the elasto-plastic constitutive matrix $[D]_{ep}$ is given as

$$[D]_{ep}=[D]_e+\frac{-[D]_e\left\{\frac{\partial f}{\partial\{s\}}\right\}\left\{\frac{\partial f}{\partial\{s\}}\right\}^T[D]_e}{A+\left\{\frac{\partial f}{\partial\{s\}}\right\}^T[D]_e\left\{\frac{\partial f}{\partial\{s\}}\right\}} \quad (3)$$

where

$$\frac{\partial f}{\partial\{s\}}=C_1\frac{\partial I_1}{\partial\{s\}}+C_2\frac{\partial\sqrt{J_2}}{\partial\{s\}}+C_3\frac{\partial J_3}{\partial\{s\}} \quad (4)$$

In which

$$C_1=\frac{\partial f}{\partial I_1}=\frac{1}{3}(a-1)\left[\sin(q+\frac{p}{3})+b\cos(q-\frac{p}{6})\right] \quad (5)$$

$$C_2=\frac{1}{2\sqrt{3}}\left\{a\left[\sin(2q+\frac{p}{3})+\frac{\sqrt{3}}{2}\right]+b\left[\sqrt{3}-\sin(2q+\frac{p}{3})+ab(\sqrt{3}-\sin 2q)\right]+(\sin 2q+\frac{\sqrt{3}}{2})\right\}\frac{1}{\sqrt{J_2}}$$
$$+\frac{3\sqrt{3}}{4\sin 3q}\frac{J_3}{\sqrt{J_2^5}}\left\{\begin{array}{l}\frac{I_1}{3}(a-1)\left[\cos(q+\frac{p}{3})-b\sin(q-\frac{p}{6})\right]+\frac{\sqrt{J_2}}{\sqrt{3}}\left[\begin{array}{l}2a\cos(2q+\frac{p}{3})-2b\cos(2q+\frac{p}{3})- \\ 2ab\cos 2q+2\cos q\end{array}\right] \\ -(1+b)\cos(q+\frac{p}{3})f_t\end{array}\right\} \quad (6)$$

$$C_3=-\frac{\sqrt{3}}{2\sin 3q}\frac{1}{\sqrt{J_2^3}}\frac{I_1}{3}(a-1)\left\{\begin{array}{l}\left[\cos(q+\frac{p}{3})-b\sin(q-\frac{p}{6})\right]+\frac{\sqrt{J_2}}{\sqrt{3}}\left[\begin{array}{l}2a\cos(2q+\frac{p}{3})-2b\cos(2q+\frac{p}{3})- \\ 2ab\cos 2q+2\cos 2q\end{array}\right] \\ -(1+b)\cos(q+\frac{p}{3})f_t\end{array}\right\} \quad (7)$$

Based on the above constitutive model，subroutines embedded in the ANSYS for its secondary development are coded. In order to verify the validity of the subroutine coded, the numerical resolutions are compared with the theoretical resolutions in reference[7] for a circular tunnels under the static pressure.

As shown in Fig.1, the 1/4 part of a circular tunnel is taken as the computation model, in which the calculation parameters are: the radius of the tunnel is 1 meter, the dimensions of wall rock mass are 15 meters in length and width, the Young's modulus of wall rock is 280 GPa, the Poisson's ratio is 0.3, the cohesion is 3.0MPa, the internal friction angle is 30°, the expansion coefficient is 0.012, the initial void ratio is 0.6, the tensile to compressive strength ratio of the wall rock is 0.1, the support pressure is 8.5MPa, the static original rock stresses in X and Y directions are both 30MPa, and the gravity stress of the wall rock is neglected[8].

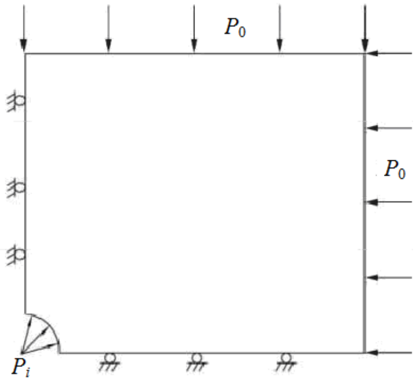
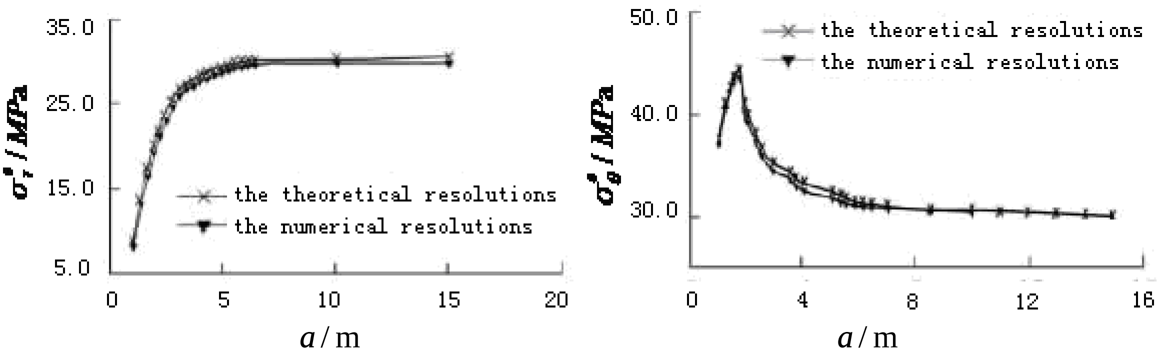


Fig1. Computation model of a circular tunnel

Comparisons between the theoretical and numerical radial stresses and tangential stresses are shown in Fig.2, in which a is the distance from the calculating points to the tunnel center. s_r^e is the radial stresses and s_q^e is the tangential stresses.

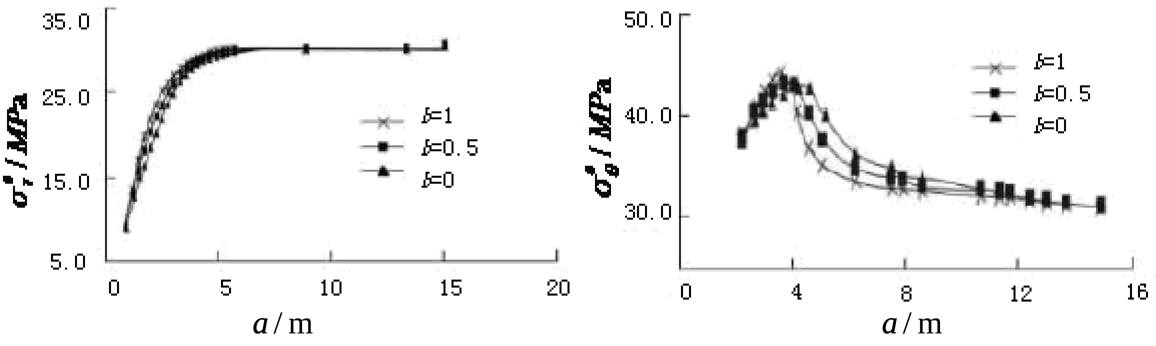


(a) radial stresses (b) tangential stresses

Fig 2. Comparison of the results between theoretical and numerical solutions for radial stress and tangential stress

Fig 2 shows the coincidence between the theoretical and the numerical resolutions. The maximum relative error is 0.285% and the average relative error is 0.125% for the radial stress, and the maximum relative error is 0.261% and the average relative error is 0.153% for the tangential stress. The reasons of error are that it took nearly two times for the main parts of the numerical calculations. Although bringing stress back to the yield surface and modifying the respective stress, It is impossible to eliminate the error completely. The calculation precision having met the requirements in engineering means that the proposed constitutive model and the subroutine codes are convincible.

The numerical resolutions of the radial and tangential stresses are also influenced by the intermediate principal stress effect parameter b, shown in the Fig.3, in which the radial stresses increase with b and the tangential stresses increase with b in the plastic zone but decrease with b in the elastic zone.



(a) radial stress (b) tangential stress

Fig 3. Comparison radial stress and tangential stress with different b

Case Study: A Landslide Analysis

The profile of the landslide analysis is shown in Fig.4, in which the calculation parameters for the bedrock are: E=19GPa, $u = 0.25$ and $g = 26KN / m^3$, and the calculation parameters for sliding mass are shown in Tab.1. The slope stability is analyzed with the secondary developed ANSYS software.

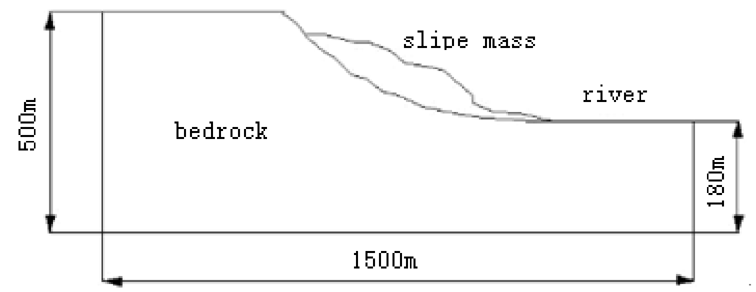


Fig 4. computation model of landslide

Tab. 1 calculation parameters of landslide

the Young's modulus (Gpa)	the Poisson's ratio	cohesion (*k*Pa)	the internal friction angle (°)	expansion coefficient	intermediate principal stress effect parameter *b*
3	0.34	60	35	0.01	0

The computation model of landslide can be simplified as a plane strain problem[9-11]. The strength reduction method is applied and the reduction factor F would be used in judging the stability of slope. When the b=0, the triple-shear unified yield criterion can degenerate to the Mohr-Column Criterion. Judging the stability of slope by reduction factor F. Calculation convergence is gained with cohesion c and the internal friction angle j , if the calculation convergence, sliding mass is stable; Otherwise Continue to increase the reduction factor F, Until the procedure was not convergence. The results are shown in Tab.2

Tab. 2 Comparison of the results

reduction factor *F*	**The X axial displacement** (m)	**plastic strain of the maximum**
1.0	0.0105	
1.2	0.0105	0.434E-03
1.4	0.0107	1.979E-03
1.6	0.0169	4.308E-03
1.65	0.0250	5.721E-3
1.7	0.0466	7.341E-3

When transfixion to a partial or all of the sliding mass in plastic zone, the reduction factor F is the stability of sliding mass. The stability of sliding mass with F=1.65 was analyzed. The sliding mass is only the upper part in yield state with F=1.66 whereas the lower sliding mass is still in a relatively stable state.

In order to reflect how the stability safety factors of sliding mass are influenced by the intermediate principal stress, we take different b to calculate the stability safety factors. The results are shown in Tab 3.

Tab. 3 Comparison of the results

b=0.05		b=0.10		b=0.15		b=0.20		b=0.25	
F=1.66	F=1.67	F=1.68	F=1.69	F=1.69	F=1.70	F=1.71	F=1.72	F=1.73	F=1.74
CT*	T**	CT*	T**	CT*	T**	CT*	T**	CT*	T**

* Coming Transfixion

** Transfixion

The Tab.3 shows that when b=0.05, 0.10, 0.15, 0.20 and 0.25, the corresponding stability safety factors of sliding mass are F=1.66, 1.68, 1.69, 1.71 and 1.73. It indicates that the stability safety factors of sliding mass increase with the intermediate principal stress parameter.

Conclusions

(1) With the triple-shear unified yield criterion and the associated flow rule, the triple-shear unified elasto-plastic constitutive relations is proposed. The secondary development to the ANSYS with the elasto-plastic constitutive model is coded to analyze the complicated practical engineering problems.

(2) According to the constitutive model, the subroutines for the secondary development to ANSYS are coded. The coincidence between the theoretical and the numerical resolutions for a circular tunnel under the static pressure indicates that the proposed constitutive model and the subroutine codes are convincible.

(3) The slope stability is analyzed with the developed ANSYS software. The strength reduction method is applied and the reduction factor F would be used in judging the stability of slope. It indicates that the stability safety factors of sliding mass increase with the intermediate principal stress parameter.

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